Some Remarks about General Relativity Principle

Elmo Benedetto

Mathematics and Computer Science Department
University of Salerno and Department of Engineering
University of Sannio
Piazza Roma 21, 82100 Benevento, Italy

Abstract

In most textbooks that introduce the General Relativity (GR), it is often written that this theory is a generalization of Special Relativity (SR) in which Einstein extends the principle of relativity to all motions. Students who specialize in another branch of physics or mathematics, even after degree did not understand that general covariance is profoundly different from the special relativity principle. This short note reviews this fundamental difference that is, in our opinion, all too often not analyzed in depth. We emphasize that it can be didactically very useful for students to realize how the request of covariance does not necessarily imply a principle of relativity.

Copyright © 2014 Elmo Benedetto. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Keywords: relativity principle, equivalence principle

1 Introduction

Let us repeat the words of Salviati in Dialogue Concerning the Two Chief World Systems "The fish in their water will swim toward the front of their bowl with no more effort than toward the back, the butterflies and flies will continue their flights indifferently toward every side and if smoke is made by burning some incense, it will be seen going up in the form of a little cloud, remaining still and moving no more toward one side than to the other". Everything happens as if the ship were not moving at all and certainly Galileo Galilei
was the founder of the principle of relativity. After nearly three centuries, Einstein extended, in SR, the Galilean relativity principle to the whole physics. It is not possible to determine the absolute motion of an inertial frame and there is no physical way to differentiate between a reference frame moving at a constant speed and an immobile reference frame. The uniform motion is relative and it can not be perceived except in reference to an external point. Velocity is a relative physical quantity and this is the core of the relativity principle. Therefore in SR, whether or not an object possesses velocity depends on frame of reference. Contrariwise acceleration is absolute and whether or not an object possesses acceleration does not depend on frame of reference. Newton, in his Philosophiae Naturalis Principia Mathematica, tried to show that the acceleration is absolute. Indeed in his famous example of the rotating bucket filled with water, he demonstrates that the centrifugal forces arise only when the water is in absolute rotation with respect to the absolute space. George Berkeley in his De Motu and later Mach proposed a deep criticism of Newton’s thought[1]. In particular Mach concluded that the inertia would be an interaction that requires other bodies to manifest itself and it has no sense in a Universe consisting of only one mass [2]. Therefore the concept of absolute motion should be substituted with a totally relative motion. The idea of Mach played an important role in the development of GR. Einstein, following Mach, wanted to make the acceleration a relative magnitude. The starting point was the equivalence between inert mass and gravitational charge and this equivalence principle allowed him to create a new theory of gravity that revolutionized our thinking about the nature of space and time. Gravity is simply geometry of spacetime and the gravitational force becomes a metric force, resulting from the local curvature of spacetime and above all cosmology has been enclosed in the physical disciplines and not only in the philosophical ones when the field equations were applied to the whole universe. All this is wonderful and the theory is now supported by many experimental tests, but none longer ask if Einstein was able to create a fully relativistic theory.

2 The Equivalence Principle

The equivalence principle emerged in the late 16th and early 17th centuries, when Galileo expressed experimentally that the acceleration of a test mass due to gravitation is independent of the amount of mass being accelerated. We have the well know relation

$$\vec{a} = \vec{g} \frac{mG}{m_i}$$

(1)
If \( \frac{ma}{m_i} \) is a constant independent of the body, the acceleration is the same for all body. There is no a priori reason, in Newtonian physics, why inert and gravitational masses should be equal but they are found to be identically so, to extremely high precision\[3\]. Newton itself tested the equivalence principle studying the motion of pendulum of different composition and equal length, finding no difference in their periods and from Eötvös to present day the experiments are very accurate and therefore we have, without any doubt, \( m_i = m_G \) \[4\]. This formulation is called the Weak Equivalence Principle and it implies that it is impossible to distinguish between the effects of an uniform gravitational field from those experienced in an uniformly accelerated frame, using the simple observation of the free-falling particles behavior. The previous equivalence, extended to all physical laws, is known as the Strong Equivalence Principle (SEP). Therefore the SEP states that there is no experiment that we can make to distinguish between a local inertial frame which is free falling in a uniform gravitational field and an inertial frame which is in a region far from any gravitating masses. Vice versa we can simulate gravity by giving the reference frame an acceleration which will be indistinguishable from a uniform gravitational field\[5\],\[6\]. The gravitational force affects all objects in the same way and this is a characteristic only of the gravitational field. In fact if we consider for example the electromagnetic force we get

\[
\vec{a}^i = \frac{q}{m_i} (\vec{E}^i + \vec{v}^i \wedge \vec{B})
\]

and different objects will experience different accelerations. In conclusion we can say that the theory of uniform gravitational field is a general relativity theory. Indeed the effects due to an uniform gravitational field are physically indistinguishable from the effects due to uniform acceleration. In this context, the acceleration is a relative physical quantity and we obtain a perfect generalization of the special relativity principle. Instead, it is well known that gravitational fields are not uniform and so it is crucial to ask whether we can extend the principle of relativity to accelerated reference frames. Can acceleration be made relative? Quite often the answer seems affirmative, and it is based on the general covariance principle. It is well know that general covariance is the invariance of the form of physical laws under arbitrary differentiable coordinate transformations and in many textbooks this principle is described as a general relativity principle. It is frequently written that the reason to call the Einstein gravitational theory "general relativity" is the fact that this theory generalizes the special relativity principle and this generalization is realized in principle of general covariance through which all frames of reference are equivalent. In our opinion it is crucial immediately to present the Kretschmann reflection that the general covariance does not make any assertions about the physical content.
of the physical laws, but only about their mathematical formulation, and Ein-stein entirely concurred with his view [7],[8],[9][10],[11],[12],[13],[14],[15]. We
can give a generally covariant formulation for the Newtonian mechanics and
the special relativity too and this clearly shows that the notion of general co-
variance is completely independent of relativity. Simple examples shows us
how a non-covariant equation can be transformed into another equivalent and
covariant [4]. This is the point that we believe is not sufficiently emphasized.
Moreover there are generally covariant gravitational theories that at any point
are reduced to classical mechanics. Pauli pointed out “The generally co-
variant formulation of the physical laws acquires a physical content only through
the principle of equivalence”. Indeed the foundation of Einstein theory is the
following alternative version of the principle of equivalence:

A physical law is true if:

1) it is true in the absence of gravity, i.e. it reduces to the laws of special
relativity when \( g_{\mu\nu} \rightarrow \eta_{\mu\nu} \), and the Christoffel symbols vanish

2) In order to preserve their form under an arbitrary coordinate transforma-
tion, all equations must be generally covariant. This means that all equations
must be expressed in a tensor form.

It is instructive to show that this formulation of general covariance follows
from the principle of equivalence but it should be stressed that it does not in
any way imply the physical equivalence of all reference frame similarly to what
happens in special relativity. The previous two statements describe only the
effects of gravitation.

3 Absolute or Relative Acceleration?

In Minkowski spacetime there is a difference between straight trajectories with
no force present ( \( \frac{d^2x}{dt^2} = 0 \)) and curved trajectories ( \( \frac{d^2x}{dt^2} \neq 0 \)). In Einstein
gravitational theory there is a difference between straight trajectories with
no force present ( \( \frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = 0 \)) and curved trajectories ( \( \frac{d^2x^\mu}{ds^2} +
\Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} \neq 0 \)). So, in Einstein gravitation, the key point is that inertial
reference frames include objects at rest, in constant motion, or gravitationally
accelerating. Therefore whether or not you are gravitationally accelerating
depends on your frame of reference. Instead this is not true for other types
of acceleration and non-gravitational acceleration is still absolute and it is
independent of frame of reference. For a given gravitational field, vectorially
subtracting the gravitational acceleration at the origin of a free falling reference
frame from the gravitational acceleration at a given point, we obtain the tidal
acceleration at a point with respect to the origin. If we consider the particle
\( m \) at the point \((0,0,z)\), we have
Some remarks

\[ a_{\text{tidal}} = -\frac{GM}{(r + z)^2} + \frac{GM}{r^2} \]  \hspace{1cm} (3)

for small values of \( z \) we get

\[ a_{\text{tidal}} \approx 2z\frac{GM}{r^3} \]  \hspace{1cm} (4)

and

\[ F_{\text{tidal}} \approx 2z\frac{GMm}{r^3} \]  \hspace{1cm} (5)

At the point \((x, y, z)\) we have

\[ F_{\text{tidal}} = -\frac{xGMm}{r^3}i - \frac{yGMm}{r^3}j + \frac{2zmGMm}{r^3}k \]  \hspace{1cm} (6)

If we have a rod in free fall, the tidal force generates a torque that produces an angular acceleration. The torque around the \( x \) axis is

\[ M_x = \frac{3GM}{r^3}I_{yz} \]  \hspace{1cm} (7)

where \( I_{yz} \) is the component \( y - z \) of the product of inertia. For the \( x \)-component of angular momentum we have

\[ \frac{dL_x}{dt} = \frac{3GM}{r^3}I_{yz} \]  \hspace{1cm} (8)

If we consider the length \( l \to 0 \), both the torque and the angular momentum tend to zero but this is not true for the angular acceleration. Therefore we can use the angular acceleration to determine, even locally, the presence of a gravitational field. There are many ways to measure in an infinitesimal neighborhood the tidal field. For details you can read [4] In conclusion we notice the presence of a gravitational field, but we can not calculate any absolute gravitational acceleration. The equality of gravitational and inertial mass has as consequence only the physical indistinguishability (relativity principle) between an a constant accelerated frame of reference and a uniform gravitational field. Perhaps the confusion stems from the fact that Lorentz covariance correspond to the notions of physical equivalence and relativity. The covariance of Maxwell equations does not imply that the speed of light is the same in each reference but it is the invariance obtained by Lorentz covariance that leads to the principle of relativity.
4 Conclusion

In this brief paper we have stressed that the introductory textbooks on GR do not deepen the principle of general covariance and do not express clearly the difference between invariance and covariance. Lorentz invariance is a relativity principle but general covariance is not a general relativity principle. GR is not a fully relativistic theory but a geometric theory of gravitation built thanks to the principle of equivalence. Despite its name, the theory has failed in its original purpose and that is to create a totally relativistic physics. As long as the Mach principle will not be incorporated into GR, we are forced to admit that Newton was right. The acceleration (non-gravitational) is absolute.

References


Received: March 17, 2014