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Static Hydrodynamic Equation in $4d$ BSBM Theory

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Abstract

In this paper, we construct static hydrodynamic equation known as Tolman-Oppenheimer-Volkof (TOV) equation for Bekenstein-Sandvik-Barrow-Magueijo (BSBM) theory in four dimensions. Our starting point is to consider static spcetimes that admit spherical symmetry. At the end, we discuss a simple toy model where one can obtain the standard result in general relativity.

Keywords: BSBM Theory, TOV equation, Static Spacetimes

1 Introduction

One could wonder why such as the electromagnetic fine-structure constant, $\alpha = e^2/\hbar c$ should be constant in the nature in the context of standard quantum physics. Recently, there are some clues that this quantity may vary in the universe. For example, many high-redshift observations of quasar absorption spectra [1, 2, 3, 4, 5] strongly indicate that this quantity may depends locally in spacetimes. These facts have been becoming a great interest for some author such as Jordan [6, 7] Teller [8], Garrow [9], Dicke [10], and Stanyukovich [11] who firstly made a step to consider the fine-stucture constant is time dependent. The other theory, for example, that test and accomodate the spacetime dependent α was proposed by Sandvik, Barrow and Maguieijo (BSBM theory)[12] known as Bekenstein, Sandvik, Barrow and Maguieijo (BSBM) theory.

The spacetime dependent α can be ascribed from variation of the unit electric charge e or the speed of light, since it might be solved some cosmological problems such as inflation [13]. However, in BSBM theory the quantities such as c and $\hbar$ are taken to be constant, while the charge $e(x) = e_0\, e^{\psi}$ with e_0 is the present value of the unit charge and $\psi \equiv \psi\,(x)$.

Our goal in this paper is to derive static hydrodynamic equation known as Tolman-Oppenheimer-Volkof (TOV) equation [14, 15, 16] which the equation of states that relates the density and the pressure in a hydrodynamic system such as a star. The structure of the paper can be mentioned as follows. We shortly review some aspects of BSBM theory, see for example [17] , in section 2. Then, we derive TOV equation for static hydrodynamic system in section 3. In section 4 we give a simple toy model of a star. Finally, we put the conclusions in section 5.

2 BSBM Theory

Let us now consider the action

$$S = S_g + S_m + S_\psi + S_{em} \ , \tag{2.1}$$

where,

$$\begin{aligned} S_g &= \frac{1}{16\pi G}\int d^4x\sqrt{-g}R \ , \\ S_m &= \int d^4x\sqrt{-g}\mathcal{L}_{\rm m} \ , \\ S_\psi &= -\frac{\omega}{2}\int d^4x\sqrt{-g}\partial_\mu\psi\partial^\mu\psi \ , \\ S_{em} &= -\frac{1}{4}\int d^4x\sqrt{-g}f_{\mu\nu}f^{\mu\nu}e^{-2\psi} \ . \end{aligned}$$

The field strength tensor $f_{\mu\nu}$ is defined as

$$f_{\mu\nu} \equiv \partial_\mu a_\mu - \partial_\nu a_\mu = e^{\psi}F_{\mu\nu} \ , \tag{2.2}$$

where $a_\mu = e^{\psi}A_\mu$. The quantity R is the Ricci scalar, whereas ω is a coupling constant. Lagrangian $\mathcal{L}_{\rm m}$ contains some additional matter fields.

Then, we could derive equations of motions in order. First, the variation of the metric $g_{\mu\nu}$ we get Einstein equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G\left(T^{(m)}_{\mu\nu} + T^{(\psi)}_{\mu\nu} + T^{(em)}_{\mu\nu}\right) \ , \tag{2.3}$$

where

$$T^{(\psi)}_{\mu\nu} = \omega\left(\partial_\mu\psi\partial_\nu\psi - \frac{1}{2}g_{\mu\nu}g^{\lambda\rho}\partial_\lambda\psi\partial_\rho\psi\right) \ , \tag{2.4}$$

$$T^{(em)}_{\mu\nu} = \left(f^\lambda_\mu f_{\lambda\nu} - \frac{1}{4}g_{\mu\nu}f^{\lambda\rho}f_{\lambda\rho}\right)e^{-2\psi} \ , \tag{2.5}$$

with $T^{(m)}_{\mu\nu}$ is additional matter tensor energy-momentum and $R_{\mu\nu}$ is Ricci tensor. In the case where the radiation of electromagnetic are dominant, the total density is the sum of Coulomb energy density $\zeta\rho_m$ and the radiation energy density ρ_r, where $-1 \leq \zeta \leq 1$. So, $T^{(em)}_{\mu\nu}$ can be regarded as a perfect fluid [17]:

$$T^{(em)}_{\mu\nu} = (|\zeta|\rho_m + \rho_r + p_m + p_r)\,e^{-2\psi}u_\mu u_\nu - (p_m + p_r)\,e^{-2\psi}g_{\mu\nu} \ , \tag{2.6}$$

whereas the dilaton varies with respect to equations of motions

$$\omega\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\psi\right) - \frac{1}{2}\sqrt{-g}f_{\mu\nu}f^{\mu\nu}e^{-2\psi} = 0 \ . \tag{2.7}$$

3 TOV Equation in BSBM Theory

Let us turn our attention to a specific model, namely static spacetimes whose metric is given by

$$ds^2 = -e^{2\Phi(r)}dt^2 + e^{2\Lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \ , \tag{3.1}$$

where we have used a local coordinate (t, r, θ, ϕ) with r is the radial coordinates whereas (θ, ϕ) are the angle coordinates on 2-sphere.

Then, for perfect fluids in a star, the four velocity has the form

$$u^\alpha = \left(e^{-\Phi}, 0, 0, 0\right) = e^{-\Phi}\xi^\alpha, \tag{3.2}$$

where ξ is a timelike Killing vector. The conservation of energy momentum in BSBM theory is given by

$$\begin{aligned} T^{\mu\nu}_{\ \ ;\nu} \ &= \ \nabla_\nu\big[(\rho_m\,(1 + \zeta\exp\,[-2\psi]) + \rho_r\exp\,[-2\psi])\,u^\mu u^\nu \\ &\quad + (p_m\,(1 + \zeta\exp\,[-2\psi]) + p_r\exp\,[-2\psi])\,(g^{\mu\nu} + u^\mu u^\nu)\,\big] = 0 \end{aligned} \tag{3.3}$$

where we already took a case of

$$\rho_B = (\rho_m (1 + \zeta \exp[-2\psi]) + \rho_r \exp[-2\psi]) \ , \tag{3.4}$$

and

$$p_B = (p_m (1 + \zeta \exp[-2\psi]) + p_r \exp[-2\psi]) \ . \tag{3.5}$$

Moreover, we are also taking the assumption that in the interior the electromagnetic radiation is so densed such that it could be considered as a sea of pure radiation in which we have $f_{\mu\nu} f^{\mu\nu} \to 0$ [17]. Thus, the dilaton of (2.7) can be chosen to be

$$\psi(r) = C_0 \int \frac{e^{\Lambda-\Phi}}{r^2} dr + C_1 \ , \tag{3.6}$$

for real C_0 and C_1. After some computation, (3.3) gives TOV equation

$$\frac{dp_B}{dr} = -(\rho_B + p_B) \frac{d\Phi}{dr} \ , \tag{3.7}$$

together with the pressure transverse equation

$$\frac{d\Phi}{dr} = \frac{m + 4\pi r^3 p_B}{r(r-2m)} \ , \tag{3.8}$$

where

$$m(r) = 4\pi \int r^2 \rho_B dr \ . \ , \tag{3.9}$$

which is coming from Einstein field equation (2.3).

4 A Simple Case of Interior Solutions

To have a simple interior model, we simply take that the density $\rho_B = \rho_0$ is uniform in an area less than the maximum radius R and the outside is vacuum. It follows that the matter density ρ_m and the radiation density ρ_r should be

$$\begin{aligned} \rho_m &= \rho_{0m} (1 + \zeta \exp[-2\psi])^{-1} \ , \\ \rho_r &= \rho_{0r} \exp[2\psi] \ , \end{aligned} \tag{4.1}$$

respectively and both ρ_{0m} and ρ_{0r} are real constants. The mass of the star is $m = \frac{4}{3}\pi\rho_0 r^3$ and TOV equation (3.7) simplifies to

$$\frac{dp_B}{dr} = -\frac{4\pi r (\rho_0/3 + p_B)(\rho_0 + p_B)}{1 - 8\pi\rho_0 r^2/3} \ , \tag{4.2}$$

which gives

$$p_B = \rho_0 \frac{\sqrt{(1-8\pi\rho_0 r^2/3)\,/\,(1-8\pi\rho_0 R^2/3)}-1}{3-\sqrt{(1-8\pi\rho_0 r^2/3)\,/\,(1-8\pi\rho_0 R^2/3)}}. \tag{4.3}$$

The pressure at the center of the star is given by

$$p_c = \frac{3M}{4\pi R^3}\frac{1-\sqrt{1-2M/R}}{3\sqrt{1-2M/R}-1}\ , \tag{4.4}$$

where the condition

$$\frac{M}{R} < \frac{4}{9}\ , \tag{4.5}$$

must be fulfilled in order to avoid singularity when

$$\frac{M}{R} = \frac{4}{9}\ . \tag{4.6}$$

5 Conclusions

We have constructed TOV equation for static hydrodynamic system in four dimensional BSBM theory. To simplify the case, we take the assumption that the density of electromagnetic radiation is quite large such that it could be thought of as a sea of pure radiation. Then, we find that the dilaton depends only on the radial coordinate r. At the end, we discussed a simple toy model where the density ρ_B is constant and then, obtain $M/R < 4/9$ in order to avoid singularity at the center.

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