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Abstract

The paper must have abstract. The objective of this study is the modeling of storage installations for non dangerous waste for improving the management of bioreactors. We have studied the physico-chemical-biological behavior of waste in the anaerobic phase. For this, we presented the equations conservation of matter, energy and thermodynamic functions are adopted the model of Van Genuchten. We
have developed a numerical model to simulate the coupled bioreactors behavior using the finite volume method.

**Keywords:** non dangerous waste, Van Genuchten model, anaerobic phase, numerical model, reactive mass transfer, heat transfer

**Nomenclatures**

- \( K_{ip} \) intrinsic permeability\((m^2)\)
- \( K_{ip} \) relative permeability\((m^2)\)
- \( \mu_p \) dynamic viscosity of each phase (liquid, gas)
- \( \rho_p \) density of each phase (liquid, gas) \((kg/m^3)\)
- \( P_p \) pressure of each phase (liquid, gas) \((Pa)\)
- \( S_l \) liquid saturation
- \( S_r \) residual saturation
- \( S_e \) effective saturation
- \( M_g \) molecular mass\((kg / mol)\)
- \( R \) universal gas constant \((J / mol.K)\)
- \( n \) porosity of the medium
- \( A_i \) fraction of the waste
- \( C_g \) potential the production of biogas \((m^3 / Kg)\)
- \( \alpha_g \) term the production of biogas
- \( V_p \) velocity of moving fluid\((m/s)\)
- \( Q \) volume flow \((Wm^-3)\)
- \( (\rho C_p)_f \) volumetric heat capacity \((Jm^-3.k^-1)\)
- \( T \) Temperature, K
- \( (\rho C_p)^* \) volumetric heat capacity\((Jm^-3.k^-1)\)

**1 Introduction**

The management of non dangerous waste, especially in large cities, has become a complex problem, both politically, scientifically and technically. We have studied the behavior Hydro-thermo-biological waste in the anaerobic phase. This is the longest period during which degradation leachate and biogas are produced in large quantities.

Mathematical models could be used to simulate the coupled biological and hydraulic behavior of bioreactor landfills and help to a better understanding of processes taking place over stabilization time and during leachate recirculation.
These models could make it possible to predict gas and leachate production and to optimize the time and cost of operating bioreactor landfills by optimizing the volume of injected leachate, the number and spacing of injection devices and the duration of recirculation.

The mathematical models which have been developed previously (El Fadel and Findikakis, 1996 [8]; Vavilin et al., 2002[3]) are usually based on constant values of temperature in controlled laboratory conditions or controlled industrial digesters and so they can hardly be directly applied to landfills where temperature is the result of coupled processes. In these models the growth and decay rates of biomass are supposed to be constant, at a given temperature.

In this work we have developed a numerical model based on the finite volume method (Benard and Eymard, 2005[2]; Eymard et al., 2000[5]; Gueraoui and al. 1996[17]; Ghouli and al., 2009[18]) incorporating basic concepts from hydrodynamics and microbiology to simulate the hydraulic, thermal and biological behavior of anaerobic bioreactor landfills during the methanogenic phase. This model is composed of a two-phase flow model of leachate and biogas based on Darcy’s law, coupled with a biological model of heat and gas generation, considering the effects of saturation and temperature changes on the biological behavior.

This gas production is controlled essentially by two factors which are the temperature and humidity. This last parameter appears as being the key factor of the degradation.

We consider a bioreactor as a porous medium where there production of biogas and heat. This porous medium heterogeneous (for the composition of the waste, the water content and size of the components) consists of three main phases: a solid phase constituted by the waste, the liquid phase that is the leachate and the gaseous phase which the biogas is considered a mixture of methane and carbon dioxide in the methanogenic anaerobic phase. The equations that govern our problem are the equations of conservation of mass and energy and the thermodynamic functions we adopted the model of Van Genuchten.

2 Mathematical formulation

The main objective is the modeling of two phase flows without inter-phase changes. For this, we present the equations of conservation of matter, energy and the thermodynamic functions.

3 Generalized Darcy law

The authors’ affiliation should appear at end of the paper. The velocity of the fluid phases flowing in a porous medium can be expressed by using the generalized
Darcy law Aran, C. (2000)[4]:

\[ V_p (m/s) = -\frac{K_{ip} K_{r,p}}{\mu_p} (\nabla P_p - \rho_p g) \]  \hspace{1cm} (1)

The relation between the saturation and capillary pressure is that a given material may be calculated solely experimentally. We can then parameterize these results and we are getting models liquid-gas capillary. The best known are the law of Van Genuchten [15]. This approach are using, instead of the saturation of the liquid, the effective saturation \( S_e \) defined by:

\[ S_e(P_c) = \left[ 1 + \left( \frac{P_c}{P_r} \right)^{\frac{1}{m}} \right]^{-m} \]

Where the parameter \( m = 1 - 1/x \) (\( x \gg 1 \)) represents the curvature of the capillary pressure function, and therefore the thickness of the capillary fringe (transition zone between the saturated Zone and the unsaturated Zone) and \( P_r \) parameter designates the elevation of the capillary plate. You can also write:

\[ P_c(S_e) = P_r \left[ S_e - \frac{1}{m} \right]^{\frac{1}{x}} \]

Mualem [13] proposes a simple analytical model to predict the relative permeability curves of water from the capillary pressure curves as a function of the effective saturation, associated with the model from Van Genuchten [16] gives:

\[ K_{r,l} = S_e^{\frac{1}{2}} \left[ 1 - \left( 1 - S_e^{\frac{1}{m}} \right)^{m} \right]^{2} \]

Parker [25] shows that we can also define a gas relative permeability, such as:

\[ K_{r,g} = \left[ 1 - S_e \right]^{\frac{1}{2}} \left[ 1 - S_e^{\frac{1}{m}} \right]^{2m} \]

Or the effective saturation \( (S_e) \) is defined by:

\[ S_e = \frac{S_l - S_r}{1 - S_r} \]

The density of gas can be defined as follows:

\[ \rho_g = \frac{M_g P_g}{RT} \]

4 Conservation of mass

Consider the case where the porous medium is composed of the solid matrix, a liquid and a gas. The conservation of each constituent (liquid, gas) writes (Shabnam Gholamifard (2009)) [1]:
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\[
\begin{aligned}
\frac{\partial m_i}{\partial t} + \nabla \cdot (\rho_i V_i) &= 0 \\
\frac{\partial m_g}{\partial t} + \nabla \cdot (\rho_g V_g) &= \alpha_g
\end{aligned}
\]  \hspace{1cm} (2)

with: \( m_i = n \rho_l \) and \( m_g = n(1 - \rho_l) \rho_g \), where \( n \) is the porosity of the medium, and \( \rho_l, \rho_g \) the density of liquid and gas phases. \( V_p \) the rate of filtration of \( P \) the phase defined by Darcy’s law and \( \alpha_g \) the term the production of biogas. This latter term is defined from the biological model of degradation and biogas production.

Therefore the equation (2) and Darcy’s law leads to the following equations:

\[
\begin{aligned}
\frac{\partial S_l}{\partial t} &= -\frac{K_{ul}}{r \mu_l} \frac{\partial}{\partial r} (r K_{ul} f_i) - \frac{K_{ul}}{\mu_l} \frac{\partial}{\partial z} (K_{ul} f_i) \\
(1 - S_i) \frac{\partial \rho_g}{\partial t} + r \rho_s W' + &\frac{K_{ig}}{\mu_g} \frac{\partial}{\partial r} (r \rho_s K_{ig} f_g) + \frac{K_{ig}}{\mu_g} \frac{\partial}{\partial z} (\rho_s K_{ig} f_g) = \alpha_g - U'
\end{aligned}
\]  \hspace{1cm} (1)

with:

\[
\begin{aligned}
f_i &= \nabla P_l - \rho_l g = \frac{\partial P_l}{\partial r} + \frac{\partial P_l}{\partial z} \\
f_g &= \nabla P_g = \frac{\partial (P_g)}{\partial r} + \frac{\partial P_g}{\partial z}
\end{aligned}
\]

we put:

\[
\begin{aligned}
W' &= \frac{K_{ul}}{r \mu_l} \frac{\partial}{\partial r} (r K_{ul} f_i) + \frac{K_{ul}}{\mu_l} \frac{\partial}{\partial z} (K_{ul} f_i) \\
U' &= \frac{g K_{ig}}{\mu_g} \frac{\partial}{\partial z} (\rho_s^2 K_{ig}) + \frac{g K_{ig}}{r \mu_g} \frac{\partial}{\partial r} (r \rho_s^2 K_{ig})
\end{aligned}
\]

5 Production of biogas

The production of biogas depends on different parameters, such as the waste composition, the water content, temperature and density of the medium to be taken into account in the kinetic models biogas production. The production of biogas changes depending of degradable components waste and quantity. The production of biogas is defined by exponential law proposed by Halvadakis[9]. This law is malfunctioning for the first years degradation of a young waste, but applies very well to the anaerobic phase (Finidikakis and al) [12]:

\[
\alpha_g = c_g \sum_{i=1}^{n} \frac{A_i(t + dt) - A_i(t)}{dt} \hspace{1cm} (4)
\]

Where \( A_i \) is the fraction of the waste, \( C_g \) the potential the production of biogas
m$^3$/Kg and $\alpha_g$ is the rate of biogas generation.

6 Conservation of energy

The macroscopic law of conservation of energy in the porous medium, taking into consideration the phenomenological Fourier law, is written:

$$\left( \rho C_p \right)^* \frac{\partial T}{\partial t} + \nabla \cdot \left( \rho C_p \right)_{fl} \nabla T = \nabla \left( \lambda \nabla T \right) + Q \quad (5)$$

Where:
- $T$ the temperature (the solid phase and fluids phases),
- $\mathbf{V_p}$ The velocity of moving fluid, given by Darcy’s law as follows:

$$\mathbf{V_p} = V_{p1} \hat{e}_r + V_{p2} \hat{e}_z$$

Where:
- $V_{p1}, V_{p2}$ velocity components,
- $\left( \rho C_p \right)_f$ the volumetric heat capacity (Jm$^{-3}$·k$^{-1}$) of the moving fluid,
- $Q$ (Wm$^{-3}$) of energy produced or consumed by the chemical reactions.

The projection system of equations (5) in the cylindrical coordinates system while noting that:

$$\left( \rho C_p \right)^* \frac{\partial T}{\partial t} + \left( \rho C_p \right)_f \left( V_{p1} \frac{\partial T}{\partial r} + V_{p2} \frac{\partial T}{\partial z} \right) = \frac{\lambda^*}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \lambda^* \frac{\partial^2 T}{\partial z^2} + Q$$

7 Method of resolution

The equations obtained previously do not admit analytical solutions, so the use of numerical methods appears mandatory. We opted for this purpose the finite volume method.

In this simulation, we chose to use the implicit finite volume method. This method requires two steps: mesh and discretization [22].
8 Discretization

Equations of conservation of mass:

By integrating equation (1) after the volume control \((rdrdz)\) and along the length of time interval \(dt\), it follows that:

\[
\iiint_t^{t+dt} nr \frac{\partial S_i}{\partial t} dr dz dt = \iiint_t^{t+dt} \frac{K_{ii}}{\mu_i} \frac{\partial}{\partial r} (rK_{ri}f_i) dr dz dt + \iiint_t^{t+dt} \frac{K_{il}}{\mu_l} \frac{\partial}{\partial z} (K_{rl}f_l) r dr dz dt
\]

Ultimately:

\[
S_{i(i,j)}^{dt+dt} = S_{i(i,j)}^{dt} + \frac{K_{ii}}{r_p \mu_i} \left[ \frac{\partial}{\partial r} (rK_{ri}f_i) \right]_{(i,j)}^{(i+1,j)} \Delta t + \frac{K_{il}}{\mu_l} \left[ \frac{\partial}{\partial z} (K_{rl}f_l) \right]_{(i,j)}^{(i,j)} \Delta t
\]

By integrating equation (2) after the volume control \((rdrdz)\) and along the length of time interval \(dt\), it follows that:

\[
\iiint_t^{t+dt} nr(1 - S_i) \frac{\partial \rho_g}{\partial t} dr dz dt + \iiint_t^{t+dt} r \rho_g W dr dz dt + \frac{K_{ig}}{\mu_g} \iiint_t^{t+dt} r \frac{\partial}{\partial r} (\rho_gK_{rg}f_g) dr dz dt
\]

\[
+ \frac{K_{ig}}{\mu_g} \iiint_t^{t+dt} r \frac{\partial}{\partial z} (\rho_gK_{rg}f_g) dr dz dt = \iiint_t^{t+dt} r(\alpha_g - \bar{U}) dr dz dt
\]

The equation can therefore be written in the following form:

\[
A(i,j) (\rho_g)_{(i+1,j)}^{dt+dt} + B(i,j) (\rho_g)_{(i,j+1)}^{dt+dt} + C(i,j) (\rho_g)_{(i-1,j)}^{dt+dt} + F(i,j) (\rho_g)_{(i,j)}^{dt+dt} + G(i,j) (\rho_g)_{(i,j-1)}^{dt+dt} = D(i,j)
\]

with:

\[
A(i,j) = \frac{K_{ig} \Delta z \Delta t}{\mu_g} \left( rK_{rg}f_g \right)_E^{dt+dt}
\]

\[
B(i,j) = r_p \frac{K_{ig} \Delta r \Delta t}{\mu_g} \left( K_{rg}f_g \right)_N^{dt+dt}
\]

\[
C(i,j) = - \frac{K_{ig} \Delta z \Delta t}{\mu_g} \left( rK_{rg}f_g \right)_O^{dt+dt}
\]

\[
F(i,j) = nr_p \left( 1 - S_{i,j}^t \right) \Delta r \Delta z
\]

\[
G(i,j) = -r_p \frac{K_{ig} \Delta r \Delta t}{\mu_g} \left( K_{rg}f_g \right)_S^{dt+dt}
\]

\[
D(i,j) = \Delta r \Delta z r_p \left( n \left( 1 - S_{i,j}^t \right) - W_{p}^{dt+dt} \Delta t \right) (\rho_g)_p^t + r_p (\alpha_g - \bar{U})_p^{dt+dt} \Delta r \Delta z \Delta t
\]

Equations of energy balance:

By integrating equation (4) after the volume control \((rdrdz)\) and along the length of time interval \(dt\), it follows that:
The equation can therefore be written in the following form:

\[ A(i,j)\Delta T_{(i+1,j)}^{dt} + B(i,j)\Delta T_{(i,j)}^{dt} + C(i,j)\Delta T_{(i-1,j)}^{dt} + F(i,j)\Delta T_{(i,j)}^{dt} + G(i,j)\Delta T_{(i-1,j)}^{dt} = D(i,j) \]  

(6)

with:

\[
A(i,j) = \left( \frac{r_p}{\rho C_p} \right)_p \left( U_{p1} \right)_p^{dt} - \frac{\lambda^*(2r_p + \Delta r)}{2\Delta r} \Delta z \Delta t
\]

\[
B(i,j) = r_p \Delta t \Delta r \left( \frac{\rho C_p}{f} \right)_p \left( U_{p2} \right)_p^{dt} - \frac{\lambda^*}{\Delta z} \Delta z \Delta t
\]

\[
C(i,j) = -\left( \frac{r_p}{\rho C_p} \right)_p \left( U_{p1} \right)_p^{dt} + \frac{\lambda^*(2r_p - \Delta r)}{2\Delta r} \Delta z \Delta t
\]

\[
F(i,j) = r_p \left( \frac{\rho C_p}{f} \right)_p \Delta r \Delta z + \lambda' r_p \frac{\Delta z \Delta t}{2\Delta r} + 2r_p r_p \frac{\Delta r \Delta t}{\Delta z}
\]

\[
G(i,j) = -r_p \Delta t \Delta r \left( \frac{\rho C_p}{f} \right)_p \left( U_{p2} \right)_p^{dt} + \frac{\lambda^*}{\Delta z}
\]

\[
D(i,j) = r_p \left( \frac{\rho C_p}{f} \right)_p \Delta r \Delta z T_p^{dt} + Q r_p \Delta r \Delta z \Delta t
\]

The algebraic equations obtained are solved using the dual scanning method [22].

9 Convergence Test

The convergence test of the solution for the studied problem concerns the saturation. If \( m \) is the number of computes cycles and \( \varepsilon \) the approximation fixed in advance, then we impose to saturation to verify [17]:

\[
\text{Sup} \left| \frac{S_{m+1} - S_m}{S_m} \right| \leq \varepsilon
\]

10 Resolution process primed

The calculations will be initiated by an initial profile that whatever may be provided that meets the boundary conditions. However, to reduce the computation time, we choose an initial profile that is fairly close to the actual profile. Using these values allows the determination of profiles of temperature and saturation [23][24].
11 Results

Fig 1: Saturation profile as a function of time and the axial variable

Fig 2: Temperature profile as a function of time and the axial variable
Fig 3: mass profile as a function of time and the axial variable

Fig 4: volume profile as a function of time and the axial variable
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Figure 1 shows the variation of the saturation according to the axial variable, the radial variable and time. Note that this quantity decreases slowly with the axial variable and increases over time, makes sense, given that saturation increases from its minimum value and increased to the residual saturation of the medium.

Figure 2 illustrates the variation in temperature depending on the axial variable and time. It is found that the temperature increases with time. This can be explained by the fact that over time the component was introduced, and therefore the interaction between them increases and therefore the temperature.

Figure 3 shows the evolution of the mass a function of time biogas. We found that the mass is increased with time. This can be explained by the fact that with time, the component is deposited, therefore, increases the biogas.

Figure 4 shows the evolution of the volume as a function of the axial variable and time. On finds that the volume increases with time, this result can be interpreted in that the component is deposited.

Figure 5 illustrates the variation of the density of biogas according to the axial variable and time. It is found that the density of the biogas decreased with the axial variable and time, this may be explained by the fact that the evolution of the mass is very low compared to the volume of biogas.

![Graphs showing variation in biogas density and volume with axial variable and time.](image-url)
12 Conclusion

This article begins with the mathematical coupled model and the numerical method used to solve the conservation equations and using a finite volume method for modeling bioreactors. The coupled model contains a model of two-phase flow based on Darcy's law and a biological model based on the simplified model of Monod [19], [20], [21] and given the biogas degradation of biodegradable solid waste and VFA production.

References


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