Expanding Cosmology with Back Reaction Term
Caused by Quantum Particle Creation

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Abstract

An extended Standard cosmological Model is considered with a time dependent cosmological term of the form $\lambda_0(R/\dot{R})/R$, $R(t)$ the cosmic scale factor. Such term, that is proportional to the number of quantum particles created by Universe expansion, is introduced to take into account the back reaction to particle creation. The model is first studied when the equation of state $p = w\rho$ holds between the energy density $\rho$ and pressure $p$ of the Universe. Exact solutions are determined in the flat space-time case for arbitrary $w$. The curved space-time cases are solved for $w = -1, -1/3$. The solutions show the existence of a large variety of possible dynamics of the Universe. The model is studied also when the energy density has the sufficiently general form $\rho = \rho_0(R/R_0)^{-\alpha}$. Solutions are given by quadrature. In the flat space-time case case the quadrature is performed exactly. This shows the existence of an inflationary big-bang at time $t = 0$ and an accelerated expansion for large $t$ with corresponding particle production. Moreover a numerical determination of the particle creation parameter $\Omega_{cr}$ is obtained. In the matter dominated case the value of $\Omega_{cr}$ is near to the value of $\Omega_{\Lambda}$ while in the radiation dominated case $\Omega_{cr}$ is near to $\Omega_{\Lambda} + \Omega_C$ where $\Omega_{\Lambda}, \Omega_C$ are the numerical values of the experimental dark energy and cold dark matter parameters at present time. The model is solved also for $\rho = 0, p \neq 0$. For such unusual situation the value of the "radius of the Universe" at the present time is calculated

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1 Introduction

As it was originally found by Parker \cite{5, 6, 7}, creation of particle is possible in curved space-time as a consequence of field quantization and Universe expansion. At present the problem can be found widely discussed in many books \cite{2, 1, 8}.

Existence of creation of particles of a given field is generally proved by the non vanishing of the Bogoliubov coefficients that connect "in" and "out" asymptotic normal modes of the field. For specific applications to spin 0, 1/2 field in Robertson-Walker (RW) space-time see, e. g., \cite{4} and references therein.

In \cite{11, 12, 13} a different procedure has been proposed that leads, in the context of the RW space-time, to an evaluation of the number of the instantaneous creation of spin 0, 1/2, 1 particles. Accordingly, one finds that the number $n_{sk}(t)$ of created particles per unit time, per mode $k$ and spin $s$ is given by $n_{sk}(t) = \pm 6 \frac{\dot{R}(t)}{R(t)}$, $R(t)$ the scale factor of RW metric. (A similar calculation has been performed also for spin 1 field in the context of the Lemaître-Tolman-Bondi cosmologies \cite{14}). The $\pm$ sign does not refers to creation and annihilation but to two different procedures to obtain the result. If one chooses the sign plus then $\dot{R} > 0$ corresponds to particle creation and $\dot{R} < 0$ to particle annihilation. Therefore in correspondence to an inflationary phase of the Universe expansion a great amount of creation of particles is expected. Unfortunately, by that argument, it is not possible to establish in what place in the universe particles are created because this is consequence of a pure quantization procedure independent of the physical context. Moreover, since the result is mode independent, the total number of created particle with arbitrary mode is divergent. It seems however plausible that the process may be catalyzed or inhibited by the physical context of the Universe. Therefore by an overall mediation one is led to assume that the total number of created quantum particles of arbitrary mode and spin due to universe expansion is of the form

$$n(t) = \sum_s \int \gamma_s(k, t) \frac{\dot{R}}{R} dk = \gamma(t) \frac{\dot{R}(t)}{R(t)}$$

(1)

$\gamma_s(k, t)$ is a $k$-integrable function. The presence of created particle modifies the gravitational dynamics. Therefore one has to take into account of it in the formulation of a cosmological model. The problem seems very difficult to formulate. An elementary answer to the problem is of considering the Standard Cosmological Model where the energy density $\rho$ of the universe is re-defined.
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by the substitution

\[ \rho \rightarrow \rho + \gamma(t) \frac{\dot{R}}{R} \]  

(2)

The choice \( \gamma(t) = \rho_0/R^3 \) was first studied qualitatively in a Standard Cosmological model without cosmological term \([15]\) and then numerically \([16]\). The study in \([16]\) indicates that the choice \( \gamma(t) = \rho_0/R^3 \) is problematic for flat radiation dominated model. The choice \( \rho = \alpha = \text{const.} \) was considered in the flat cosmological model without cosmological term \([17]\). The value of the particle creation parameter \( \Omega_{cr} \) determined in \([17]\) is greatly improved in \([18]\). In the flat case, both for radiation and matter dominated model, the numerical results are in agreement with the experimental data for reasonable assignments of the vacuum energy parameter. In the mentioned papers a study lacks of the simultaneous presence of matter and radiation terms in the curve space-time case. Such study seems, as far as the author understands, a difficult analytical problem (but numerically solvable). (The difficulties are not fewer if one tries to include particle creation term in a cosmological generalized Lemaître-Tolman-Bondi model \([19]\)).

On account of the mentioned difficulty and problematic aspects in taking into account quantum particle creation by Universe expansion in the standard cosmological model, it does not seem useless an alternative formulation of the problem.

In the present paper the back reaction to particle production is simulated in the Standard Cosmological Model by a time dependent cosmological term \( \approx \dot{R}(t)/R(t) \), that is just proportional to the number of created particle at time \( t \). The model is first studied for the state equation \( p = w\rho, \; (w \in \mathbb{R}) \). Exact solutions are determined for both \( R(t) \) and \( \rho(t) \) in the flat space-time case \( (a = 0) \) and in case \( w = -1, -1/3 \) for all curvature cases \( (a = 0, \pm 1) \). The solutions exhibit different kinds of dynamical evolution with initial big-bang and final accelerated expansion. Then the case of an energy density of the form \( \rho = \rho_0(R/R_0)^{-\alpha}, \; \alpha \in \mathbb{R} \) is considered. The corresponding cosmological equation is solved by quadrature for every \( \alpha \). In the flat space-time case the quadrature can be performed analytically for every \( \alpha \). For such situation the solution \( \dot{R}(t) \) shows an inflationary big-bang at \( t = 0 \) and grows exponentially for large \( t \). For \( \alpha = 3 \) (matter dominated model) and \( \alpha = 4 \) (radiation dominated model) and in presence of particle creation, a numerical analysis is developed. This leads to explicit numerical values for the particle creation parameter \( \Omega_{cr} \). It seems of some interest the fact that, in the matter dominated case, \( \Omega_{cr} \) results compatible with the value of \( \Omega_\Lambda \) and in the radiation dominated case with \( \Omega_\Lambda + \Omega_C, \; \Omega_\Lambda, \; \Omega_C \) being the dark energy and cold dark matter experimental parameters at the present time \([20]\) respectively.

Finally the case of a flat Universe with particle creation, without energy density but with non trivial pressure, is considered. For such unusual model
two negative values for \( \Omega_{cr} \) are found. They enable a calculation of the "radius of the Universe" \( R(t_0) \) at the present time. One of them results larger and the other smaller, but of the the same order of magnitude, of \( R_c = ct_0 \), \( t_0 \) being the Age of the Universe and \( c \) the light velocity. As a conclusion, the present formulation of a cosmology that take into account quantum particle creation produced by its own expansion seems to be in agreement with the request of the existence of a big-bang and of a late accelerated expansion of the Universe. Moreover it seems more simple than that previously considered [15, 16, 17]). A final acceptance of the scheme requires however a further numerical analysis with a simultaneous consideration of all the cosmological parameters and possibly also in the curved space-time cases.

2 Cosmology with back reaction to particle creation

The following considerations are developed within the Robertson-Walker (RW) space-time metric whose structure follows from homogeneity and isotropy assumptions of the Universe [9, 10]. The corresponding line element is given by

\[
ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dt^2 - R^2(t) \left[ \frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \right], \quad K = 0, \pm 1
\]

where \( x^0 = t, x^k = r, \theta, \varphi \). The object is to study the cosmological model, as suggested in the introduction, that is characterized by the Einstein equation

\[
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = k T_{\mu\nu} - k \lambda_0 \frac{\dot{R}}{R} g_{\mu\nu} \quad (k = 8\pi G)
\]

\[
T_{\mu\nu} = (\rho + p)U^\mu U^\nu - pg_{\mu\nu}, \quad U^0 = 1, \ U^k = 0, \ k = r, \theta, \varphi
\]

where it has been set \( c = 1 \), \( \dot{R} = dR/dt \); \( \rho, p \) respectively the energy density and pressure of the Universe that depend, a priori, on the space-time point. The expression \( \lambda_0 \dot{R}/R \) is interpreted as due to back reaction to particle creation. By exploiting (4) and its consistency condition in the metric (3) one gets:

\[
3 \frac{\ddot{R}}{R} = -\frac{k}{2} (3p + \rho) - k \lambda_0 \frac{\dot{R}}{R}
\]

\[
\frac{\ddot{R}}{R} + 2 \frac{\dot{R}^2}{R^2} + 2 \frac{a}{R^2} = -\frac{k}{2} (p - \rho) - k \lambda_0 \frac{\dot{R}}{R}
\]

\[
\dot{\rho} + 3 \frac{\dot{R}}{R} (\rho + p) = k \lambda_0 \frac{d}{dt} \left( \frac{\dot{R}}{R} \right)
\]

\[
\partial_j \rho = 0; \quad j = r, \theta, \varphi
\]
From (9), (6) both \( p \) and \( \rho \) do in fact depend only on the time coordinate: \( p = p(t), \rho = \rho(t) \).

A first case of interest is when it is possible to give an equation of state in the simple form

\[
p(t) = w \rho(t)
\]

(10)

\( w \) a real constant. Under such condition one can extract from (6), (7) a closed equation for \( R \):

\[
2 \frac{\ddot{R}}{R} + \left( \frac{\dot{R}}{R} + \frac{a}{RR} \right)(1 + 3w) = -k\lambda_0(1 + w)
\]

(11)

If \( a = 0 \), \( w \neq -1 \) it is possible to solve exactly the last equation. One obtains

\[
R(t) = \left( d - \alpha e^{-\beta t} \right)^\gamma, \quad \alpha = \frac{3}{k\lambda_0} e^h, \quad \beta = \frac{k\lambda_0}{2}(1 + w), \quad \gamma = \frac{2}{3(1 + w)}
\]

(12)

with \( h, d \) real integration constants. By using the expression (12), one can integrate also (8) to obtain

\[
\rho(t) = \frac{\delta + e^{-\beta t + h'}}{(d - \alpha e^{-\beta t})^2}, \quad e^{h'} = 3e^{2h}
\]

(13)

\( \delta \) a real integration constant. The asymptotic behaviors of \( R(t) \) and \( \rho(t) \) are then for \( w + 1 > 0, \lambda_0 > 0 \):

\[
R(t) \xrightarrow{t \to \infty} d \frac{2}{3(1 + w)}, \quad R(t) \xrightarrow{t \to 0} (d - \frac{3}{k\lambda_0} e^h)^\frac{2}{3(1 + w)}
\]

(14)

\[
\rho(t) \xrightarrow{t \to \infty} \frac{\delta}{d^2}, \quad \rho(t) \xrightarrow{t \to 0} \frac{\delta + 3e^{2h}}{(d - \frac{3}{k\lambda_0} e^h)^2}
\]

(15)

In order to have a big-bang at \( t = 0 \), \( R(0) = 0 \), it must be \( d = 3e^h/(k\lambda_0) \). This also implies \( \rho(0) = \infty \) if \( \delta + 3e^{2h} > 0 \).

Instead if \( w + 1 < 0, \lambda_0 > 0 \) one has from (12), (13)

\[
R(t) \xrightarrow{t \to \infty} 0, \quad R(t) \xrightarrow{t \to 0} (d - \frac{3}{k\lambda_0} e^h)^\frac{2}{3(1 + w)}
\]

(16)

\[
\rho(t) \xrightarrow{t \to \infty} 0, \quad \rho(t) \xrightarrow{t \to 0} \frac{\delta + e^{h'}}{(d - \frac{3}{k\lambda_0} e^h)^2}
\]

(17)

Note that \( d = 3e^h/(k\lambda_0) \) implies \( R(0) = \infty \) and \( \rho(0) = \infty \) if \( \delta + e^{h'} > 0 \). The cases with \( \lambda_0 < 0 \) can be discussed analogously.

For \( w = -1 \) the equation (12) can be solved for any value of the curvature parameter and one has

\[
R(t) = p^{-1/2} e^{\pm \sqrt{pt + p_0}}, \quad p \in \mathbb{R}^+, \quad (a = 0)
\]

(18)

\[
R(t) = p^{-1/2} \cosh(\pm \sqrt{pt + p_1}), \quad p \in \mathbb{R}^+, \quad (a = 1)
\]

(19)

\[
R(t) = p^{-1/2} \sinh(\pm \sqrt{pt + p_{-1}}), \quad p \in \mathbb{R}, \quad (a = -1)
\]

(20)
p, p_0, p_1 real integration constants, p_{-1} real if p > 0 and p_{-1} = i p'_{-1} with p'_{-1} real if p < 0. Therefore for a = 0, 1, w = -1 a big-bang situation cannot occur at any finite time, while this is possible for a = -1 under a suitable choice of the integration constants. Correspondingly, since from (8), \( \rho = k \lambda_0 \dot{R}/R \) up to an integration constant, one has also

\[
\rho = k \lambda_0 \sqrt{p} = \text{constant}, \quad p > 0, \quad (a = 0) \tag{21}
\]

\[
\rho = k \lambda_0 \sqrt{p} \tanh(\sqrt{pt} + p_1), \quad p > 0, \quad (a = 1) \tag{22}
\]

\[
\rho = k \lambda_0 \sqrt{p} \coth(\sqrt{pt} + p_{-1}), \quad p > 0, \quad (a = -1) \tag{23}
\]

\[
\rho = k \lambda_0 \sqrt{|p|} \tanh(\sqrt{|p|t} + p'_{-1}), \quad p < 0, \quad (a = -1) \tag{24}
\]

The case \( w = -1/3 \) can be treated in a similar fashion. For example, suppose \( \lambda < 0 \), then from (11), (8)

\[
R = \frac{3}{k|\lambda_0|} e^{t k|\lambda_0|/3}, \quad \rho = e^{-\frac{2}{3} k|\lambda_0| t + \beta} \tag{25}
\]

with \( \alpha, \beta \) integration constants.

### 3 Energy density as a power of the scale factor

By combining (6) and (7) one can report the study to a cosmological equation that depends on \( \rho \) but not on \( p \). Such equation is studied for a specific expression of \( \rho \) in terms of \( R \). Precisely the object is now the solution of the sufficiently general model given by

\[
\frac{\dot{R}^2}{R^2} + \frac{a}{R^2} = \frac{k}{3} \rho - \frac{k}{3} \lambda_0 \frac{\dot{R}}{R} \quad a = 0, \pm 1 \tag{26}
\]

\[
\rho = \rho_0 \left( \frac{R}{R_0} \right)^{-\alpha}, \quad \alpha \in \mathbb{E} \tag{27}
\]

where \( \rho_0 = \rho(t_0), R_0 = R(t_0), t_0 \) a fixed given time. The expression (26) contains the matter (\( \alpha = 3 \)) and radiation (\( \alpha = 4 \)) dominated universe case. Once \( R(t) \) has been determined, the solution for \( p \) follows from (6) (or (7)). The equation (8) results automatically satisfied because it is indeed a consequence of (6) an (7) (e. g. [21]). By defining as usual [3]

\[
x = \frac{R}{R_0}, \quad \Omega_0 = \frac{\rho_0}{H_0^2}, \quad \Omega_{cr} = -k \frac{\lambda_0}{3 H_0^2}, \quad \Omega_a = -a \frac{H_a}{R_0^2 H_0^2}, \quad a = 0, \pm 1 \tag{28}
\]

\((H_0 = \dot{R}(t_0)/R(t_0))\) the scheme (25), (26) leads to the equations

\[
\dot{x}^2 - (\Omega_{cr} H_0 x) \dot{x} + H_0^2 (\Omega_0 + \Omega_{cr} - 1 - \Omega_0 x^{-2}) = 0 \tag{29}
\]

\[
\Omega_0 + \Omega_{cr} + \Omega_a = 1 \tag{30}
\]
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that can be integrated in the form

$$H_0 t = 2 \int_0^x \frac{dx}{\Omega_{cr} x \pm \sqrt{\Omega_{cr}^2 x^2 - 4(\Omega_0 + \Omega_{cr} - 1 - \Omega_0 x^{\alpha - 2})}}$$

(31)

An exact analytical calculation of the integral does not seem easy. A numerical evaluation is possible but it appears quite cumbersome due to the presence of many independent parameters. In the flat space-time case the scheme can be further developed.

3.1 The flat case: solution, asymptotic expansion and numerical estimates

It is possible to perform the integration in the flat Universe case. Suppose indeed $a = 0$ so that $\Omega_a = 0$, $\Omega_0 + \Omega_{cr} = 1$. Accordingly one gets

$$2 \alpha H_0 t = - \frac{\Omega_{cr}}{\Omega_0} x^\alpha + \frac{1}{\Omega_0} \sqrt{\Omega_{cr}^2 x^{2\alpha} + 4\Omega_0 x^{\alpha} \pm 4 \Omega_0 + \Omega_{cr} x^{\alpha/2}} \log \left( \frac{\sqrt{\Omega_{cr}^2 x^{\alpha} + 4\Omega_0 + \Omega_{cr} x^{\alpha/2}}}{2\sqrt{\Omega_0}} \right)$$

(32)

Considering now the limit for large and small $x$ in the last equation for the upper sign case and then reversing the relation with $t$ one obtains:

$$x(t) \xrightarrow{t \to 0} (\alpha H_0 \sqrt{\Omega_0} t)^{2/\alpha}, \quad x(t) \xrightarrow{t \to \infty} \left( \frac{\Omega_{cr}}{\Omega_0} \right)^{2/\alpha} e^{H_0 \Omega_{cr} t}$$

(33)

There is then a big-bang at $t = 0$, $R(0) = R_0 x(0) = 0$ with initial inflationary phase ($\alpha > 2$) (see, e. g., [3]) and great amount of particle production because $\dot{R} \sim t^{(2-\alpha)/2}$ and $\frac{\dot{R}}{R} \to \frac{2}{\alpha}$, respectively, for $t \to 0$. Moreover for large $t$ there is an accelerated expansion of the Universe with constant amount of particle production proportional to $H_0 \Omega_{cr}$. Correspondingly $\rho(t) \propto t^{-2}$ for $t \to 0$ and $\rho(t) \propto \exp(-\alpha H_0 \Omega_{cr} t)$ for $t \to \infty$.

For $t = t_0$, $x = 1$. By setting $\Omega_{cr} = y$ and taking into account that $\Omega_0 = 1 - y > 0$, from (31) one has

$$2 \alpha H_0 t_0 = - \frac{y}{1 - y} \pm \frac{2 - y}{1 - y} \mp \log(1 - y), \quad y < 1$$

(34)

In correspondence to the $\pm$ signs one has respectively

$$\alpha H_0 t_0 = 1 - \frac{1}{y} \log(1 - y) = F(y)$$

(35)

$$\alpha H_0 t_0 = - \frac{1}{1 - y} + \frac{1}{y} \log(1 - y) = G(y)$$

(36)

Suppose now $t_0$ the Age of the Universe. Since $H_0 > 0$ and $G(y) < 0$ for $y < 1$, the equation (34) can be satisfied only for $\alpha < 0$. Even if this gives
raise to a very improbable energy density in (26), negative values of $\alpha$ cannot in principle be excluded if particle creation is admitted.

Instead (33) is possible since $F(y)$ is an everywhere positive increasing function in $(-\infty, 0)$. By taking $H_0 = 0.068588 \times 10^{-9} \text{yr}^{-1}$ and $t_0 = 13.819 \times 10^9 \text{yr}$ (see e.g. [20]), (33) gives, for $\alpha = 3, 4$ respectively,

$$3H_0 t_0 \approx 2.84 = F(y) \implies \Omega_{cr} \equiv y \approx 0.75 \quad (37)$$

$$4H_0 t_0 \approx 3.79 = F(y) \implies \Omega_{cr} \equiv y \approx 0.925 \quad (38)$$

These values of $\Omega_{cr}$ show that particle creation is relevant in the present model.

### 3.2 Pure creation case: solution, numerical estimates

Suppose now $\rho_0 = 0$ so that $\Omega_a + \Omega_{cr} = 1$. The integration of (30) gives now

$$tH_0 = \frac{1}{4\Omega_{cr}(1 - \Omega_{cr})} \left[ \Omega_{cr} x \left( \sqrt{\frac{\Omega_{cr}^2 x^2 + 4(1 - \Omega_{cr})}{\Omega_{cr}^2 x^2 + 4(1 - \Omega_{cr}) + \Omega_{cr} x}} + 4(1 - \Omega_{cr}) \log \left( \frac{\sqrt{\Omega_{cr}^2 x^2 + 4(1 - \Omega_{cr}) + \Omega_{cr} x}}{2\sqrt{1 - \Omega_{cr}}} \right) \right) \right] \quad (39)$$

There follows that it must be $\Omega_{cr} \equiv 1 - \Omega_a < 1$, or $\Omega_a \equiv -a/(H_0^2 R_0^2) > 0$ and hence $a = -1$. The situation of pure particle creation is therefore compatible only with the open Universe model. By setting again $t = t_0$, $\Omega_{cr} = z < 1$ one has, according to the sign $\mp$ in (39)

$$t_0 H_0 = \frac{1}{2} - \frac{1}{2z} \log(1 - z) = f(z) \quad (40)$$

$$t_0 H_0 = \frac{1}{2(1 - z)} - \frac{1}{2z} \log(1 - z) = g(z) \quad (z < 1) \quad (41)$$

By choosing $t_0 H_0$ as in the previous case a straightforward numerical analysis (or graphical representation of the functions $f(z)$, $g(z)$) gives

$$H_0 t_0 \approx 0.948 = f(z) \implies \Omega_{cr} \equiv z \approx -0.242 \quad (42)$$

$$H_0 t_0 \approx 0.948 = g(z) \implies \Omega_{cr} \equiv z \approx -0.074 \quad (43)$$

Therefore, in absence of matter and radiation, the particle creation parameter takes negative values. By using the definition of $\Omega_{-1}$ in (27) one has $R_0 = H_0^{-1}(1 - \Omega_{cr})^{-\frac{1}{2}}$. From the value $H_0^{-1} \approx 14.48 \times 10^9 \text{yr}$ ([20]) in correspondence to (40), (41) one has respectively $R_0 = 12.99 \times 10^9 \text{ light - yr}$ and $R_0 = 13.97 \times 10^9 \text{ light - yr}$. If one compares these value with the present value $ct_0 \approx 13.819 \times 10^9 \text{ light - yr}$ one sees that the mean speed of the Universe expansion is near to the light speed but it can be greater or smaller of it. The choice $\rho_0 = 0$ does not imply $p = 0$. Once $R = R(t)$ is made explicit from (37), $p$ can be calculated from (8).
4 Discussion and Comments

In the previous Sections the Standard Cosmological model has been studied with a time dependent cosmological term that mimic the back reaction to the quantum particle production due to the expansion of the Universe and that is just proportional to the "number" of created particles. In case energy density and pressure of the universe are in the state equation \( p = w\rho \) the scheme has been integrated in the flat space-time case for arbitrary value of \( w \). For \( w = -1, -1/3 \) also the curved cases have been integrated. In correspondence to the solutions there is a great variety of cosmological dynamics according to different choices of the integration constants. There is the situation where the universe starts with a big-bang and evolves asymptotically to a finite (e.g. (15)) radius \( R \). There is also a configuration with an initial finite or infinite size and a final collapse at \( t = +\infty \) (e. g. (16)). It may also happen that a big-bang behavior is not possible at any time as in (18) and (19). The solution (20) with \( p_{-1} = 0 \) admits a big bang at \( t = 0 \) with large production of particles, because \( \frac{\dot{R}}{R} \approx \frac{\sqrt{\rho}}{t} \) for \( t \to 0 \), and an exponential acceleration for \( t \to +\infty \).

The model has been studied also in case of an energy density of the form \( \rho = \rho_0 (R/R_0)^{-\alpha} \). The solution of the cosmological equation has been performed exactly in the flat case and for \( \alpha > 0 \) it shows a dynamical evolution of the Universe with initial inflationary big bang with great amount of particle production and a final exponentially accelerated expansion.

The exact solution of the flat case allows some numerical evaluation and interpretation of the quantum particle creation parameter \( \Omega_{cr} \). There results that:

i) in the matter dominated case (\( \alpha = 3 \)) one has \( \Omega_{cr} \approx 0.75 \) a value that is near the value of the experimental dark energy parameter \( \Omega_\Lambda = 0.6825 \) ([20]). This suggest the possible interpretation of \( \Omega_\Lambda \) as due to particle creation.

ii) in the radiation dominated case (\( \alpha = 4 \)) one has \( \Omega_{cr} \approx 0.925 \). If one also considers the experimental data \( \Omega_C \approx 0.2671, \Omega_C \) the cold dark matter experimental parameter ([20]), one has \( \Omega_\Lambda + \Omega_C \approx 0.95 \). The dark energy plus cold dark matter could be then interpreted as due to particle creation.

iii) with the choice \( \rho_0 = 0 \) the scheme represents a curved cosmological model without matter and radiation, but with pressure and with particle creation. This very hypothetical context allows anyhow a calculation of the radius of the Universe.

The above considerations have put into evidence some positive aspects of the cosmological model with particle creation proposed in the paper. What however lacks is a discussion of the time evolution of the Universe in intermediate times. This should be possible in the flat space-time case, by tabulating \( t = t(x) \) in (32) and then inverting the relation to have \( x = x(t) \). However, to have a better description of the universe and possibly confirm the suggested
interpretations given above, one should try to integrate the scheme in correspondence to an energy density assumption of a form like $\rho \sim \xi R^{-3} + \eta R^{-4}$. By proceeding as is the previous Section one would be left with a cosmological equations that generalize (29)-(31). Even if the exact analytical integration of the general case seems quite hard, a numerical integration should not be impossible even in the presence of many parameters. In this connection also a further analysis on the existence of an initial inflationary phase would be interesting.

Finally, one can compare the present cosmological formulation of the back reaction to particle creation with the one proposed in [17, 18, 16]. It is evident that the present one is simpler and that the numerical values of the cosmological parameters obtained here seem to better fit the experimental data. It must be remarked however that a complete systematic study of both scheme is lacking. Improvements both in formulation and solutions of the models would be useful to establish the effective validity of the schemes and to discriminate between them.

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