The Effect of Core-Polarization on Nucleon-Nucleon Realistic Potential Parameters in Doubly-Magic Nucleus $^{40}$Ca

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Abstract

To study the core-polarization effect on realistic nucleon –nucleon potential parameters, the inelastic electron scattering form factors in the doubly magic nucleus $^{40}$Ca based on microscopic theory is employed. Higher energy excitations from 1s-shell core orbits and also from the valence 1p-shell to higher allowed orbits up to $2 \hbar \omega$ are considered for core-polarization calculations. The realistic Michigan three Yukawa potential for N-N interactions for the core-polarization theoretical calculations to be considered.

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1. Introduction

The origin of the nuclear force is a central problem of nuclear physics. It has a long history since Yukawa (H. Yukawa,1935) proposed the static one-pion exchange potential (OPEP) of the range of the pion Compton wave length.
The electron scattering process can be explained according to the first Born-approximation as an exchange of a virtual photon carrying a momentum $q$ between the electron and the nucleus (T. Deforest, Jr. and J Walecka, 1966; J.O. Newton, 1975). According to this approximation, the interaction of the electron with the charge distribution of the nucleus is considered as an exchange of a virtual photon with zero angular momentum along the direction of the momentum-transfer $q$, this is called Coulomb or longitudinal scattering. On the other hand, the interaction of the electron with the spin and current distributions of the nucleus gives rise to the transverse scattering. For the latter type, the process is considered as an exchange of a virtual photon with angular momentum $\pm 1$ along $q$ direction. As a consequence of parity and time reversal invariance, only electric multipoles can have longitudinal component, while both electric and magnetic multipoles can have transverse components. Rand et al. have measured the magnetic form factors of the $1p$-shell nuclei ($^6$Li, $^7$Li, $^9$Be, $^{10}$B, $^{11}$B, and $^{14}$N) by light-energy electron scattering at higher momentum transfer (Rand et al., 1973). The scattering from those nuclei was dominated by the octupole momentum. The inelastic electron scattering form factor of isoscalar and isovector in nuclei $^{12}$C and $^{16}$O has been studied by (Majid Abusini et al., 2011). The results indicate that to reproduce the experimental data the admixture from higher orbits and the core polarization effect should be introduced.

Electron scattering form factor with core-polarization effect for negative and positive parity states of $^{13}$C has been studied (Hussain, 2001). In the low $q$-region the results show a good agreement with the experimental data. Nadia (N. Adeeb, 2002) has studied elastic and inelastic electron scattering form factors for the similar parity states of $1p$-shell nuclei ($^6$Li, $^7$Li, $^9$Be, $^{10}$B, and $^{11}$B) in the frame work of the many-particles shell model. Energy excitations up to $6\hbar \omega$ are considered for core-polarization calculation, with MSDI as a residual interaction. The results showed a good agreement with the experimental data in low $q$-region and the average of Coulomb form factor $(C2)$ values become about 90% of the average experimental value when the core-polarization effects are included.

Core-polarization effects on Coulomb form factor $(C2)$ in several selected $1p$-shell nuclei ($^6$Li, $^{10}$B, $^{12}$C, $^{13}$C) have been discussed by Radhi (R.A. Radhi et al., 2001) Core-polarization calculation is found essential in both the transition strengths and momentum transfer dependence of form factors. A good agreement with the measured data was obtained.

The goal of the present paper is to use the realistic effective nucleon-nucleon (NN) interaction as a residual interaction to calculate the core-polarization (cp) effects thought a microscopic theory which combines shell model wave functions and highly exited states using adjustable NN parameters.

We will discuss the core-polarization effects on the inelastic electron scattering longitudinal form factors for the low lying states of doubly magic nucleus $^{40}$Ca. The NN interaction adopted in the present study is a realistic interaction between two nucleons, namely the Michigan three Yukawa potential. This interaction is used as the residual interaction between nucleons in the core, and nucleons excited to higher orbits.
The derivation of cp effects with higher configuration in the first order perturbation theory and the two-body matrix elements of three part of the realistic interaction: central, spin orbit and strong tensor force will be introduced.

A computer program is written using FORTRAN 90 language to include realistic interaction M3Y in the original code (Uberal, 2009) which calculates the model space form factors (zero-order).

### 2. Formalism

For closed-shell nuclei, the structure of the low-lying excited states are studied mainly through particle-hole models which are Tamm-Dancoff Approximation (TDA). According to TDA, the ground state is treated as independent particle model (closed shell), while the excited states of closed-shell nuclei are described as a linear combination of particle-hole excitation which is created by excitation of a nucleon from the closed (filled) shell to a higher unoccupied shell leaving a hole within the closed shell.

The residual interaction used in this paper is the Michigain sum of three-range Yukawa (M3Y) potential taken from ref. (Bertsch et al., 1977). This potential is derived from the fitting of parameterized three Yukawa radial dependent form of central, spin-orbit and tensor forces with the harmonic oscillator matrix element of Reid soft-core potential (Reid, R. 1968).

\[
V_c = \sum_{i=1}^{3} V_i Y(r/R_i) \\
V_{so} = \sum_{i=1}^{3} V_i Y(r/R_i) \vec{S} \cdot \vec{\ell} \\
V_t = \sum_{i=1}^{3} V_i r^2 Y(r/R_i) S_{12}
\]

where \( V_c, V_{so} \) and \( V_t \) are central, spin-orbit and tensor parts of the M3Y potential, respectively. The variable \( r \) is a relative coordinates for the distance between the interacting two particles, and \( Y(r) \) is Yukawa function, \( S_{12} \) is the tensor operator and \( V_i, R_i \) parameters that take special values according to the M3Y potential component and the channel of the two-body interaction, given in table (1) with the parameters of Reid taken from ref. (Anantaraman et al., 1983).

The nuclear longitudinal and transverse form factors are expressed in term of reduced matrix elements of electron scattering operator as (Brown et al, 1985)

\[
|F_j(q)|^2 = \frac{4\pi}{Z^2} \frac{1}{2J_f+1} \sum_{T_f} (-1)^{\tau_f} \left( \begin{array}{ccc} T_f & T \omega \\ -T_{fJ} & M_f \end{array} \right) \langle J_f T_f || \hat{T}_j(q) || J, T \rangle^2 \\
\times |F_{cm}(q)|^2 |F_{Jf}(q)|^2
\]
where the triple-bar means that the matrix elements are reduced both in angular momentum \( J \), and isospin \( T \), and \( \eta \) select the longitudinal (L), transverse electric (el), and transverse magnetic (m) operators, respectively.

The last two term in eq.(10) are the correction factors of the center of mass (c.m) and finite nucleon size (f.s) given by (Willy, 1963), respectively

\[
F_{c.m}(q) = e^{-rac{a_p q^2}{A}} \quad (5a)
\]

\[
F_{f.s}(q) = e^{-a_p q^2} \quad (5b)
\]

where \( a_p = 0.4 \, \text{fm}^2 \), \( b \) is the oscillator length parameter (or size parameter), and, \( A \) is nuclear mass number.

For electromagnetic interactions, \( T=1,0 \) and \( T_f=T_l=T_z \) which gives \( M_l=0 \), where

\[
T_z = \frac{Z-N}{2} \quad (6)
\]

The many-particle states matrix element reduced in spin-isospin space can be written in terms of single-particle matrix element as (Donnelly and Sick, 1984)

\[
\langle J, T, \hat{T} \left| a \right| b \rangle = \sum_{J_{ab}^T} \chi_{ab}^{JT} \langle a \left| \hat{T} \right| b \rangle \quad (7)
\]

where \( \chi_{ab}^{JT} \) is the eigenvector that obtained from the diagonalization of the (TDA) Hamiltonian matrix element in the presence of the M3Y potential. The states \( \left| b \right\rangle \) and \( \left| a \right\rangle \) are the single-particle wave functions of initial and final states, respectively.

The longitudinal (Coulomb) scattering comes from the interaction of the electron with the charge distribution of the nucleus. The longitudinal operator is defined as (Sato et al., 1994)

\[
T_{JM}^L(q) = \int M_{JM}^L(q\vec{r}) \hat{\rho}(\vec{r}) d\vec{r} \quad (8a)
\]

with \( J=0,1,2,... \)

\[
M_{JM}^L(q\vec{r}) = j_J(qr) Y_{JM}(\hat{r}) \quad (8b)
\]

where \( j_j(qr) \) is the spherical Bessel function, \( Y_{JM}(\hat{r}) \) is the spherical harmonics functions, and \( \hat{\rho}(\vec{r}) \) is the nuclear charge density operator which is given by

\[
\hat{\rho}(\vec{r}) = e(t_z) \delta(\vec{r} - \vec{r}_0) \quad (9a)
\]

with
The reduced single-particle matrix element of the longitudinal operator can be written as

\[
\langle n_a \ell a | \hat{T}_{L,t}^z | n_b \ell j_b \rangle = e(t_z) \langle n_a \ell a | j_j(qr) | n_b \ell b \rangle \\
\times \left( \ell_a \frac{1}{2} j_a \right) Y_j(\hat{r}) \left( \ell_b \frac{1}{2} j_b \right)
\]

where the reduced matrix elements of the spherical harmonics is given by

\[
\langle \ell_a j_a | \ell_b j_b \rangle = (-1)^{j_a + j_b} \frac{\sqrt{(2j_a + 1)(2j_b + 1)(2J + 1)}}{4\pi} \times \left( \begin{array}{c} j_a \\ 1/2 \\ 0 \\ -1/2 \end{array} \right) \left( \begin{array}{c} J \\ j_b \end{array} \right) \frac{1}{2} \left[ 1 + (-1)^{\ell_a + J + \ell_b} \right]
\]

So, eq.(10) can be written as

\[
\langle n_a \ell a | j_j(qr) | n_b \ell b \rangle = e(t_z) P_{j_j} \langle \ell \ell_a \ell b | C_j(j_a, j_b) \\
\times \langle n_a \ell a | j_j(qr) | n_b \ell b \rangle
\]

The matrix elements of The radial single-particle wave function \(| n \ell \rangle\) can be solved analytically as (De Forest and Walecka, 1966)

\[
\langle n_a \ell a | j_j(qr) | n_b \ell b \rangle = \frac{2^j}{(2J + 1)!} y^{j/a} e^{-y} \sqrt{(n_b - 1)!} \sqrt{(n_a - 1)!} \Gamma(n_b + \ell_a + 1/2) \Gamma(n_a + \ell_a + 1/2) \\
\times \sum_{m_a=\pm} \sum_{m_b=\pm} \frac{(-1)^{m_a + m_b}}{m_a^! m_b^!} (n_b - m_b - 1)! (n_a - m_a - 1)! \Gamma(m_b + \ell_a + 3/2) \\
\times \frac{1}{\Gamma(m_a + \ell_a + 3/2)} \\ F\left( \frac{1}{2} (J - \ell_b - \ell_a - 2m_b - 2m_a); J + \frac{3}{2}; y \right)
\]

where \( y = \frac{bq}{2} \), \( \Gamma \) is the gamma function and \( F \) is the confluent hypergeometric function, defined as (Arfken, 1970)

\[
F(\alpha; \beta; y) = 1 + \frac{\alpha + 1}{\beta + 1} + \frac{\alpha + 1}{\beta + 1} y^2 + \cdots
\]

In our calculation of the form factor, the ground state wave function is modified to include admixture from higher configurations, which is very important to add more degree of collectivity, as follows (Talmi, 1989)

\[
| b \rangle = \gamma | n_b l_b j_b \rangle + \delta | (n_b + 1) l_b j_b \rangle
\]

where \( \gamma \) and \( \delta \) are mixing parameters with \( \gamma^2 + \delta^2 = 1 \).

This extension in the ground state wave function will modify only the radial part of the reduced single-particle matrix element, while the angular part is still unchanged.
3. Results and Discussion

The ground state of the doubly magic nucleus $^{40}$Ca is at $J^\pi, T = 0^+, 0$ assumed to form the closed $1s_{1/2}, 1p_{3/2}, 1p_{1/2}, 1d_{5/2}, 2s_{1/2}$ and $1d_{3/2}$ shells. The states expected to be most strongly excited from closed-shell are linear combinations in which one nucleon has been raised to a higher shells $1f_{7/2}, 2p_{3/2}, 1p_{1/2}$ and $1f_{5/2}$, forming pure single-particle-hole state.

The resulting values of the amplitudes are used to calculate the reduced many-particle matrix elements of the electron scattering form factors which are functions of the momentum transfers $\tilde{q}$ according to equation (7). The reduced single-particle matrix element of longitudinal operator is calculated according to eq. (12). The squared form factors for the longitudinal is calculated according to eq. (4), which takes into consideration the corrections for the center of mass motion and the finite size of the nucleon.

Our TDA calculations for the negative parity ($T=0$) states at $J^\pi_n, E_x (MeV)$ $3^-, 6.58$ and $5^-, 4.49$, predict the energy eigenvalues at 9.7133, and 5.0128 MeV, respectively. The dominant particle-hole configuration for these states is $(1f_{7/2})(1d_{3/2})^{-1}$.

Fig.(1) shows the longitudinal C3 form factor for the isoscalar $3^-$ state at $Ex=6.58$ MeV in $^{40}$Ca nucleus. The calculation of the form factor in the framework of TDA is shown in the dashed curve. The solid curve shows the effect of the addition of a more collectivity to this state by the modification of the ground state wave function to include 3p-2f shells with $\gamma=0.42$, as well as taking the core polarization effects into account by introducing effective charges $(e_{eff}(p)=\delta(p)e_{free}(p))$, $(e_{eff}(n)=\delta(n)e_{free}(n))$, where $\delta(p)=1.08$ and $\delta(n)=0.16$) with adjustable NN parameters of M3Y in table (2) which leads to a very well description with the experimental data taken from ref. (Itoh et al., 1970).

Fig.(2) shows the longitudinal C5 form factor for the isoscalar $5^-$ state at $Ex=4.49$ MeV in $^{40}$Ca nucleus. The dashed curve shows the calculation of the form factor using the values of the free proton and neutron charges, in the framework of TDA. Admixture of higher configurations from 3p-2f shells with $\gamma=0.96$ as well as taking the core polarization effects into account by introducing effective charge of proton only$(e_{eff}(p)=\delta(p)e_{free}(p))$ with adjustable NN parameters of M3Y in table (2), produces an overall agreement with the data taken from ref. [24](Itoh et al., 1970) as shown in the solid curve.

As seen in Figs 1 and 2, according to TDA model the ground state is uncorrelated. In our work, the correlation in the ground state is added by including admixture from higher harmonic oscillator orbits using a mixing parameter, $\gamma$.

From the present work we conclude that the form factor results of Tamm-Dancoff Approximation require some enhancement, therefore the calculation of form factor for the ground state wave function is modified to include admixture from higher configurations using a mixing parameter, $\gamma$. Also, to enhance our calculation for the
longitudinal form factors, core polarization effects are taken into account by giving the nucleons an effective values to their charges and g-factor which are different from that of their free values which lead to adjust the NN potential parameters. This modification in the ground state wave function enhances the form factor theoretical curves excellently.

In the further work we planning to study the electron scattering form factors of sum closed-shell nuclei by using TDA in the presence of a recent interaction using other NN-realistic potential such as CD-Bonn and Paris potentials.

References

### Table (1): The parameters of M3Y potential which fit it to the oscillator matrix elements of Reid soft-core taken from ref. [Anantaraman et al, 1983] (*R3=0.7 fm (for tensor part only)

<table>
<thead>
<tr>
<th>M3Y component</th>
<th>Interaction channel</th>
<th>R1=0.25 fm</th>
<th>R2=0.4 fm</th>
<th>R3*=1.414 fm</th>
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<td>Vc</td>
<td>Singlet-even</td>
<td>12455</td>
<td>-3835</td>
<td>-10.463</td>
</tr>
<tr>
<td></td>
<td>Singlet-odd</td>
<td>29580</td>
<td>-3464</td>
<td>31.389</td>
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<tr>
<td></td>
<td>Triplet-even</td>
<td>21227</td>
<td>-6622</td>
<td>-10.463</td>
</tr>
<tr>
<td></td>
<td>Triplet-odd</td>
<td>12052</td>
<td>-1990</td>
<td>3.488</td>
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<tr>
<td>Vso</td>
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<td>0.0</td>
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<tr>
<td></td>
<td>Triplet-odd</td>
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<td>0.0</td>
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<tr>
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<td>-1260</td>
<td>-28.4</td>
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<tr>
<td></td>
<td>Triplet-odd</td>
<td>0.0</td>
<td>263</td>
<td>13.8</td>
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</table>

Table (2): The adjustable parameters of M3Y potential which fit to the oscillator matrix elements for all interaction channels.

<table>
<thead>
<tr>
<th>M3Y component</th>
<th>Interaction channel</th>
<th>R1=0.22 fm</th>
<th>R2=0.34 fm</th>
<th>R3*=1.385 fm</th>
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<td>14077</td>
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<tr>
<td></td>
<td>Triplet-odd</td>
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</tr>
<tr>
<td>Vt</td>
<td>Triplet-even</td>
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<td>-1317</td>
<td>-29.9</td>
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<tr>
<td></td>
<td>Triplet-odd</td>
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<td>302</td>
<td>14.1</td>
</tr>
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</table>
Fig(1) The longitudinal C3 form factor for $3_3^-(T=0)$ state at $E_x = 6.58$ (MeV) in $^{40}$Ca nucleus including the effect of core polarization with adjustable NN parameters-Tab 2 (solid line), where the dashed line is the TDA calculation. Data are taken from ref. (Itoh et al., 1970)

Fig(2) The longitudinal C5 form factor for $5_1^-(T=0)$ state at $E_x = 4.49$ (MeV) in $^{40}$Ca nucleus including the effect of core polarization with adjustable NN parameters-Tab 2 (solid line), where the dashed line is the TDA calculation. Data are taken from ref. (Itoh et al., 1970)

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