Magnetic Field of Celestial Objects

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Abstract

The paper discusses the possibility of interpreting the SWB conjunction in the framework of a unified field theory. Using the generalized field theory MWT, a direct relation between the polar magnetic field, the angular velocity and the gravitational potential of the body considered, is obtained. This theoretical formula shows that the field is generated as a result of rotation of massive objects. The Schuster-Wilson-Blackett form of non-minimal gravitational-electromagnetic coupling (NMGEC) is obtained from MWT.

Keywords: Unified field theory, Magnetic fields, Astrophysics, Relativity

1 Introduction

The question of the origin of large scale magnetic fields first across when Gilbert, four hundred years ago, introduced the concept of the existence of a terrestrial magnetic field. The existence of magnetic field in almost all cosmical bodies (e.g. the Sun, Planets, Stars and Galaxies) makes the idea more attractive that the field origin is universal. Different theories that have been constructed to account for the origin of large scale magnetic fields, are the hydromagnetic oscillator, battery, fossil and dynamo theories. The most acceptable ones are the dynamo and fossil theories.
The fossil field is the relic of a primitive magnetic field not yet completely dissipated. The fossil theory alone, without assistance of some other mechanisms of field generation, cannot explain the observed nonstationary fields of the Earth, the Sun, convective stars and some other objects. However, the fossil theory is believed to hold for the field of chemically peculiar stars, Kasumov and Allahverdiev [1]. The origin of the so called previous field or of the magnetic fluctuation is still controversial. They are often believed to be either of cosmological origin, if the presence of a primordial magnetic field at the time of the Big- Bang is assumed, or else to issue from a Bierrmann battery effect. The battery effect generates a seed of magnetic field induced by faint electric currents which are emanated, for instance, from pressure or temperature gradients left out in the MHD approximation (cf. Asseo and Sol [2]). Battery theories now seem unlikely to be effective sources of the magnetic field, Dolginov [3], and magnetic oscillator theories appear incapable of reproducing the observed field variations, Parker [4] and Moss [5]. The Bierrmann effect is able to create only a seed field. It is connected with the observed local chemical inhomogeneity in Cp stars, and may be a cause of the field generation.

The dynamo theory is a theory concerning amplification and maintenance of an initial field, and not the generation of a new field. This theory is based on the idea of Larmor, in 1919, that magnetic fields of celestial bodies could be due to dynamo effects. There is no general agreement on the validity of these concepts in their original form. Dolginov [6] summarizes most of difficulties encountered by the dynamo theory; also, Beck [7] discussed the origin of magnetic fields in galaxies, by using observational tests with the square kilometer array.

According to Grand unified theory GUT’s monopoles will occur in cosmology during the early stages of the hot big-bang universe, and estimations give rise to high concentration at that time. High values of the abundance of monopoles would contradict the existence of the present day large scale magnetic fields. The predictions of GUT’s (if it is correct) thus appear to be incompatible with astronomical observation Zeldovich et al. [8]. The Schuster-Wilson-Blackett (SWB) conjecture, suggests that the magnetic fields in planets and stars arise due to their rotation. Blackett [9] suggested that there may be a fundamental relation between the magnetic field of a rotating massive body and its angular momentum. In this scenario, neutral mass currents generate magnetic fields, implying the existence of a non-minimal coupling between gravitational and electromagnetic fields in MWT, Mikhail and Wanas [10].

In section 2, we discuss the formula of Schuster – Wilson – Blackett’s conjecture and in section 3, we discuss a solution of MWT. In section 4, we are going to examine S-W-B statement in MWT. Our conclusions and discussion are presented in section 5.

2 Schuster – Wilson – Blackett’s conjecture

Since 1891 many authors have discussed a gravitational origin of the magnetic
field of rotating celestial bodies. In a series of papers, Sutherland (1900-8) assumed that the earth’s magnetic field is due to its possession of a positive surface charge compensated by a negative surface charge. Schuster [11] gave a survey of the possible theories and their difficulties, and studied in some detail one similar to that of Sutherland, depending on the assumption of unequal forces between like and unlike atomic particles. Brunt [12] surveyed many different theories and also showed that the separation of positive and negative charges in the sun’s gravitational field could not produce more than 10-15 of the sun’s magnetic field. Schuster [11] should that, the magnetic fields of planetary and stellar bodies arise solely from their rotation. That is, that electrically neutral mass current, at least in the case of rotation, generate magnetic fields.

Wilson [13] following a suggestion of Schuster, showed that one may obtain the right order of magnitude for the fields of the earth and the Sun if he assumed that a moving mass element M, measured in gravitational units, has the same magnetic effect as a moving negative charge Q measured in e.s.u. A further work excluded translational motion from the above assessment leaving only rotational motion as a possible source of magnetic fields along this line of thought. Wilson [13] made a laboratory experiment with a swinging iron bar to test his hypothesis that a moving mass with velocity v, even though electrically neutral, produces a magnetic field. He has shown experimentally that the value of the field as given by

\[ B = (-\frac{M}{c^2})V \cdot r = (-\frac{m}{c^2})e \cdot r \]  

(2.1)

was unsatisfactory. Blackett [9] has pointed out that the ratios of magnetic moment to angular momentum for Earth, the Sun, and the star, 78 Virginis, are nearly equal and of a magnitude which suggests a gravitational origin for the magnetic fields of these bodies. He was the first to claim the existence of a correlation between magnetic moment and angular momentum in astrophysical objects. It has been known for a long time, particularly from the work of Schuster, Sutherland and Wilson, though lately little regarded, that the magnetic moment P and the angular momentum U of the earth and sun are nearly the square root of the gravitational constant G divided by the velocity of light c. We can write, in fact,

\[ P = \beta \frac{\sqrt{G}}{c} U \]  

(2.2)

where \( \beta \) is a constant of the order of unity. The magnetic moments P of the earth and the sun are nearly proportional to their angular momenta U, calculated from the expression,

\[ U = \frac{1}{2} \omega M R^2 \]

For a uniformly dense sphere of mass M, radius R and angular velocity \( \omega \), measured values of the polar magnetic fields are also given. These are related to the magnetic moments by the expression,

\[ H_p = \frac{2P}{R^3} \]

Experiments on rotating masses in the laboratory in order to test (2.2) have been performed by Blackett [14] and others Surdin [15], Sirag [16], Woodward [17], de Sabbata and Gasperini [18]. Available observations and theoretical considerations
with respect to the relation (2.2) and other explanations of the origin of the magnetic field of celestial bodies have been reviewed by Biemond [19]. This shows that there is a need to search for such a relation. Existing theories don’t explain the correlation between magnetic and angular momenta. Dynamo and fossil neither predict nor explain the general correlation between magnetic and angular momenta of celestial bodies, which seems to exist as shown in Fig.1. Figure (1) is similar to that given by Dolginov [6]. The curve is plotted using the values of table (1) since Dolginov did not tabulate the observed data he used. The correlation coefficient between magnetic and angular momenta is found to be $r = 0.99$ and the equation of the straight line is: $y = a + bx$, where $a, b$ are calculated using least square method applied to the data of table (1).

![Graph](image)

**Fig.(1):** The relation between the angular momentum ($U$) and the magnetic moment ($P$) relative to those of the Earth.

After his laboratory experiment testing this hypothesis yielded negative results, Blackett [14], gravitational magnetism became largely ignored. Recent magnetic field measurements of the moon, Mercury, and Jupiter, Sirag [16], however, yield ratios quite close to that considered by Blackett. Moreover, Russell [20], who emphasizes only the phenomenology of these ratios and eschews any physical model, adds Venus and Saturn to the list, although these data are tentative and will become firm only with the completion of probes to Venus and Saturn. Blackett himself declared that there may be some hope to discover this relation through one of the unified field theories, which discusses gravitational and electromagnetic effects on the same equal footing. An early attempt to make a theory that encompasses the S-W-B conjecture was made by Pauli [21]. Latter, attempts were made by Papapetrou [22], Luchack [23], Tonnelat [24], McCrea [25], Ahluwalia and Wu [26], Barut and Gornitz [27]. The majority of these
It is the purpose of the present paper to explore this possibility. We are going to use the solution of the generalized field theory, constructed by Wanas [28], for this purpose. We are not claiming that Blackett’s effect, if it finds an interpretation within the framework of any unified field theory, will interpret the magnetic field observed in all celestial objects. But it may interpret the generation of a seed field that evolved, due to another mechanism, to the observed one. In other words Blackett’s effect may represent a mechanism that contributes, with other mechanisms, to produce the observed large scale magnetic fields of astronomical objects.

3 A Solution of the generalized field theory MWT

We use one of the solutions of the generalized field theory, constructed by Wanas [28]. It is given in the form

\[ ds^2 = \frac{\gamma(R) dT^2}{\gamma(R)} - \frac{dR^2}{R^2} - R^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \]  

(3.1)

where

\[ \gamma(R) = 1 - \frac{\alpha}{R} - \frac{4\alpha_i}{R^2} + \frac{2\alpha_i^2}{R^4}. \]  

(3.2)

To give a physical meaning for the constants \( \alpha, \alpha_i \) of the solution (3.1), let \( \alpha_i = 0 \), then expression (3.2) reduced to form the well-known Schwarzschild exterior metric with \( \alpha = 2m \), where \( m \) is the geometric mass of the object (the mass in relativistic units, \( c = G = 1 \)). Compare equation (3.1) with the well-known Reissner-Nordstrom metric for a charged point mass to get,

\[ ds^2 = \frac{\gamma'(R) dT^2}{\gamma'(R)} - \frac{dR^2}{R^2} - R^2 (d\theta^2 + \sin^2 \theta d\varphi^2), \]  

where

\[ \gamma'(R) = 1 - \frac{2m}{R} - \frac{Ke^2}{2R^2}, \]  

(3.3)

for sufficiently large values of \( R \). Thus \( \alpha \) represents the geometric mass, and \( \sqrt{\alpha_i} \) represents the charge of the source in relativistic units (we are going to call it the geometric charge); with \( \alpha = 2m, \alpha_i = \frac{K}{8} e^2 \), where \( m \) is the geometric mass, \( e \) is the electric charge. Taking \( m = \frac{GM}{c^2}, K = \frac{8\pi G}{c^4} \), we get (\( M \) is the mass in grams),

\[ \gamma(R) = 1 - \frac{2GM}{c^2 R} - \frac{4\pi Ge^2}{c^4 R^2}. \]  

(3.4)

Setting the constant \( \alpha_i = 0 \), into the electromagnetic potential, we get
\[ F^{01} = -\frac{9\alpha^{1/2}}{4R^{3/2}}, \]  

and

\[ f^{01} = -\frac{9}{4} \frac{\alpha}{R} \sin \theta. \]  

We study the effect of translation on the present model, to show whether translation is capable of giving any magnetic field or not. We noted that, the translational motion will not give rise to any magnetic components. This result is in agreement with Wilson’s experiment.

By using the angular velocity \( \omega \), then the non-vanishing components are: \( F^{\text{13}} \) (the component of electromagnetic field tensor) defined by,

\[ \vec{F}^{\text{31}} = -\vec{F}^{\text{13}} = \frac{9}{4R^2} \sqrt{\frac{2M}{R}} \omega \]  

and \( \vec{f}^{\text{13}} \) (the component of electromagnetic tensor density) defined by,

\[ \vec{f}^{\text{31}} = -\vec{f}^{\text{13}} = \frac{9}{4} \sqrt{\frac{2M}{R}} \omega \sin \theta \]  

where \( M \) is the mass of body in grams. To distinguish between these quantities we found that the dimensions of \( \vec{F}^{\text{13}} \) is \( \text{cm}^{-1} \) in relativistic units and \( g^{1/2} \text{cm}^{-5/2} \text{sec}^{-1} \) in cgs units. The dimensions of \( \vec{f}^{\text{31}} \) are \( \text{cm}^{-1} \) in relativistic units and \( g^{1/2} \text{cm}^{-1/2} \text{sec}^{-1} \) in cgs units. However, the required quantity (the strength of the magnetic field) should have the dimensions \( \text{cm}^{-1} \) in relativistic units or \( g^{1/2} \text{cm}^{-1/2} \text{sec}^{-1} \) in cgs units. Therefore \( F^{\text{13}} = -\vec{F}^{\text{31}} \) should be ruled out, and we have to use \( \vec{F}^{\text{13}} \) to represent the observed magnetic field since it has the proper dimensions. So, the surface \( (R = R_0) \) polar \( (\theta = \pi/2) \) magnetic field for a rotating body of mass \( M \) grams, radius \( R_0 \) cm, and angular velocity \( \omega \text{ sec}^{-1} \) is given, using (3.8), by

\[ B_p = \frac{9}{4} \sqrt{\frac{2M}{R}} \omega \]  

This formula may give a possible interpretation of a seed magnetic field which will develop to produce the large scale magnetic field observed for celestial objects.

**4 SWB conjecture in MWT**

Wilson [13] assumed that a mass \( m \) moving with velocity \( v \), even though electrically neutral, produces a magnetic field given by the empirical relation

\[ B = \frac{-\sqrt{(G)m}}{Cr^2} v \cdot r. \]
This hypothesis is certainly untrue, as stated by Wilson, if $\nu$ is interpreted as a translational velocity, but probably, there exists a magnetic field due to a rotating body. Blackett [14] introduced a hypothesis about the magnetic field of massive rotating bodies, but this hypothesis was rejected by Blackett himself, as a result of some experiments. The Schuster-Wilson-Blackett conjecture relates the angular momentum $U$ to the magnetic dipole moment $P$:

$$P = \left[ \beta \frac{\sqrt{G}}{2c} \right] U$$  \hspace{1cm} (4.1)

where $\beta$ is approximately a constant, on the order of unity, $G$ is the Newtonian constant of gravitation, and $c$ is the speed of light. The angular momentum $U$ is $U = I \omega$, where $\omega = 2\pi p^{-1}$ is the angular velocity, $p$ the rotational period, and $I$ is the moment of inertia. The dipole moment $P$ is related to the magnetic field $B$ by:

$$B = \frac{2P}{R^3}$$ \hspace{1cm} (4.2)

where $R$ is the distance from $P$ to the point at which $B$ is measured. Since electric charges may move in different ways in rotating bodies, it is to be expected that $\beta$ in (4.1) is not a universal constant. Indeed, different results for $\beta$ were found for fourteen different rotating bodies: metallic cylinders in the laboratory, moons, planets, stars and galaxies. Comparing relation (3.9) and Blackett’s formula (4.1), we get

$$45 \frac{C}{\varphi} \beta M \approx \frac{5}{5R}$$ \hspace{1cm} (4.3)

where

$$\beta = \frac{45C}{8G \varphi}$$ \hspace{1cm} (4.4)

So $\beta$ is not a constant but a parameter depending on the gravitational potential ($\varphi = 2M/R$) of the body. In units of solar mass ($M_0$) and solar radius ($R_0$), we have

$$\beta \approx 2730 \left[ \frac{R_0 M_0}{R M} \right]^{1/2}$$ \hspace{1cm} (4.5)

For a black hole, $M/R$ is approximately a constant and $\beta \sim 5$. For a 2 solar mass neutron star, $\beta \sim 10$.

### 5 Discussions and Conclusion

Different theories have been constructed to account for the origin of large scale magnetic fields, such as the dynamo and fossil theories, Asseo and Sol [2]. We believe that these theories can account only for the amplification and maintenance of the magnetic field, but not for the creation of the original field.
itself. The fossil field is the relic of a primitive magnetic field not yet completely dissipated. The field generated by dynamo results from the amplification of a small seed magnetic field. It is often believed that the seed field is either of cosmological origin, if the presence of a primordial magnetic field at the time of the Big-Bang is assumed, or else to be originated from a Bierman battery effect, Moss [5]. The battery effect generates a seed magnetic field induced by faint electric currents.

Although Blackett’s hypothesis has been collapsed many years ago, some author’s still believe that a fundamental relation must exist between the magnetic moment and angular momentum of a celestial object, or between the magnetic field and rotation of that object (cf. Dolginov [6], McCrea [25], Barut and Gornitz [27], Wanas [28]). This shows that there is a need to search for such a relation. Existing theories do not explain the correlation between magnetic and angular momenta of celestial objects. Dynamo and fossil theories neither predict nor explain the general correlation between magnetic and angular momenta of such objects, which seems to exist as shown in Fig.1. Besides, existing theories neither predict nor explain the general correlation between the magnetic field and the product of the angular velocity and $\sqrt{\varphi}$ (where $\varphi$ is the gravitational potential), which is illustrated in Fig. 2.

![Fig. (2): Dependence of the polar magnetic field on angular velocity and the gravitational potential, (Observed line). The theoretical relation (3.9), (Theoretical line); where $\varphi \equiv \varphi / \varphi_\odot$, $\omega \equiv \omega / \omega_\odot$, $\varphi_\odot$ is the value for the Earth.](image-url)
Thus, the SWB conjecture, equation (3.9), is obtained from MWT. As shown above, formula (3.9) may give a possible interpretation of a seed magnetic field which will develop to produce the large scale magnetic field observed for celestial objects. This formula shows that the origin of the magnetic field is generated as a result of rotation of the massive object.

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References


Table 1: Observed data relevant for the magnetic field.

<table>
<thead>
<tr>
<th>Objects</th>
<th>Angular momentum ( U = \frac{2}{5} \omega M r^2 )</th>
<th>Magnetic moment ( P = 0.5 R B^2 )</th>
<th>( \log(\frac{U}{M}) )</th>
<th>( \log(\frac{P}{R^2}) )</th>
<th>( \log(\frac{\phi}{\omega}) )</th>
<th>( 0.5 \log(\frac{\phi}{\omega}) + \log(\frac{\omega}{\phi}) )</th>
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</thead>
<tbody>
<tr>
<td>White dwarf (WD)</td>
<td>120.69x10^{49}</td>
<td>635.173x10^{77}</td>
<td>10.23</td>
<td>13.89</td>
<td>7.902</td>
<td>2.427</td>
</tr>
<tr>
<td>Neutron Stars (NS)</td>
<td>1.449x10^{48}</td>
<td>0.5x10^{30}</td>
<td>07.31</td>
<td>03.79</td>
<td>12.203</td>
<td>11.488</td>
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<td>Peculiar Star (AP)</td>
<td>903.74x10^{48}</td>
<td>9219.042x10^{77}</td>
<td>10.10</td>
<td>11.05</td>
<td>3.748</td>
<td>1.056</td>
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<tr>
<td>Millisecond Pulsar (Ps)</td>
<td>2.5144x10^{48}</td>
<td>0.5x10^{31}</td>
<td>10.88</td>
<td>04.79</td>
<td>13.2</td>
<td>9.9498</td>
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<td>Galaxy (G)</td>
<td>265.47x10^{75}</td>
<td>280.758x10^{69}</td>
<td>34.57</td>
<td>36.53</td>
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<td>-9.04</td>
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<td>Sun (SUN)</td>
<td>115.68x10^{47}</td>
<td>8934.569x10^{30}</td>
<td>08.21</td>
<td>08.04</td>
<td>1.928</td>
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<td>Mercury (MER)</td>
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<td>5.184x10^{22}</td>
<td>-03.90</td>
<td>-03.20</td>
<td>-1.922</td>
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<td>Venus (VEN)</td>
<td>203.37x10^{46}</td>
<td>0.140x10^{24}</td>
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<td>Earth (E)</td>
<td>715.23x10^{48}</td>
<td>82.051x10^{24}</td>
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<td>0.0</td>
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<td>Mars (MAR)</td>
<td>2.118x10^{39}</td>
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<td>Jupiter (J)</td>
<td>67024.94x10^{41}</td>
<td>1557.22x10^{77}</td>
<td>04.97</td>
<td>04.28</td>
<td>1.162</td>
<td>1.124</td>
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<td>40.932x10^{77}</td>
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<td>02.70</td>
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<td>Uranus (UR)</td>
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<td>01.68</td>
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We have taken the average for all: Mass, Radius, Angular velocity and Polar magnetic field; where \( \phi = 2M / r \) is the gravitational potential and \( \omega = 2\pi / \text{period of rotation} \), \( B_p = 2P / R^3 \).

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