On the Origin of the Universe of Matter and Light in a Combined Four-Manifold of Particle-Waves

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Abstract


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1 Introduction

This paper continues to work on our previously proposed spacetime geometry of a "combined 4-manifold $M[3]$" (see [7, 8, 9, 10, 11]), where the universe $M[3]$ of particles coincides with the universe $B \subset M[2]$ of electromagnetic waves, $B$ being a cosmic black hole contained in $M[2]$; here we focus on the birth of $B \subset M[2]$ and how the center of mass, singularity $0 \in B$ opened up $M[3]$ in the process of $M[2] \rightarrow Big\ Bang\ M[1] \times B \subset M[1] \times M[2]$ (for the formal recognition of the Big Bang theory, see [12]; for its latest quantum version that has become known as the Big Bounce, see, e.g., [1, 5, 6]). A brief summary of our theoretical development of the model is in order (with the analytical details presented in Section 2):
(1) We began with two independent sets of Einstein Field Equations, each with its own gravitational constant, \( G[i] \), \( i = 1, 2 \); we unified the two sets of gravitational motions by

\[
\]

via a consideration of the form-invariance of

\[
g_{11}^{[i]} = 1 - \frac{2G[i]M[i]}{rc^2}, \quad \forall i = 1, 2, 3.
\]

(2) We distributed the energy \( E[3] \) carried by a (particle, wave) into

\[
\left( \frac{3}{4} E[3], \frac{1}{4} E[3] \right)
\]

by a consideration of Feynman’s analysis of the electromagnetic mass.

(3) We revised the Planck energy formula into

\[
\hat{E} = h\nu + \frac{h}{\nu \sec^2}, \quad \text{where} \quad \hat{E} = \frac{10}{16} E[3];
\]

in the process we settled

\[
G[2] = \frac{c^5 \cdot \text{second}^2}{1.6h} \quad (h \equiv \text{the Planck constant})
\]

\[
\approx 2.3 \times 10^{75} \times \frac{\text{meter}^3}{\text{kilogram} \times \text{second}^2} \approx 10^{85} G[3] \approx 10^{85} G[1],
\]

which led to a Schwarzschild radius \( r_B \) for \( B \) at least \( 10^{108} \) meters \( \gg 10^{26} \) meters = the radius of \( \mathcal{M}[1] \) and thus \( \mathcal{M}[1] \) can be embedded in \( B \).

(4) We established the spinning motion of a free electron-wave to be that of a path formed by two semi-circles that are connected perpendicularly by solving the Dirac equation; in the process we settled the diffeomorphic relation between \( \mathcal{M}[1] \) and \( B \) to be (for the idea of using a mirror world to restore parity violation, see [4])

\[
\mathcal{M}[3] = \{(t + ti, x + yi, y + zi, z + xi)\}.
\]

In addition, we showed how an electron and a positron could annihilate each other.

To a large extent, our present paper will build on the above (4) to show how photons could create electrons and positrons at Big Bang (for Big Bang nucleosynthesis, see, e.g., [2]).
To provide the background of our analysis, we would like to make a verbal description of the evolution of

$$\mathcal{M}^2 \rightarrow \text{Big Bang} \mathcal{M}^1 \times \mathcal{B} \subset \mathcal{M}^1 \times \mathcal{M}^2.$$  

Prior to Big Bang (for other pre-Big-Bang theories, see, e.g., [14]) there had been a universe $\mathcal{M}^2$ of electromagnetic waves. By the astronomically large $G^2$ a cosmic black hole $\mathcal{B}$ came into being. $\mathcal{B}$ attracted electromagnetic waves in $\mathcal{M}^2$ toward its center of mass $0 \in \mathcal{B}$. We consider a wave $\lambda$ traveling from the exterior of $\mathcal{B}$ to the interior of $\mathcal{B}$ and finally to $0 \in \mathcal{B}$. By the Schwarzschild metric: (1) as $r_\lambda \rightarrow r_\mathcal{B}$, the proper time $t_0^2(\lambda)$ slows down (relative to the frame $\{(t, r)\}$ in Equation (2), with $g_{11}^2 = \left( \frac{\partial t_0^2}{\partial t} \right)^2$); (2) at $r_\lambda = r_\mathcal{B}$, $\left( \frac{\partial t_0^2}{\partial t} \right) = 0$; (3) when $r_\lambda \in (0, r_\mathcal{B})$, $t_0^2(\lambda)$ begins to run faster and faster in the unit of $\sqrt{-1} \cdot \text{second}$, where

$$1 \ i \cdot \text{sec} \equiv 1 \ \text{cycle} = \frac{1}{\nu}, \ \text{and}$$

$$1 \ i \cdot \text{meter} = (3 \times 10^8)^{-1} \frac{c \cdot i \cdot \text{sec}}{\nu} \equiv \frac{\text{meter}}{\nu \cdot \text{sec}} \quad \text{(9)}$$

$$= (3 \times 10^8)^{-1} \frac{\text{meter}}{\nu \cdot \text{sec}} \equiv \lambda \quad \text{(11)}$$

we may imagine $\lambda$ as a vehicle traveling on a highway from "Exit 1" to "Exit 30" but only to have "Exit 30" $\equiv$ "Exit 1," i.e., while $\lambda$ is traveling linearly from "Province 1" to "Province 2" to ... to "Province $n$," the fact of the matter is that "Province $m - 1$" $\equiv$ "Province $m$." As for $\sqrt{-1} \cdot t$, our daily identification of $24 : 00 \equiv 0 : 00 \ \text{hour}$ serves as an example. Now at some point in time the center of mass of $\lambda$ hit $0 \in \mathcal{B}$, with a unique wave length

$$\lambda^* = 2.56 \times 10^{-35} \ \text{meter} (= \text{Planck length}, \ \text{Equation (46) below}).$$

Then the infinite energy density at $0 \in \mathcal{B}$ transformed $\lambda^*$ into a (photon $\gamma^*$, wave $\lambda^*$), with $\gamma^*$ at $0 \in \mathcal{B}$. By the spherical symmetry surrounding $0 \in \mathcal{B}$, there must have been a whole host of photons coinciding at $0 \in \mathcal{B}$, opening $\mathcal{M}^1$ (Big Bang) and thus the combined 4–manifold $\mathcal{M}^3 = \mathcal{M}^1 \times \mathcal{B}$. (As a parenthetical note, this transfusion of wave energies from $\mathcal{B}$ into $\mathcal{M}^1$ through $0 \in \mathcal{B}$ by logic must be a continuing process from Big Bang into the future; cf. e.g., [3,13], for the confirmed accelerated expansion of $\mathcal{M}^1$). A photon of energy $\frac{2}{3} E[3]$ located at $0 \in \mathcal{B}$ made a spacetime singularity of $\mathcal{M}^1$ as a mini black hole $b$ of radius

$$r_b^* = \frac{\lambda^*}{e} \quad \text{(in Equation (48) below).}$$
Then $E^{[1]} = \frac{3}{4}E^{[3]}$ resided in $(-r^*_b, r^*_b)$, and $E^{[2]} = \frac{1}{4}E^{[3]}$ resided in $[-er^*_b, -r^*_b] \cup [r^*_b, er^*_b]$.

The above introductory description will be presented analytically in the following Section 2.1, followed by an analysis of the pair-creation process of electrons and positrons at Big Bang in Section 2.2. Section 3 will then make a summary remark.

2 Analysis

2.1 The electromagnetic wave length at Big Bang

To provide a self-contained presentation, we will first repeat (compactly) our previously derived results and label them as lemmas that will lead to our Proposition in this paper.

Lemma 1 The recognized gravitational constant

$$G^{[3]} = \frac{G^{[1]}G^{[2]}}{G^{[1]} + G^{[2]}},$$ (12)

where $G^{[i]}, i = 1, 2$, comes from Einstein Field Equations

$$R^{[i]}_{\mu\nu} - \frac{1}{2}R^{[i]}g^{[i]}_{\mu\nu} = -\frac{8\pi G^{[i]}}{c^2} T^{[i]}_{\mu\nu}.\quad (13)$$

Proof. $\forall i = 1, 2, 3, \quad g^{[i]}_{11} = 1 - \frac{2G^{[i]}M^{[i]}}{rc^2}$, so that

$$g^{[3]}_{11} = 1 - \frac{2G^{[3]}M^{[3]}}{rc^2}$$

$$= \left(\frac{G^{[2]}}{G^{[1]} + G^{[2]}}\right) \left(1 - \frac{2G^{[1]}M^{[1]}}{rc^2}\right)$$

$$+ \left(\frac{G^{[1]}}{G^{[1]} + G^{[2]}}\right) \left(1 - \frac{2G^{[2]}M^{[2]}}{rc^2}\right)\quad (14)$$

implies that

$$G^{[3]} = \frac{G^{[1]}G^{[2]}}{G^{[1]} + G^{[2]}} \quad \text{and} \quad M^{[3]} = M^{[1]} + M^{[2]}.$$
Remark 1 By Feynman’s analysis of the electromagnetic mass, we set the energy carried by a particle-wave

\[ E^{[3]} = E^{[1]} + E^{[2]} = \frac{3}{4}E^{[3]} + \frac{1}{4}E^{[3]} \tag{15} \]

Lemma 2 A laboratory-measured energy

\[ \hat{E} = \frac{10}{16}E^{[3]} \tag{16} \]

Proof. \( m^{[3]}a^{[3]} = -\left[ \left( \frac{G^{[2]}}{G^{[1]}+G^{[2]}} \right) \left( \frac{G^{[1]}M^{[1]}m^{[1]}}{||r||^2} \right) + \left( \frac{G^{[2]}}{G^{[1]}+G^{[2]}} \right) \left( \frac{G^{[2]}M^{[2]}m^{[2]}}{||r||^2} \right) \right] \left( \frac{r}{||r||} \right), \)

or

\[ a^{[3]} = -\frac{G^{[3]}M^{[3]}}{||r||^2} \cdot \left( \frac{M^{[1]}m^{[1]}}{m^{[3]}} + \frac{M^{[2]}m^{[2]}}{m^{[3]}} \right) \left( \frac{r}{||r||} \right) \]

\[ = -\frac{G^{[3]}M^{[3]}}{||r||^2} \cdot \left[ \left( \frac{3}{4} \right)^2 + \left( \frac{1}{4} \right)^2 \right] \left( \frac{r}{||r||} \right) \]

\[ \equiv -\frac{G^{[3]}M^{[3]}}{||r||^2} \cdot \left( \frac{r}{||r||} \right) \tag{17} \]

i.e.,

\[ \hat{M} = \frac{10}{16}M^{[3]}, \text{ or } \hat{E} = \frac{10}{16}E^{[3]} \]

\[ \square \]

Remark 2 Accordingly,

\[ E^{[3]} = 1.6\hat{E}, \quad E^{[1]} = 1.2\hat{E} \equiv \phi^{[1]}\hat{E}, \quad \text{and } E^{[2]} = 0.4\hat{E} \equiv \phi^{[2]}\hat{E}. \tag{18} \]

Lemma 3 (cf. [9])

\[ \hat{E} = h\nu + \frac{h}{\nu \text{sec}^2}, \tag{19} \]

where \( \frac{h}{\nu \text{sec}^2} \) = the heat energy from \( \lambda \equiv \frac{C}{\nu} \).

Proof. Let a reference frame S be given. Consider an electromagnetic wave of length \( \lambda \). By the periodicity of \( \lambda \), we identify \([0, \lambda] \equiv [0, 2\pi]_\theta\) and form a circle \( S^1 \) of radius \( \frac{\lambda}{2\pi} \) on the \( y - z \) plane. Then the wave can be represented
by \((ct, \frac{\lambda}{2\pi} \cos \omega t, \frac{\lambda}{2\pi} \sin \omega t)\), which contains energy \(\hat{E}\). Define an equivalence relation \(\sim_{\theta} \) on \(S^1\) by

\[
\theta_p \sim_{\theta} \theta_q \text{ if } \cos \theta_p = \cos \theta_q, \tag{20}
\]

and partition \(S^1\) into \(S^{1*} := \{\{\theta_p, \theta_q\} \mid \theta_p \sim_{\theta} \theta_q \in S^1 \equiv [0, 2\pi]\}\). Then a motion from \(\theta = 0\) to \(2\pi\) is equivalent to a motion from \(\theta = 0\) to \(\pi\) and back to 0. As such, we have a simple harmonic motion \(\xi(t)\) of any representative point mass \(\hat{m} = \frac{\hat{E}}{c^2}\) that moves as a pendulum between \(\theta = 0\) and \(\pi\). At \(\theta = \frac{\pi}{2}\), \(\hat{m}\) is at \((y, z) = \left(0, \frac{\lambda}{2\pi}\right)\) and at \(\theta = 0, \pi\), \(\hat{m}\) is at \((y, z) = \left(\pm \frac{\lambda}{2\pi}, 0\right)\), so that the maximum displacement from \(\theta = \frac{\pi}{2}\) is \(\|\left(0, \frac{\lambda}{2\pi}\right) - \left(\pm \frac{\lambda}{2\pi}, 0\right)\| = \sqrt{2} \cdot \frac{\lambda}{2\pi}\). Thus,

\[
\xi(t) = \sqrt{2} \cdot \frac{\lambda}{2\pi} \cos \omega t, \tag{21}
\]

with the associated potential energy

\[
PE(\xi(t)) = \frac{1}{2} \hat{m} \omega^2 \xi^2(t) = \frac{1}{2} \hat{m} \omega^2 \left(\sqrt{2} \cdot \frac{\lambda}{2\pi} \cos \omega t\right)^2 = \hat{m} c^2 \cos^2 \omega t = \hat{E} \cos^2 \omega t, \tag{22}
\]

implying that in particular the average potential energy

\[
PE_{avg} = \frac{\hat{E}}{2}. \tag{23}
\]

Yet

\[
\frac{2PE(\xi(t))}{\xi^2(t)} = \hat{m} \omega^2 = \hat{m} (2\pi \nu)^2, \tag{24}
\]

so that

\[
\nu^2 = \frac{2PE(\xi(t))}{4\pi^2 \hat{m} \xi^2(t)}. \tag{25}
\]

Since

\[
PE\left(\xi(t) = \frac{\lambda}{2\pi}\right) = \frac{1}{2} \hat{m} \omega^2 \left(\frac{\lambda}{2\pi}\right)^2 = \frac{1}{2} \hat{m} \left(\frac{\omega \lambda}{2\pi}\right)^2 = \frac{1}{2} \hat{m} c^2 = \frac{\hat{E}}{2} = PE_{avg}, \tag{26}
\]

we have

\[
\nu^2 = \frac{2PE_{avg} \hat{m}}{4\pi^2 \hat{m} \left(\frac{\lambda}{2\pi}\right)^2} = \frac{2PE_{avg}}{\hat{m} \cdot \left(\frac{\nu}{\nu}\right)^2}, \tag{27}
\]
so that

\[ PE_{avg} = \frac{1}{2} \left( \frac{\hat{m}c^2}{\nu} \right) \cdot \nu = \left( \frac{\hat{E}/2}{\nu} \right) \cdot \nu. \quad (28) \]

Since \( \left( \frac{\hat{E}/2}{\nu} \right) \) has a unit of \((\text{joule} \cdot \text{sec})\), it is a relativistic invariant = \((\gamma_L \cdot \text{joule})\) \((\gamma_L^{-1} \text{sec})\) through a Lorentz factor \(\gamma_L\). Also, \( \left( \frac{\hat{E}}{\nu} \right) \) is the energy contained in one \((\text{cycle} \cdot \text{second})\). By these two considerations, \( \left( \frac{\hat{E}/2}{\nu} \right) \) is a constant and we identify it with Planck constant \(h\).

Moreover, since the average kinetic energy \(KE_{avg} = PE_{avg} = \frac{\hat{E}}{2}\), we have

\[ KE_{avg} = h\nu = \frac{h}{\nu} \cdot \nu^2 = \frac{h}{\nu \text{sec}^2} + \left( h\nu - \frac{h}{\nu \text{sec}^2} \right) \quad (29) \]

\[ = \text{Boltzmann } k \cdot \text{Kelvin } T \]

\[ = \text{the heat energy from one cycle} \]

\[ + \text{a heat-converted mechanical energy used up in preserving } \nu. \]

As such,

\[ \hat{E} = PE_{avg} + KE_{avg} = h\nu + (\text{heat energy} + \text{energy loss}) \]

\[ = h\nu + \frac{h}{\nu \text{sec}^2}. \quad (30) \]

\[ \blacksquare \]

**Lemma 4**

\[ G^{[2]} = \frac{c^5 \text{sec}^2}{1.6h}. \quad (31) \]

**Proof.** \( \forall \nu \) we have

\[ \left( \frac{\partial h^{[2]}_0}{\partial \hat{t}^{[1]}_0} \right)^2 = \left( \frac{\nu}{1/i \text{sec}} \right)^2 = -\nu^2 \text{sec}^2 \quad (\text{cf. [9] for the equality here}) \quad (32) \]

\[ = g_{11} = 1 - \frac{2G^{[2]} \phi^{[2]} \hat{E}}{\lambda^2 \cdot c^4}, \quad \left( \text{recall } \phi^{[2]} = 0.4, \frac{\lambda}{2} = r \right) \quad (33) \]

\[ = 1 - 1.6G^{[2]} \cdot \left( h\nu + \frac{h}{\nu \text{sec}^2} \right) \nu \quad (34) \]

\[ = 1 - \left( \frac{1.6G^{[2]}h}{c^5} \right) \nu^2 - \frac{1.6G^{[2]}h}{c^5 \text{sec}^2}. \quad (35) \]
thus,

\[
G^{[2]} = \frac{c^5 \text{sec}^2}{1.6h}.
\]

\[\blacksquare\]

Remark 3 In our previous paper [7] we derived

\[
g_{11} = \left(\frac{\partial t_0}{\partial t}\right)^2 \approx \left(1 - \frac{GM}{rc^2}\right)^2, \tag{36}
\]

where \(G\) and \(M\) are referred to by any parameter domain \(S \equiv \{(t, r = \sqrt{x^2 + y^2 + z^2}) \mid (t, x, y, z) \in \mathbb{R}^{1+3}, r = 0\text{ being the center of } M\}\) that calculates the proper time \(t_0\) of a frame \(F\) falling freely to \(r = 0\). We now re-derive it below:

Fix a light source at \(r = r^* \equiv ct^* \approx \infty\) of proper frequency \(\omega_0\) and let frame \(F\) fall freely at \(r^*\) with initial velocity \(v(t^*) = 0\) relative to \(S\). Then by the Doppler effect for light there exists a small neighborhood of \(r^*\) wherein one has

\[
g_{11} = \left(\frac{\partial t_0}{\partial t}\right)^2 \approx \left(\frac{\omega(t)}{\omega_0}\right)^2 \approx \left(\frac{1 + \frac{v(t)}{c}}{\sqrt{1 - \left(\frac{v(t)}{c}\right)^2}}\right)^2 \tag{37}
\]

\((= \lambda^2, \lambda = \text{an eigenvalue of the Lorentz transformation, cf. [8] for a derivation of } g_{11} = \lambda^{\pm 2})\) so that

\[
g_{11} \approx \left(1 + \frac{v(t)}{c}\right)^2 \approx \left(1 + \frac{at}{c}\right)^2 = \left(1 - \frac{GM}{r^2c} \right)^2 = \left(1 - \frac{GM}{rc^2}\right)^2, \tag{38}
\]

as desired \((\approx 1 - \frac{2GM}{rc^2} \text{ as } r \to \infty)\). Thus, we have

\[
\left|\frac{\partial t_0}{\partial t}\right| \approx \left|1 - \frac{GM}{rc^2}\right| = \left|1 - \frac{GE}{rc^4}\right|. \tag{39}
\]

Remark 4 A photon-wave \((\gamma, \lambda)\) with energy \((E^{[1]}, E^{[2]}) = \left(\frac{3}{4}E^{[3]}, \frac{1}{4}E^{[3]}\right)\) punctures \(\mathcal{M}^{[1]}\) with a mini black hole \(b\) by the energy \(E^{[1]}\) of \(\gamma\). Denoting the radius of \(b\) by \(r_b\), we have the wave energy \(E^{[2]}\) contained in \([-\frac{1}{2}, -r_b] \cup [r_b, \frac{1}{2}]\) (as
projected onto \( \mathbb{R}_x \). At Big Bang the initial combined manifold was \( \mathcal{M}^{[3]} = b \times B \), i.e., \( \mathcal{M}^{[1]} = b \); as such, there existed a unique reference frame \( S^* \) over \( \mathcal{M}^{[1]} \) such that the proper time of \( S^* \) was
\[
t_0^{[1]} = 0 \text{ at } x = r_b^*; \tag{40}
\]
to emphasize, \( r_b^* \) in the unit of meter was absolute, since there could be just one reference frame \( S^* \) in \( M^{[1]} \) at Big Bang.

Remark 5 Before Big Bang, the very last half-cycle \( [\pi, 2\pi] \) of \( \lambda \) coming into \( \left[ -\frac{\lambda}{2}, \frac{\lambda}{2} \right] \subset \mathbb{R}_x \) from the "left" commenced at the point when \( \lambda_{\text{front}} \) arrived at \( x = 0 \equiv \pi \) radian (with \( \lambda_{\text{center}} \) lagging behind by \( \frac{\lambda}{2} \equiv \pi \) radian at \( x = -\frac{\lambda}{2} \equiv 0 \) radian). Denoting by \( A \equiv \left[ -\frac{\lambda}{2}, -r_b^* \right] \cup \left[ r_b^*, \frac{\lambda}{2} \right] \), we partition \( A \) by an equivalence relation \( \sim_{\pi} \) defined by: \( \forall x_\alpha > x_\beta \in A \), one has \( x_\alpha \sim_{\pi} x_\beta \) if \( x_\alpha = \lambda_{\text{front}}(t) \) and \( x_\beta = \lambda_{\text{center}}(t) \); i.e., \( A \) is now decomposed into a quotient space \( A^* = \{ \{x_\alpha, x_\beta \} | x_\alpha \sim_{\pi} x_\beta \in A \} \). Correspondingly on the \( y-z \) plane we have the quotient space
\[
S^1_{\pi} := \{ \{\theta_\alpha, \theta_\beta \} | \theta_\alpha = \theta_\beta + \pi \in S^1 \} \tag{41};
\]
in particular, we have \( (\theta = 0) \sim_{\pi} (\theta = \pi) \sim_{\pi} (\theta = 2\pi) \), so that one \( 2\pi - \) cycle as in the definition of the frequency \( \nu \) contains two \( \pi - \) cycles. As we are investigating the energy \( E^{[2]} \) contained in the one \( \pi - \) cycle for \( \lambda_{\text{front}} \) to move from \( x = 0 \) to \( x = \frac{\lambda}{2} \), or equivalently from \( \theta = \pi \) to \( \theta = 2\pi \) \( (\sim_{\pi} \pi) \), we must apply \( \left( \frac{\nu}{2} \right) \) to
\[
E^{[2]} = \hat{\phi}^{[2]} E = 0.4 \hat{E} = 0.4 \left( h \cdot \left( \frac{\nu}{2} \right) + \frac{h}{\left( \frac{\nu}{2} \right) \sec^2} \right) = 0.2h + \frac{0.8h}{\nu \sec^2}; \tag{42}
\]

Remark 6 In the above equation, recall that the first term \( 0.2h \nu \) was the average potential energy of \( E^{[2]} \) in \( [r_b^*, \frac{\lambda}{2}] \). Since potential energy is associated with a point position in space and at Big Bang \( \left( t_0^{[1]} = 0 \right) \) the center of mass of \( E^{[2]} \) in \( A \) was \( 0 \in \mathcal{M}^{[3]} = b \times B \), the pertinent gravitational constant for \( 0.2h \nu \) was \( G^{[3]} \). On the other hand, the second term \( \left( \frac{0.8h}{\nu \sec^2} \right) \) being the heat energy generated from the motion of half a cycle of \( \lambda \) was associated with the average velocity of the harmonic oscillator over \( \left[ 0, \frac{\lambda}{2} \right] \) before Big Bang, when \( G^{[3]} \) had not yet existed; as such, the pertinent gravitational constant for \( \left( \frac{0.8h}{\nu \sec^2} \right) \) was \( G^{[2]} \).
Remark 7 Collecting the above remarks, we have $\forall t_0^{[1]} \in [-\frac{\lambda}{2c}, 0]$
\[
\left| \frac{\partial t_0^{[2]}}{\partial t_0^{[1]}} \right| \approx 1 - \frac{G E^{[2]}}{r c^4} \quad (by \ Equation \ (39))
\]
\[
= 1 - \frac{G^{[3]} \phi^{[2]} h \cdot \left(\frac{\nu}{2}\right)}{r c^4} - \frac{G^{[2]} \phi^{[2]} \cdot \left(\frac{h}{(\nu/2) \sec^2}\right)}{r c^4}
\]
(by Equation 42 and the preceding remark)
\[
= 1 - \frac{0.2G^{[3]} \nu h}{r c^4} - \frac{1.6G^{[2]} \hbar}{c^5 \sec^2}
\]
\[
= \frac{0.2G^{[3]} \nu h}{r c^4} \quad (by \ Equation \ (31)). \quad (43)
\]

The above equation (43) shows that the clock $t_0^{[2]}$ ran faster and faster as $r \to 0$. Equivalently, as $t_0^{[1]}$ turned backwards to Pre-Big Bang from $t_0^{[1]} = 0$ to $t_0^{[1]} = -\frac{\lambda}{2c}$, $t_0^{[2]}$ would have had to be running slower and slower. That is, setting the unit of time $t_0^{[1]}$ by $\left(\frac{r_b^*}{c}\right)$ (recall that $r_b^*$ was unique from special note (40)), we have
\[
1 = \int_0^1 \frac{1}{(r_b^*/c)} dt_0^{[1]} = \int_1^e \frac{1}{t_0^{[1]}} dt_0^{[1]} = \int_{r_b^*}^{er_b^*} \frac{d(c t_0^{[1]})}{c t_0^{[1]}} = \int_{r_b^*}^{er_b^*} \frac{1}{r} dr. \quad (44)
\]

Proposition 1 The photon-waves at Big Bang ( $t_0^{[1]} = 0$) had a unique wave length $\lambda^* \equiv c/\nu^*$, with
\[
\nu^* = \sqrt{\frac{c^5}{0.4G^{[3]} \hbar}} \approx 1.17 \times 10^{43}/sec \quad and \quad \lambda^* = 2.56 \times 10^{-35} \text{meter} \quad (= \text{Planck length.}) \quad (45, 46)
\]

Proof.
\[
\frac{1}{2} \text{cycle} : = \int_0^{\frac{\lambda}{2c}} \left| \frac{\partial t_0^{[2]}}{\partial t_0^{[1]}} \right| dt_0^{[1]} = \int_{r_b^*}^{er_b^*} \frac{0.2G^{[3]} \nu h}{r c^4} dr \quad (by \ Equations \ (40, 43))
\]
\[
= \int_{r_b^*}^{er_b^*} \left( -\frac{1}{2} \text{cycle} \right) \frac{1}{r} dr \quad (by \ Equation \ (44)). \quad (47, 48)
\]
Thus,

\[ -\frac{1}{2} \text{ cycle} = -\frac{0.2G[3]h\nu}{\text{meter} \cdot c^4}, \quad \text{or} \]

\[ \frac{\lambda}{2} = \frac{0.2G[3]h\nu}{c^4} = \frac{c}{2\nu}, \quad \text{i.e.,} \]

\[ \nu^* = \sqrt{\frac{c^5}{0.4G[3]h}}. \]

\[ \square \]

### 2.2 Pair-creation of electron and positron

Since we have established the unique electromagnetic wave length \( \lambda^* = 2.56 \times 10^{-35} \text{meter} \) at \( t_0^{[1]} = 0 \), we now proceed to show how photon-waves could create electron-waves. As this section depends greatly on our forthcoming paper [11], we will present our arguments in a series of remarks more verbally.

**Remark 8** In Proposition 4 in [10], we showed that the motion of an electromagnetic wave \( A \) with energy \( E^{[3]} \) (before Big Bang) had the following equation:

\[
\begin{pmatrix}
  x(t) \\
y(t) \\
z(t)
\end{pmatrix}_A = \begin{pmatrix}
  ct + \xi \lambda \\
  \rho r \cos\left(\frac{ct + \xi \lambda}{r}\right) \\
  \rho r \sin\left(\frac{ct + \xi \lambda}{r}\right)
\end{pmatrix},
\]

where \( r = \frac{\lambda}{2\pi} \) and \( \xi, \rho \in [0, 1] \).

For our present purpose, we re-set \( \xi \in \left[-\frac{1}{2}, \frac{1}{2}\right] \), so that at \( t = 0 \) we have

\[ x_A(0; \xi) = \xi \lambda \in \left[-\frac{\lambda}{2}, \frac{\lambda}{2}\right], \]

\[ y_A(0; \xi) = \rho r \cos\left(\frac{\xi \lambda \cdot 2\pi}{\lambda}\right) = \rho r \cos(2\pi \xi), \quad \text{and} \]

\[ z_A(0; \xi) = \rho r \sin 2\pi \xi. \]

Now by the spherical symmetry around \( 0 \in B \) there must have existed another wave \( B \) with

\[
\begin{pmatrix}
x(t) \\
y(t) \\
z(t)
\end{pmatrix}_B = \begin{pmatrix}
  -(ct + \xi \lambda) \\
  \rho r \cos\left(\frac{ct + \xi \lambda}{r}\right) \\
  \rho r \sin\left(\frac{ct + \xi \lambda}{r}\right)
\end{pmatrix},
\]

\[ \square \]
so that at $t \equiv t_0^{[1]} = 0$ waves A and B summed to

$$
\begin{pmatrix}
  x(0) \\
  y(0) \\
  z(0)
\end{pmatrix}_A +
\begin{pmatrix}
  x(0) \\
  y(0) \\
  z(0)
\end{pmatrix}_B =
\begin{pmatrix}
  0 \\
  2\rho r \cos 2\pi \xi \\
  2\rho r \sin 2\pi \xi
\end{pmatrix}_{A \cup B},
$$

$$
= \frac{\rho \lambda}{\pi} \begin{pmatrix}
  0 \\
  \cos \theta \\
  \sin \theta
\end{pmatrix}_{A \cup B}, \theta \in [0, 2\pi], \quad (57)
$$

i.e., a disk of energy $2E^{[3]}$ centered at $0 \in B$ with a radius of $\frac{\lambda}{\pi}$ on the $y-z$ plane.

Yet again by the spherical symmetry around $0 \in B$ there must have existed another pair of waves C and D such that $C \cup D$ resulted in

$$
\frac{\rho \lambda}{\pi} \begin{pmatrix}
  \cos \theta \\
  \sin \theta \\
  0
\end{pmatrix}_{C \cup D}. \quad (58)
$$

Before we show how $(A \cup B) \cup (C \cup D)$ could result in a pair of electron and positron, we must add in a background remark below.

**Remark 9** *In our forthcoming paper [11] we have shown that the electromagnetic wave associated with an electron spins along two semi-circles that are perpendicularly connected to each other by solving the Dirac equation for the two eigenvectors of the spin operator for a free electron. We found that Pauli matrices

$$
\begin{pmatrix}
  0 & 1 \\
  1 & 0
\end{pmatrix}_{(x,y)},
\begin{pmatrix}
  1 & 0 \\
  0 & -1
\end{pmatrix}_{(x,z)},
\begin{pmatrix}
  0 & -i \\
  i & 0
\end{pmatrix}_{(x,y)}
$$

showed the directions of the momenta of the eigenvectors $\Lambda_A$ and $\Lambda_B$. Focusing first on the first columns of these three matrices, we see that $\Lambda_A$ moves from $(x, y, z)$:

- *West* $\equiv (-1, 0, 0)$ to
- *North* $\equiv (0, 1, 0)$ to
- *East* $\equiv (1, 0, 0)$ to
- *Top* $\equiv (0, 0, 1)$.

Next the second columns of the three matrices showed that $\Lambda_B$ moves from $(x, y, z)$:

- *Top* $\equiv (0, 0, 1)$ to
- *East* $\equiv (1, 0, 0)$ to
- *Bottom* $\equiv (0, 0, -1)$ to
- *South* $\equiv (0, -1, 0)$.*
Universe origin

That is,

\[ \Lambda_A \text{ has the motion: } W \rightarrow N \rightarrow E \rightarrow T \rightarrow W, \] \hspace{1cm} (62)
\[ \Lambda_B \text{ has the motion: } T \rightarrow E \rightarrow B \rightarrow S \rightarrow T. \] \hspace{1cm} (63)

The above results relied on the matrix identification of

\[ \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}_{(x,y)} \equiv \begin{pmatrix} 0 & 0 \\ 0 & -1 \\ 1 & 0 \end{pmatrix}_{(x,y,z)}, \] \hspace{1cm} (64)

whereby we also settled the differential geometry of \( M[1] \times B \) to be

\[ M[3] = \{ (t + ti, x + yi, y + zi, z + xi) \mid (t, x, y, z) \in \mathbb{R}^{1+3} \subset M[1] \}. \] \hspace{1cm} (65)

In particular,

\[ \begin{pmatrix} 0 \\ i \end{pmatrix}_{(x,y)} = i \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{(x,y)} \equiv i \frac{\partial}{\partial y} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}_{(y,z)} \begin{pmatrix} 1 \\ 0 \end{pmatrix}_{(y,z)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{(y,z)}, \] \hspace{1cm} (66)

analogously,

\[ \begin{pmatrix} -i \\ 0 \end{pmatrix}_{(x,y)} = i \begin{pmatrix} -1 \\ 0 \end{pmatrix}_{(x,y)} \equiv -i \frac{\partial}{\partial x} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}_{(x,y)} \begin{pmatrix} -1 \\ 0 \end{pmatrix}_{(x,y)} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}_{(x,y)}. \] \hspace{1cm} (67)

As an illustration, consider the vector

\[ \mathbf{v}_p = \begin{pmatrix} 1 + 2i \\ 2 + 3i \\ 3 + 1i \end{pmatrix} \in \mathbb{C}^3 \subset T_p M[3]. \] \hspace{1cm} (68)

\( \mathbf{v}_p \) can be represented by a vector at \( p \in T_p M[3] \) as

\[ \text{Re} (\mathbf{v}_p) = 1 \mathbf{e}_x + 2 \mathbf{e}_y + 3 \mathbf{e}_z \in \mathbb{R}^3_{(x,y,z)}, \] \hspace{1cm} (69)

which however coincides with

\[ 1i \mathbf{e}_z + 2i \mathbf{e}_x + 3i \mathbf{e}_y \in i\mathbb{R}^3_{(z,x,y)}. \] \hspace{1cm} (70)

That is, \( i\mathbb{R}^3_{(z,x,y)} \) results from a \( 90^\circ \) counter-clockwise rotation of \( \mathbb{R}^3_{(x,y,z)} \) and \( \mathbb{R}^3_{(x,y,z)} \) and \( i\mathbb{R}^3_{(z,x,y)} \) coincide with each other. Simply expressed, the complex
plane $\mathbb{C}_x$ in our combined 4-manifold coincides with $\mathbb{R}^2_{(x,y)}$, so that $(x, y) = (0, 1)$ can be alternatively expressed as $i_x$ in $T_p \mathbb{B}$. In short, we have

$$\mathbb{R}^2_{(x,y)} \equiv \mathbb{C}_x, \mathbb{R}^2_{(y,z)} \equiv \mathbb{C}_y, \text{and } \mathbb{R}^2_{(z,x)} \equiv \mathbb{C}_z. \quad (71)$$

Note that without our geometry of a combined spacetime 4-manifold, the dimensionality of $\mathbb{R}^{1+3} \times \{ i \cdot t \} \times \mathbb{C}_3^{(x,y,z)}$ would be eleven, as in the M-theory by Witten.

**Remark 10** Putting the preceding two remarks together, we have waves $(A \cup B)$ and $(C \cup D)$ at $t_0^{[1]} = 0$ sum to

$$\frac{\rho \lambda}{\pi} \left[ \begin{array}{cc}
0 & \cos \theta (t_1) \\
\cos \theta (t_2) & \sin \theta (t_2) \\
\sin \theta (t_1) & 0
\end{array} \right]_{A \cup B} + \left[ \begin{array}{cc}
\cos \theta (t_2) & 0 \\
\sin \theta (t_1) & 0
\end{array} \right]_{C \cup D}, \quad \theta (t_1), \theta (t_2) \in [0, 2\pi]. \quad (72)$$

Denoting the basis vectors $(e_x, e_y, e_z)$ by $(T, E, N)$, one can reconfigure the existing two circular paths of

$$(A \cup B) : W \to N \to E \to S \to W \quad \text{and} \quad (C \cup D) : W \to B \to E \to T \to W \quad (73, 74)$$

into

$$W \to N \to E \to T \to W \quad \text{for an electron and} \quad (75)$$

$$W \to B \to E \to S \to W \quad \text{for a positron.} \quad (76)$$

Here we note: (1) Schematically the diameter of $(A \cup B)$ intersected the diameter of $(C \cup D)$ as the plus sign $+$, which was broken into $|$ (a positron) and $\|$ (an electron). (2) $\Lambda_B$ with the motion of $T \to E \to B \to S \to T$ can be obtained by a change of frame from $\Lambda_A$ (see [11]); i.e., the two eigenvectors of the spin operator on the quantum position wave function $\psi$ are of the nature "or" but not "and" (as in optical interference). This shows incidentally a drawback of the probability interpretation of waves; i.e., the superpositions of waves in some cases are "or" (as in our present situation when the electromagnetic waves are alternative representations from different spatial frames) and yet in other cases are "and." In the "Schrödinger's cat case," the superposition is "or." Fixing any particular frame, there is a specific photon-wave with the photon appearing in $\mathcal{M}^{[1]}$ of probability densities proportional to the square of the magnitudes of the electric field (see [10]) and depending on where the photon appears the cat's outcome is determined. This particular photon-wave can undergo Lorentz transformations and lose "simultaneity;" if the device is not triggered, then different observers would see different life stages of the cat; if the device is triggered, then the triggering times vary according to the frames.
**Remark 11** Note that the points \( \{W, E\} \) in the above paths of motions, being not differentiable, were singularities, where the spinning motions made abrupt changes of directions to perpendicular planes. Here, we conjecture:

1. \( \{W, E\} \) are responsible for the transmission of virtual photons to engage in electromagnetic interactions.
2. By conservation of angular momenta, the spinning motions must stop at \( \{W, E\} \) for a finite time duration before moving onto a perpendicular plane. These stops reduce the spinning frequency from \( \omega \equiv 2\pi\nu \) to \( \frac{\pi}{2} \equiv \pi\nu \equiv \frac{\pi\nu}{\lambda} \equiv 2\pi \cdot \left(\frac{c}{2\lambda}\right) \), i.e., an equivalence to electromagnetic waves traveling a distance of \( 2\lambda \). As such, the disk resulting from the scattering collision between waves A and B at \( t_0^{[1]} = 0 \) had its radius doubled, i.e.,

\[
\frac{2\lambda}{\pi} \left( \begin{array}{c} 0 \\ \cos \theta \\ \sin \theta \end{array} \right) \quad \theta \in [0, 2\pi],
\]

which was what we derived in [11]:

\[
\Lambda_A(t) = \frac{2\lambda}{\pi} \left( \begin{array}{c} \cos \left( -\frac{1}{2}\omega t \right) \\ \sin \left( -\frac{1}{2}\omega t \right) \\ 0 \end{array} \right)_{t\in \left[\frac{1}{\nu}, \frac{2}{\nu}\right]} + \frac{2\lambda}{\pi} \left( \begin{array}{c} \cos \left( \frac{1}{2}\omega \left( t - \frac{2}{\nu} \right) \right) \\ 0 \\ \sin \left( \frac{1}{2}\omega \left( t - \frac{2}{\nu} \right) \right) \end{array} \right)_{t\in \left[\frac{2}{\nu}, \frac{3}{\nu}\right]}. \quad (78)
\]

**Remark 12** At Big Bang since the disk formed by the colliding electromagnetic waves A and B contained an energy of \( 2\hat{E}_{(\gamma, \lambda)} \) and yet from the preceding remark the electron-wave \( (e, \lambda_e) \) had half of the frequency, the energy of \( (e, \lambda_e) \) was

\[
\hat{E}_{(e, \lambda_e)} = 2 \times \left( h \cdot \frac{\nu^*}{2} + \frac{h}{(\nu^*/2) \sec^2} \right) \quad \text{(recall Equation (19))} \quad (79)
\]

\[
= h\nu^* + \frac{4h}{\nu^* \sec^2} = \left( h\nu^* + \frac{h}{\nu^* \sec^2} \right) + \frac{3h}{\nu^* \sec^2}
\]

\[
= \hat{E}_{(\gamma, \lambda)} + \frac{3h}{\nu^* \sec^2} \quad (80)
\]

\[
\therefore \hat{E}_{(\gamma, \lambda)} \approx \left( 6.6 \times 10^{-34} \text{J sec} \right) \times \left( 1.17 \times 10^{43} / \text{sec} \right)
\]

\[
= \left( 8.62 \times 10^{-8} \text{kg} \right) c^2 \cdot \left( 9.11 \times 10^{-31} \text{kg} \right) \cdot \left( 9.46 \times 10^{22} \right) c^2 \quad (81)
\]

\[
= m_{e,o} c^2 \cdot \left( 1 - \left( \frac{v}{c} \right)^2 \right)^{-\frac{1}{2}},
\]
implying that the speed of the electron-wave

\[ v_{(e,\lambda_e)} \approx (1 - 0.005 \times 10^{-44}) c. \]  \hspace{1cm} (82)

**Remark 13** We contend that a union of two perpendicular circles as from four electromagnetic waves at Big Bang could result in two "bent" circles creating a pair of electron and positron because at the two intersecting points \( W, E \) = \( S^1_{A \cup B} \cap S^1_{C \cup D} \), the circular harmonic motions had zero kinetic energy. Furthermore, recall from the above Equation (80) that there was an extra kinetic energy of \( \frac{3h}{4\pi \sec^2} \); this term might serve to alter the otherwise two circular motions to that of a pair of "bent" circles (in addition to the emission of virtual photons).

### 3 Summary

Building on our previously derived results, in this paper we have calculated the unique electromagnetic wave length \( \lambda^* = 2.56 \times 10^{-35} \text{meter} \) at Big Bang, from which we have shown how a set of four photons could engender a pair of electron and positron. We note in particular that we gave a special treatment of the pervasive \( \frac{1}{4} \) problem in Equation (48). If our model of \( \{(\text{particle, electromagnetic wave})_j\} \) with energies \( \left( \frac{3}{4}E^{[3]}_j, \frac{1}{4}E^{[3]}_j \right) \) contained in \( \mathcal{M}^{[3]} = \mathcal{M}^{[1]} \times \mathcal{B} \) is correct, then we envision to contribute our combined spacetime 4-manifold \( \mathcal{M}^{[3]} = \{(t + ti, x + yi, y + zi, z + xi)\} \) to practical applications in quantum information, nanotechnology, and perhaps even teleportation.

### References


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