Natural Exit of Fresh Inflation
to a Radiation Dominated Universe

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Abstract

We examine the transition from a fresh inflationary scenario to a radiation dominated universe when the inflaton field is coupled to gravity. We show that this transition is very natural when this coupling is $\xi = 1/4$ at the end of fresh inflation. This is a very important result that shows as fresh inflation has a natural exit to the radiation dominated epoch.

1 Introduction and Motivation

Inflationary cosmology[1, 2] is the most strong candidate to explain the isotropic, homogeneous and flatness nature of the universe on cosmological scales. This fact is supported by experimental evidence[3]. In particular, the model of fresh inflation[4] attempts to build a bridge between the standard and warm inflationary models and hence can be viewed as an unification of both, chaotic and
warm inflation scenarios. In this model the universe begins from an unsta-
ble primordial matter field perturbation with energy density nearly $M^4_p$ and
chaotic initial conditions to later describing a second order phase transition
with heating and particles production. Hence, radiation energy density grows
during fresh inflation because the Yukawa interaction between the inflaton field
and other fields in a thermal bath lead to dissipation which is responsible for
the slow rolling of the inflaton field though the minimum energetic configura-
tion. Hence, slow-roll conditions are physically justified and there are not a
requirement of a nearly flat potential in fresh inflation. As a consequence of
the strong dissipation produced by the Yukawa interaction \( \Gamma \gg H \), there is no
oscillation of the inflaton field around the minimum of the effective potential[5].

A very interesting issue to study is the natural exit of fresh inflation to a
radiation dominated epoch. In this work we examine this topic in the frame-
work of a scalar field which is coupled to gravity though a coupling $\xi$. In this
context one can see that for $\xi = 1/4$ radiation dominance becomes possible
when the scalar field $\phi$ approaches to the minimum energetic configuration.

1.1 Nonminimal coupling in fresh inflation revisited

The system is considered though a Lagrangian density of an inflaton field
which is coupled to the scalar curvature $R$, plus a self-interaction Lagrangian
\( \mathcal{L}_{int} = -g^2 \phi^4 \), which describes the self-interaction of the inflaton field[6]

\[
\mathcal{L} = -\sqrt{-g} \left[ \frac{R}{16\pi G} - \frac{1}{2} \partial^\mu \phi \partial_\mu \phi + V(\phi) + \frac{1}{2} \xi R \phi^2 + \mathcal{L}_{int} \right],
\]

where $G = M^2_p$ is the gravitational constant, $M_p = 1.2 \times 10^{19}$ GeV is the
Planckian mass. During fresh inflation $\xi \ll 1/4$, but at the end of fresh
inflation it can hold to values close to $\xi = 1/4$. In this paper we are interested
to study this particular case. We shall consider a 4D Friedmann-Robertson-
Walker (FRW) metric with a line element given by $ds^2 = dt^2 - a^2(t)dr^2$, such
that $dv^2 = dx^2 + dy^2 + dz^2$. The diagonal Einstein equations are given by

\[
3H^2 = 8\pi G \left[ \left( \frac{1}{2} - 2\xi \right) \dot{\phi}^2 + V(\phi) + \rho_r \right],
\]

\[
3H^2 + 2\dot{H} = -8\pi G \left[ \left( \frac{1}{2} - 2\xi \right) \dot{\phi}^2 - V(\phi) + \frac{\rho_r}{3} \right],
\]
where \( H = \dot{a}/a \) is the Hubble parameter and \( a \) is the scale factor of the universe which is considered with zero spatial curvature. If \( \delta = \dot{\rho}_r + 3FH\rho_r = \Gamma(\theta)\phi^2 \) describes a Yukawa interaction between the inflaton field and the thermal bath, the equations of motion for \( \phi \) and \( \rho_r \), hold

\[
\ddot{\phi} + 3H\dot{\phi} + V'(\phi) + \xi R\phi + \frac{\delta}{\phi} = 0, \tag{4}
\]

\[
\dot{\rho}_r + 3FH\rho_r - \delta = 0. \tag{5}
\]

The parameter \( F = 1 + \omega \), such that \( \omega = p_t/\rho_t \) gives us the equation of state of the universe, or

\[
F = -\frac{2\dot{H}}{3H^2} = \frac{(1 - 4\xi)\dot{\phi}^2 + \frac{2}{5}\rho_r}{\rho_r + \left(\frac{1}{2} - 2\xi\right)\phi^2 + V} > 0, \tag{6}
\]

with a scalar curvature \( R = 12H^2 + 6\dot{H} \). In the early stages of inflation this parameter is very small: \( F \ll 1 \), but at the end of inflation it reaches values close to \( F = 4/3 \), so that, for couplings close \( \xi \simeq 1/4 \), inflation ends in a radiation dominated era with \( a \sim t^{1/2} \). From the inequalities in eq. (6) we obtain

\[
\dot{\phi}^2 \left[ (2 - F) \left(\frac{1}{2} + 2\xi\right) \right] + \rho_r \left(\frac{4}{3} - F\right) - FV(\phi) = 0, \tag{7}
\]

\[
H = \frac{2}{3\int F dt}, \tag{8}
\]

where, because \( \dot{H} = H^2\dot{\phi} \), one obtains

\[
\dot{\phi} = -\frac{3H^2}{2H^2}F. \tag{9}
\]

By replacing eq. (9) in eq. (7) and later in eq. (3), we obtain respectively the equations for the radiation energy density and the potential \( V(\phi) \)

\[
\rho_r = \left(\frac{3F}{4 - 3F}\right) V(\phi) - \frac{27}{4} \left(\frac{H^2}{H'}\right)^2 F^2 \left[ \frac{(2 - F)}{(4 - 3F)} \right] \left(\frac{1}{2} + 2\xi\right), \tag{10}
\]

\[
V(\phi) = \frac{3(1 - 8\pi\xi G\phi^2)}{8\pi G} \left[ \left(\frac{4 - 3F}{4}\right) H^2 + \frac{3\pi}{2} \frac{G F^2}{(1 - 8\pi\xi G\phi^2) \left(\frac{H^2}{H'}\right)^2} \right] (1 + 8\xi). \tag{11}
\]

\(^1\text{Other phenomenological interaction terms as } \delta \propto \phi^2\dot{\phi}^2[7] \text{ or } \delta \propto \phi^{\delta - 2}\dot{\phi}^4[8], \text{ has been proposed in the literature.}\)
1.2 The model

Fresh inflation is described by a global group $O(n)$ that involve a single $n$-vector multiplet of scalar fields $\phi_i$ \cite{4}, where the effective inflaton field is describes by the norm of their scalar components: $(\phi^i \phi_i)^{1/2}$. The effective potential is described by $V_{eff}(\phi, \theta) = V(\phi) + \rho_r(\phi, \theta)$, such that

$$V_{eff}(\phi, \theta) = \frac{M^2(\theta)}{2} \phi^2 + \frac{\lambda^2}{4} \phi^4,$$

where the effective squared mass $M^2(\theta) = M^2(0) + \frac{(n+2)}{12} \lambda^2 \theta^2$, $M^2(0) \approx 10^{-12} M_p^2$ and $n$ is the number of created particles due to the interaction of $\phi$ with the particles in the thermal bath

$$(n + 2) = \frac{2\pi^2}{5\lambda^2 g_{eff} \theta^2}. \quad (13)$$

During the fresh inflationary epoch $\rho_r \simeq 0$, so that $\rho_r$ increases with the time. Therefore, the number of particles created is increased dramatically because $\phi(t) = 1/(\lambda t)$, but the temperature $\theta \sim \rho_r^{1/4}$ increases with time. This means that the ratio $\frac{\theta^2}{\phi^2}$ will be increased as fresh inflation evolves.

1.3 Dynamics of the fluctuations $\delta \phi(\vec{x}, t)$

The dynamics of the background scalar field $\phi(t)$ and the fluctuations $\delta \phi(\vec{x}, t)$ are described respectively by the equations

$$\ddot{\phi} + (3H + \Gamma) \dot{\phi} + \xi R \phi + V'(\phi) = 0, \quad (14)$$

$$\ddot{\delta \phi} - \frac{1}{a^2} \nabla^2 \delta \phi + (3H + \Gamma) \delta \phi + [\xi R + V''(\phi)] \delta \phi = 0, \quad (15)$$

such that the redefined fluctuations $\chi(\vec{x}, t) = a^{3/2} e^{\frac{2}{3} \Gamma t} \delta \phi(\vec{x}, t)$ is can be written as a Fourier expansion

$$\chi(\vec{x}, t) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left[ a_k \chi_k(\vec{x}, t) + a_k^\dagger \chi_k^* (\vec{x}, t) \right], \quad (16)$$

where $a_k$ and $a_k^\dagger$ are the annihilation and creation operators that comply with the commutation rules:

$$[a_k, a_{k'}^\dagger] = \delta^{(3)}(\vec{k} - \vec{k'}), \quad [a_k^\dagger, a_{k'}] = [a_k, a_{k'}^\dagger] = 0. \quad (17)$$
The dynamics of the time dependent modes $\chi_k(t)$ being given by
\[ \ddot{\chi}_k + \left\{ \frac{k^2}{a^2} - \left[ \frac{9}{4}(H + \Gamma/3)^2 + 3 \left( 1 - 2\xi \dot{H} + \dot{\Gamma}/3 \right) - \left[ 24\xi H^2 + V''[\phi(t)] \right] \right\} \chi_k(t) = 0. \]
(18)

During inflation the infrared (IR) sector includes long wavelength modes with $k < k_0$ which are unstable.

2 Radiation after inflation

Since the decay width of the produced particles grows with time, when the inflaton approaches the minimum of the potential, there is no oscillation of $\phi$ around the minimum energetic configuration due to at the end of fresh inflation dissipation is dominant with respect to rate of expansion of the universe: $(\Gamma \gg H)$. Hence, the reheating period avoids fresh inflation. At the end of fresh inflation one would expect that the universe becomes radiation dominated.

To examine this possibility we consider the case with $F \simeq 4/3$, such that the inflaton field is coupled to gravity with $\xi = 1/4$. This will be possible at the end of fresh inflation when $V(\phi) \ll \rho_r$ and $\dot{\rho}_r + 4F\rho_r \simeq 0$. Hence the equation (6) holds
\[ F \simeq \frac{4}{3}\frac{\rho_r}{\rho_r + V} \simeq 4/3. \]
(19)

For a number of degrees of freedom for relativistic particles of the order of $g_{\text{eff}} \simeq 10^2$ one obtains that the temperature reaches values like
\[ \theta_{\text{Rad}} \equiv \theta(t)| \simeq 2.5 \times 10^{-5} \, M_p, \]
(20)

which is a value very close to the GUT-temperature value and corresponds to a radiation energy density $\rho_{\text{Rad}}^{(0)} \simeq 1.28 \times 10^{-18} \, M_p^4$. The effective friction parameter at this moment will be $\Gamma_{\text{Rad}} \simeq \frac{g_{\text{eff}}^4}{192\pi^2} \theta_{\text{Rad}}$, which can take the value
\[ \Gamma_{\text{Rad}} \simeq 4M_p. \]
(21)

After it, the radiation energy density begins to decrease as $\rho_r(t) = \rho_{\text{Rad}}^{(0)}(t/t_0)^{-2}$ and so that $\rho_r < 0$ decreases in a such manner that the temperature decreases as $\theta_r(t) = \theta_{\text{Rad}}(t/t_0)^{-1/2} \sim a^{-1}(t)$, which is that one expects in a radiation
dominated universe. At this epoch the effective equation of motion (18) for the modes is

$$\ddot{\chi}_k + \left\{ \frac{k^2 t_0}{t} - M_p^2 \right\} \chi_k(t) = 0,$$

when we have made use of the fact that $\Gamma_R^2 \gg (H^2, \dot{H}, \dot{\Gamma}, V'')_{t=t_R}$. The general solution for this equation is

$$\chi_k(t) = C_1 M \left[ \frac{k^2 t_0^2}{2M_p}, \frac{1}{2}, 2M_p t \right] + C_2 W \left[ \frac{k^2 t_0^2}{2M_p}, \frac{1}{2}, 2M_p t \right],$$

where $(M, W)$ are the Whittaker functions. In the UV sector of the spectrum the modes are stable. However, for $k < M_p \left( \frac{t_0}{t} \right)$ the this modes are unstable and the dominant part of the solution (23) is

$$\chi_k(t) \simeq C_1 M \left[ \frac{k^2 t_0^2}{2M_p}, \frac{1}{2}, 2M_p t \right].$$

However, notice that the modes $\xi_k(t) = a^{-3/2} e^{-f \Gamma_R t}$ increase at early radiation times, but after it decay to zero with time without crossing the Hubble horizon. It can be seen in the figure (1) for the values $\frac{k^2 t_0^2}{2M_p} = (0.00001; 0.1)$.

3 Final Comments

Reheating is not a minor phase at the end of standard inflation. Standard inflation may cause the monopole and domain wall nonthermal symmetry restoration[9], which could give a very inhomogeneous universe that disagree with observation. This last possibility is actually rather difficult in simple models of reheating with only two fields[10]. However, this situation can be solved in the case of multiple fields, relevant for GUT models[11]. This is the case here studied, where the number of created particles can be justified by means of the self-interaction of the inflaton. As in the case of warm inflation[5, 12, 13], the main difference between fresh inflation and standard inflation resides in that here there is not oscillation of the inflaton field around the minimum of the potential, due to dissipation is too large at the end of inflation. Here, particles creation becomes during inflation, beginning it at zero temperature.

In this work we have examined the natural exit from a fresh inflationary scenario to a radiation dominated universe when the inflaton field is coupled to gravity. In the particular case where the coupling constant is $\xi = 1/4$ it
is possible to see that this transition is very natural at the end of inflation in such that manner that the parameter $F = 4/3$ when the inflaton field reaches its minimum energetic configuration. At this moment all the modes of the inflaton field fluctuations are stable and the background temperature is close to the GUT one. However only oscillate the modes with wavelength below the Planckian scale. The modes which are in the classical limit are all stable but they increase to thereafter go down to zero [see figure (1)]. This is because at this time the inflaton field is strongly coupled to gravity and the modes decay very faster than during inflation due to they are super-damped by friction.

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References


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Figure 1: Temporal evolution of the cosmological modes $\xi_k(t)/C_1$ for different values of parameters $\frac{k^2t^2}{2M_p} = (0.00001; 0.1)$, plotted respectively with line and dots.