Mathematical Modelling of Error Contribution for Various Dimensions of Capacitive Sensors from Centimetric to Nanometric Range

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Abstract

Aim of this paper is mathematical analysis of the contribution of fringing effect in sensing performance of electrostrictive capacitive sensors. Study on capacitive sensors having separation between plates in nanometric range and area in centimetric range reveals that error due to fringing effect is of the order of $10^{-5}$ and therefore can be neglected particularly for area greater than $5 \times 10^{-4}$ sqm. In case of nanometric capacitive sensors having both area of plates and separation between them in nanometric range, the percentage error due to fringing effect has been found to be as high as 8.53% and therefore cannot be ignored for sensitivity consideration of capacitive sensors. The study also includes variation of fringing effect with distance between plates and area of plates in micrometric and millimetric capacitive sensors and it is observed that error is on higher side to be ignored. Few of capacitive sensors exhibiting minimum contribution from fringing effect have also been suggested for constructional design.
Keywords: fringing effect, compliant electrode, capacitive sensors, guarding

1 Introduction

Leakage of lines of electric flux from edges of a parallel plate capacitor is called fringing effect. This leakage of flux plays a very important role in performance of the capacitor. Estimation of error due to fringing effect will provide a guideline to the researchers in designing a capacitive sensor of different dimensional designs. A parallel plate capacitive sensor constitutes the basic design elements with simple possible configurations and its performance can be easily analyzed [3]. Capacitance of this capacitor gets affected by fringing effect as leakage of flux also reduces the electric field inside the material. This in turn reduces the performance of the capacitive sensor. In few cases, this fringing effect is negligible but for many cases it has been observed that contribution of fringing effect is significant and hence can’t be ignored.

Capacitive sensors find applications in various industrial activities and it is desirable to construct a sensor exhibiting negligible contribution from fringing effect. The contribution due to fringing effect estimated up to second term is less than 1% of total capacitance and was ignored by the researchers [6] without going into details of mathematical analysis and its effect on the performance of a capacitive sensor. In this paper an attempt has been made to prepare a mathematical model of sensor in nano and micro range, with a purpose to find constructional design details of a sensor exhibiting negligible contribution from fringing effect.

The capacitance, \( C \) of capacitor with an area \( A \) and a gap \( d \) between the plates is [6]:

\[
C = \frac{\varepsilon \varepsilon_0 A}{d} \left[ 1 + O \left( \frac{\log \left( \frac{\sqrt{A}}{d} \right)}{2 \sqrt{\frac{A}{d}}} \right) \right]
\]

(1)

Here, \( \varepsilon \) is the dielectric constant of the material between the plates, \( \varepsilon_0 \) is the dielectric permittivity of free space, \( A \) is the area of plates of sensor and \( d \) is the separation between the plates.

2 Fringing effect in Electrostrictive Capacitive Sensor-Mathematical Modeling

While preparing a mathematical model of fringing effect, error due to fringing effect up to second order has been taken in consideration and then differentiating equation (1) with respect to three different variables \( d \), \( A \), \( \varepsilon \) taking all three
variables as independent we get,

\[
\frac{\partial C}{\partial d} + \frac{\partial C}{\partial A} + \frac{\partial C}{\partial \varepsilon},
\]

(2)

\[
\frac{\partial C}{\partial C} = -\frac{\varepsilon \varepsilon_0 A}{d^2} + \frac{\varepsilon \varepsilon_0}{d} + \frac{\varepsilon \varepsilon_0 A}{d} + \frac{\varepsilon \varepsilon_0}{2} \left( \frac{1}{\sqrt{A}} - \frac{\varepsilon}{\sqrt{A}} \right) \\
+ \frac{\varepsilon_0}{2} \log \left( \frac{\sqrt{A}}{d} \right) \left( -\varepsilon + \frac{\varepsilon}{2 \sqrt{A}} + \frac{\varepsilon d}{2 \sqrt{A}} + \sqrt{A} \right) \\
+ \frac{\varepsilon_0}{4} \left( \log \left( \frac{\sqrt{A}}{d} \right) \right)^2 \left( \varepsilon + d \right).
\]

(3)

Polyphthalamide (PPA), a electrostrictive material with dielectric constant 4.4 at frequency $10^6$Hz has been used in making calculations and three ranges, Nanometric, Micrometric and Milli metric range have been considered. Percentage error has been calculated by formula:

$$((S-C)/S) \times 100$$

S is the value of capacitance without fringing effect and C is the value of capacitance with fringing effect.

Two sets of graphs have been plotted first between percentage error and area of plates of sensor and second between percentage error and separation between plates.

Two different cases have been studied for Nanometric range: 1. when only separation (d) between plates is in Nanometric range but area (A) in centimetric range. 2 When both area (A) of plates and distance between them (d) are in Nanometric range. The interface between two media i.e. solid- solid interface, solid – liquid interface or liquid-liquid interface are good examples of second case [5]. The interfacial region, here, behaves like a high energy storage capacitor or supercapacitor where the separation (d) is in nanometric range [1].
3 Other Cases Studied are When Both Area and Separation are in Micrometric Range and When Both Area and Separation are in Millimetric Range.

Figure 1: Plot between area (centimetric range) of plates and percentage error due to fringing effect for two values of separation i.e. for $d=10^{-9}$ m and $d=10^{-10}$ m.

Figure 2: Plot between area of plates in nanometric range and percentage error for two values of separation i.e. for $d=10^{-9}$m and $d=10^{-10}$m.
Figure 3: Plot between area of plates (micrometric range) and percentage error for two separations i.e. $d=10^{-5}$ m and $d=10^{-6}$ m.

Figure 4: Plot between separation between plates (micrometric range) and percentage error for two values of area i.e. for $A=10^{-8}$ sqm and $A=25\times10^{-8}$ sqm.
4 Discussion and Result

The above results are based on wide range of separation between plates (d) and area of plates (A) varying from nanometric range to centimetric range and errors due to fringing effect in case of parallel plate capacitive sensors have been categorized as follows for different values of
(a) Separation between plates in nanometric range and area of plates in centimetric range as in Fig.1.

An example of such a case is the interface between electrodes and the electrolyte. Here the capacity of the capacitor formed at the interface is very high i.e. it behaves like a supercapacitor. It has been observed that the error due to fringing effect can be neglected for all practical purposes particularly for the area of plates greater than $0.4 \times 10^{-3}$ sqm at separation equal to $10^{-10}$ m as percentage error is of the order of $10^{-5}$. Preference is to be given for area more than $0.4 \times 10^{-3}$ sqm for a separation of $10^{-10}$ m. For distance equal to $10^{-9}$ m preferable area would be above $0.5 \times 10^{-3}$ sqm.

(b) Both area and separation between plates in nanometric range as in Fig.2.

In this case of nanocapacitive sensor, where both separation and area of plates are in nanometric range, fringing effect plays an important role as the percentage error is very high. Here two curves are there for $d = 10^{-9}$ m and $d = 10^{-10}$ m. Percentage error is going up to 8.53% for $d$ equal to $10^{-9}$ m and area $10^{-17}$ sqm. Also the area of capacitor should be kept in range of $0.1 \times 10^{-17}$ to $0.2 \times 10^{-17}$ sqm for percentage error to be lowest for $d$ equal to $10^{-9}$ m for designing the sensor. For separation $10^{-10}$ m max percentage error goes above 5%. Preferred area should be above $0.6 \times 10^{-17}$ sqm for separation of $10^{-10}$ m for the construction of sensor. The nature of plots interchanges at area equal to $0.2 \times 10^{-17}$ sqm indicating typical behavior.

(c) Both area and separation in micrometric range:

In Fig.3, for separation of $10^{-6}$ m preferable area of plates should be above $1 \times 10^{-8}$ sqm as percentage error is very less. Similarly, for distance of $10^{-5}$ m, percentage error is less in the areas greater than $1 \times 10^{-8}$ sqm. In Fig.4, capacitive sensor with area $25 \times 10^{-8}$ sqm is preferable with $d$ less than $0.4 \times 10^{-6}$ m as percentage error in this range is less.

(d) Both area and separation in millimetric range as in Fig.5.

For separation equal to $5 \times 10^{-3}$ m, it is better to use capacitive sensor with area less than $1 \times 10^{-4}$ sqm. However if area is more than $1 \times 10^{-4}$ sqm, preference would be given to the capacitive sensor with $d$ equal to $2 \times 10^{-3}$ m for designing capacitive sensor. Here the two curves interchange their behavior at area = $1 \times 10^{-4}$ sqm.

In Fig.6, for capacitive sensor with area = $0.5 \times 10^{-4}$ sqm separation should be greater than $4.25 \times 10^{-3}$ m to limit the percentage error. For area = $6.25 \times 10^{-4}$ sqm separation should be less than $4.25 \times 10^{-3}$ m as percentage error is less in this range for constructional design of capacitive sensor.

There are ways, other than geometry of capacitive sensor i.e. area of plates and separation between plates, to reduce error due to fringing effect. One such way is the use of cylindrical shaped capacitors. Cylindrical shaped capacitors have less fringing effect due to little loss of flux in comparison to parallel plate capacitor and that is the reason cylindrical capacitor is preferred in many sensor applications. Use of compliant electrode is another way of reducing fringing effect. When external electric field is applied, the electroelastic material between electrodes show some strain and thus it is no longer in uniform contact with the
electrode resulting in increased fringing effect. Use of compliant electrodes [4] creates no gap between plate and material and hence reduces the fringing effect.

Fringing effect can also be reduced by use of guards in sensors [2]. For example, in cylindrical shaped capacitive sensor (Fig.7) with electroelastic material between electrodes, two circular discs of the same material as the sample are placed near the circular edges of the cylindrical sample. These discs which are working as guards are connected to the outer electrode of the capacitive sensor. Flux passes from first electrode to the sample and from the sample it has to pass to second electrode but all the flux doesn’t take this route and some flux is leaked through the edges and this leaked flux is taken by these two guards and sent to second electrode neutralizing fringing effect.

![Cylindrical capacitor](image)

Figure 7: Schematic diagram of cylindrical capacitive sensor supported by guards to reduce fringing effect.

5 Conclusion

The above study has given some range of area of plates and distance between plates in respect of parallel plate capacitors where error contribution of fringing effect is minimum. In respect of nano capacitors, where both separation between plates and area is in nanometric range, the error contribution from fringing effect is quite high and hence can’t be ignored for sensitivity mapping consideration. On the other hand, it has been observed that, for parallel plate capacitive sensor with separation in nanometric range and area in centimetric range, as in case of electric double layer capacitive sensor, the error from fringing effect is negligible and hence can be ignored. But for micrometric and millimetric range capacitive sensors’ fringing effect play an important role. Therefore, we should definitely consider fringing effect before making any mathematical modeling of capacitive sensor, keeping in mind the geometrical dimensions of the sensor as discussed above. However, it is impossible to reduce error to a zero value as a result of fringing effect due to some limitations of structure, shape and dimensions of capacitive sensor, but in view of enormous industrial applications the error can be minimized to a lowest value keeping in mind thorough mathematical analysis.
during constructional design of capacitive sensor as discussed in this paper. As discussed above, there are other methods to reduce error resulting on account of fringing effect.

References


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