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Simulation of Robust Chaotic Signal with Given Properties

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Abstract

Task simulation of dynamic chaos in the observed time series is given a lot of attention. Strange attractors are observed in many application processes. Therefore, the task of simulation is important for controlling and predicting in the dynamic behavior. In the article we consider the problem of constructing the model on the properties of the observed data. Requirements for the model are simulating signal quality with adequate properties of the original process. An efficient model that allows you to build a robust chaos developed. Computational examples are given here.

Keywords: robust chaos, simulating the chaos attractor of the Rössler

1 Introduction

Currently, chaotic signals are used in various fields of science and technology is in engineering and process automation [4], finance [9], geology [6], medicine, in agriculture and for the simulation of traffic in computer networks [3], and many others [11]. This shows the great importance of interdisciplinary research results in the theory of chaotic dynamics. In many practical applications, the researchers, it is convenient to operate under the assumption that the chaotic nature of the observed process. Attractor simply restores method delays there are many numerical methods for assessing its dimension [5].

In many applications it is necessary to solve the problem of constructing models. Developed methods for the approximate solution of the inverse problem of the dynamics, that is, the restoration of a certain type of differential equations based on experimental data [3, 6, 8, 9]. Often, however, the dynamics of which is simulated by the constructed models do not comply with the chaotic invariant characteristics of the original process.

Sometimes there are errors in the simulation methods. Possible see that in the identification of chaotic systems with a strange attractor in the phase portrait approximately modeled by differential equations with limit cycles in the phase space. This approach violates all logic simulation of chaotic processes. Also, models are not robust, which affects their sensitivity to changes in the parameters. Thus, the measurement error of the real object and the accumulated error in numerical methods give an inadequate process under study result.

In this article we solve the problem of robust simulation of chaotic signal of the processes specified time series. The object of study is the time series obtained from the experiment. Required to build a model for generating a signal with the same qualitative properties of the phase trajectory.

2 Form of a model for the simulation of robust chaotic signal

For research the time series, with the presupposed that the time series $\{y_0, y_1, ..., y_k, y_{k+1}, ...\}$ generated by the system are sampled form which has the form:

$$\begin{aligned} \mathbf{x}_{k+1} &= f(\mathbf{x}_k\ \mathbf{x}_0), \\ y_k &= g(\mathbf{x}_k), \end{aligned} \tag{1}$$

where n-dimensional point in the state space; y — the observed one-dimensional process; k — discrete time (number); f, g — the vector function.

Using discrete models allows to operate with the data obtained from the experiment and does not accumulate errors associated with the transition to the continuous model; however, this complicates the qualitative analysis of the processes.

Phase trajectory, as we know, this trajectory, showing how changes in time t the state of the dynamical system **x**. For discrete systems, are able to connect-lines, in accordance with the sequence of samples k = 1, 2, ...,. Field of the evolving (converge) all possible trajectories of the motion of systems is called an attractor. For discrete systems, as well as for continuous systems, the attractor for oscillatory systems has the form of closed cycle trajectories. For chaotic systems, there is an attractor, which is called the strange, in this case the trajectories are drawn, but not to the point, a curve, a torus, and in a subset of the phase space.

The problem considered in this paper is to construct a model of the form (1) from the observed time series.

In [1] it is shown that robust chaos can occur in piecewise smooth systems. This result simplifies the task of modeling chaotic systems. Attempts to identify a parametric model with randomly chosen types of nonlinearities, as a rule, did not find a solution. If a solution is found (which is likely a coincidence than a natural phenomenon), the solutions will naturally sensitive to small changes in parameters.

Is true that many physical phenomena, chaos appears with a slight change of the parameter, however, in technical applications using chaotic regime, as a rule, requires a search for less sensitive, i.e. robust models. This requires the very essence of the experiments with technical systems, as well as calculation errors and measurements. Furthermore, for control synthesis problems, efficiently using the chaotic properties of the systems, but, provided that the chaotic regime does not change the qualitative behavior in a certain range of parameters.

Proposed to use the model as a

$$\begin{aligned} \mathbf{x}_{k+1} &= A\mathbf{x}_k + B\Psi u_k, \\ y_k &= C\mathbf{x}_k. \end{aligned} \tag{2}$$

Here, nonlinear function $\Psi(t)$ is determined by the symmetry properties of the founding of the phase trajectory [8, 9, 11]; u_k — discretized piecewise continuous function of time.

Symmetric properties of chaotic processes devoted pleased studies that make identification in low-symmetry breaking [2]. In [9] introduced the principles of symmetry breaking in the search of the attractor, in [10] the proof and the algorithm.

This kind of model representation allows the use of parametric identification methods, such as least-squares method implemented in the System Identification Toolbox in MATLAB. The function parameters are used ident analyzed {yk} (output) and the generated piecewise continuous series { Ψu_k } (input). The output of a matrix A, B, C dimensions found. Note that the dimension of the state space of the resulting identification system is always equal to the numerical estimates originally reconstructed attractor.

This proposed method has been simulated different applied systems [9, 11].

3 Computational examples

Let a time series, shown in Figure 1.

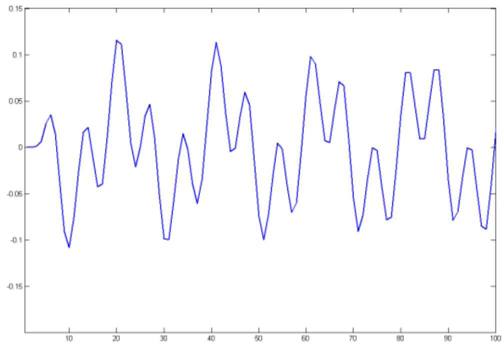

Fig. 1: The source time series

As a result of parametric identification of the system (2) using the Identification Toolbox, the following matrix equation of the system:

$$A = \begin{bmatrix} 0,7707 & -0,3206 & 0,542 & -0,004127 & -0,03038 \\ 0,5014 & 0,8746 & -0,18 & 0,05212 & 0,02569 \\ -0,3528 & 0,2858 & 0,6789 & -0,5847 & 0,1485 \\ -0,2553 & 0,2566 & 0,4667 & 0,7665 & 0,00253 \\ -0,1971 & -0,016 & -0,00074 & -0,0691 & -0,0922 \end{bmatrix};$$

$$B\Psi_0 = \left(0,6757; -0,4970; 0,4360; 0,8414\right)^T \left(4k^2 + 3k\right);$$

$$C = \left(0,4123; -0,1394;\ 0,1550; -0,0662\right).$$

Figure 2 shows a comparison of the dynamics of the model with the original time series. Adequacy of the model 84%, while the qualitative behavior of both systems is the same.

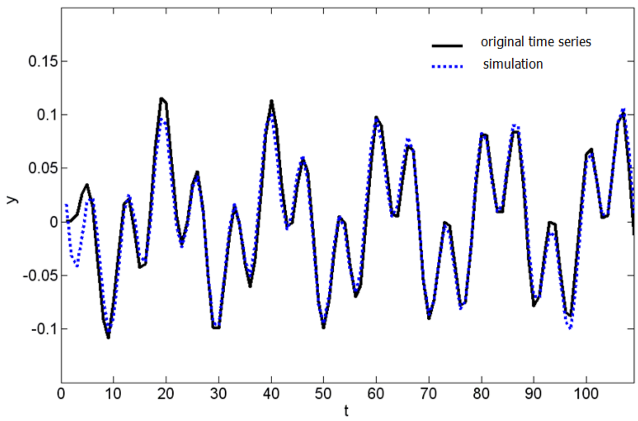

Fig. 2 Comparison of the dynamics of the model and the system (solid line — dynamics of the original system, dotted — simulated by the model signal)

Let is consider a number generated by the system Rössler. As a result of a number generated with the desired properties. Comparison of the simulated process with the original time series is shown in Fig. 3. Figure 4 shows the attractor of a simulated model. He typologically equivalent to a given attractor.

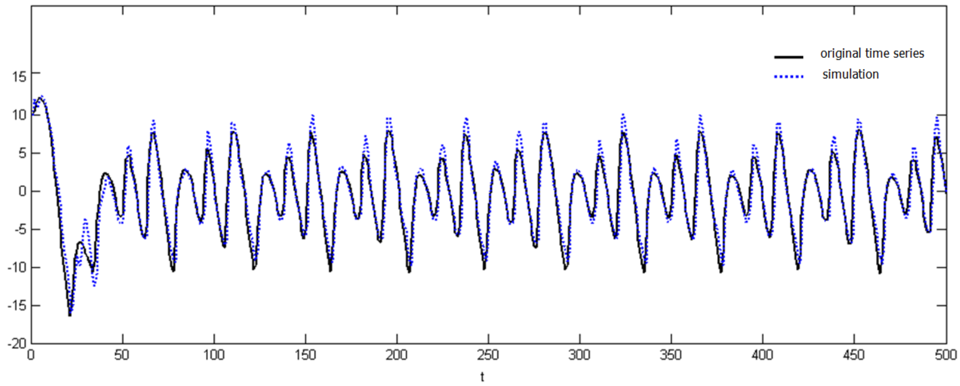

Fig. 3 Comparison of the dynamics of the test series and the constructed model

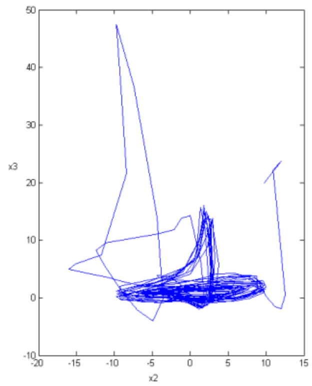

Fig. 4 Attractor simulated system

4 Conclusion

Task simulation of dynamic chaos in the observed time series is of great practical importance. Proposed in the paper models allow us to reliably identify computationally. Simulated by the model processes ensure compliance with the specified properties of the signal.

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