On a Flat Expanding Universe

Bo Lehnert

Alfvén Laboratory
Royal Institute of Technology, SE-10044 Stockholm, Sweden
Bo.Lehnert@ee.kth.se

Abstract

An earlier elaborated model of the expanding universe with its contents of dark energy, dark matter and normal matter is reconsidered and extended. The model is found to be reconcilable with the observed cosmical dimensions and with the magnitude of the present accelerated expansion. It has the form of a freely expanding cloud of zero-point-energy photons, with the inclusion of a small amount of normal matter. On a macroscopic scale, within the radius of the observable universe, the model has the character of a flat Euclidian geometry, without the need of introducing curved space effects due to General Relativity. This flat geometry is found to be stable with respect to expansive and compressive perturbations, thus suggesting the universe to possess an intrinsic mechanism which aims at flat geometry.

Keywords: Expanding universe, dark energy, dark matter, zero point energy, flat Euclidian geometry

1 Introduction

After the discovery by Hubble of the expanding universe, it was commonly expected that the attraction force due to the cosmical mass content would slow down the expansion, and even turn it to a cosmic attraction. However, as a great surprise it was soon found from observations of the type Ia supernovae that the expansion is at present accelerating, as shown by Perlmutter[7], Riess and Turner[8], Linder and Perlmutter[6] and Schmidt[10] among others. To account for the present acceleration, about 75 percent of the mass-energy content is then considered to be made of some weird gravitationally repulsive substance called “dark energy”, in the form of a “cosmological antigravity” which can drive the universe apart. The remaining 25 percent has attractive gravitational interaction, but 21 percent of this is rather some additional unknown substance called “dark matter”, and only 4 percent consist of well-known normal matter.
An attempt by the author to understand the concepts of dark energy and
dark matter has recently been based on effects due to the quantum mechanical
Zero Point Energy of the vacuum state[3,4]. The latter is not merely an empty
space, because quantum mechanics predicts that there exists a nonzero lowest
energy level. An example of the related vacuum fluctuations has been given
by Casimir[1] who predicted that two metal plates will attract each other
when being sufficiently close together. This prediction was first confirmed
experimentally by Lamoreaux[2]. It can be taken as an experimental proof
that the Zero Point Energy fluctuations act as a photon gas which exerts a
real physical pressure. In the approach by the author[3,4] the pressure gradient
force of this gas will thus act as dark energy, at the same time as the mass
equivalent of its energy density will act as a dark gravitating matter.

The present paper forms an extension of the earlier approach by the author,
with special emphasis on the free expansion of a flat universe and its stability
with respect to expansive or compressive perturbations.

2 Basis of the present Theoretical Model

To simplify the following analysis but still preserve its essential features, a
number of basic assumptions are made:

- In a first approximation the small presence of normal matter is neglected
  as compared to the mass-energy content of dark energy and dark mat-
  ter. It will later be shown that the analysis can also include normal
  matter, when its radial density distribution per solid angle is uniform in
  spherically symmetric geometry.

- The present analysis concerns the observable parts of the universe which
  are to be about flat on a scale of \( R_0 = 10^{26} \) m. This supports a simple
  Euclidian treatment of the average behaviour of an expanding universe
  on scales of the order of \( R_0 \), without the introduction of the curved space
  effects due to General Relativity.

- This does not exclude General Relativity to play an essential rôle on
  curved space at scales larger than \( R_0 \), as well as on local much smaller
  scales such as those of gravitational lensing, dark matter structures
  within galaxies, and black holes.

- The analysis is restricted to the spherical geometry of an expanding cloud
  of zero-point-energy photons. The local energy density of this cloud is
  finite, as shown in an earlier revised statistical analysis being associated
  with the Casimir effect[4].
The analysis will be mainly concentrated on a state of flat geometry in which the “antigravitational” force of dark energy is outbalanced by the gravitational force of dark matter, and on small unbalanced deviations from such a state.

3 The Local Forces due to Dark Energy and Dark Matter

With these basic assumptions a spherically symmetric cloud of zero-point-energy photons is considered, having a local energy density $u(r, t)$ in a spherical frame of reference. At a fixed time $t$ the cloud then has a radially outward directed pressure force

$$f_p = -\frac{\partial p}{\partial r} = -\frac{1}{3} \frac{\partial u}{\partial r}$$

This has the character of a radially outward directed “antigravity” force which here plays the rôle of a local contribution to dark energy.

Due to the mass-energy relation by Einstein, the local mass density of the photon gas becomes $u/c^2$, leading to an integrated mass

$$M = \left(\frac{4\pi}{c^2}\right) \int_0^r r^2 u \, dr$$

within the radius $r$. This results in an inward directed local gravitational force

$$f_g = -\frac{GMu}{c^2 r^2}$$

where $G = 6.673 \times 10^{-11}$ m$^3$/kg·s$^2$ is the Newtonian constant of gravitation. The force $f_g$ plays here the rôle of a local contribution to dark matter.

3.1 Steady Equilibrium of a Freely Expanding Flat Configuration

A state of flat geometry is defined by

$$f_p + f_g = 0$$

where the instantaneous “antigravity” and gravitational forces outbalance each other. Introducing the normalized radial coordinate $\rho = r/r_c$ with $r_c$ as a characteristic radius, and $u = u_cU(\rho, t)$ where $u_c$ stands for a characteristic photon energy density, equation (4) leads after some straightforward deductions[4] to

$$\frac{\partial^2 U}{\partial \rho^2} + 2 \frac{\partial U}{\rho \partial \rho} - \frac{1}{U} \left(\frac{\partial U}{\partial \rho}\right)^2 + 2C_0 U = 0$$
Here

\[ C_0 = 6\pi Gu_c r_c^2/c^4 = 3\pi n_c (r_c L_p)^2/\bar{\lambda} \]  

(6)
is a dimensionless parameter with \( n_c \) standing for an average equivalent photon density, \( L_p = (Gh/c^3)^{1/2} \) for the Planck length, \( \bar{\lambda} = c/\nu \), and \( \nu \) for an average frequency of the zero-point-energy photons. A particular solution is obtained for \( C_0 = 1 \) where

\[ u = u_c (r_c/r)^2 \]  

(7)

This instantaneous solution can further be extended to that of the self-consistent state of a freely expanding flat state of the photon gas in which

\[ r_c = r_{c0} (t/t_0) \quad u_c = u_{c0} (t_0/t)^2 \quad u_c r_c^2 = u_{c0} r_{c0}^2 = \text{const.} \]  

(8)

with \( t_0 \) as a characteristic time. Relations (7) and (8) thus imply that there can exist a flat state in which the radius \( r_c \) increases linearly with time, and \( u_c \) simultaneously decreases as \( 1/t^2 \), to make \( u_c r_c^2 \) constant per solid angle at any position \( r \). For this specific but physically important solution also a contribution of normal matter of constant amount per solid angle can thus be added.

The solution (7) and equation (2) further result in an integrated mass

\[ M(r) = 2c^2 r/3G \]  

(9)

with \( M(R_0) \) at the radius \( R_0 \) of the observable universe. According to Linde[5] the average density of normal matter in the present universe is \( \rho_n \cong 10^{-26} \text{ kg/m}^3 \) which would imply that the average density of dark matter can be estimated to \( \rho_d = (21/4)\rho_n \cong 5.3 \times 10^{-26} \text{ kg/m}^3 \). This implies that we also have

\[ M(R_0) = (4/3)\pi R_0^3 \rho_d \]  

(10)

Combination of equations (9) and (10) yields an estimate of the radius \( R_0 \) in terms of the present theory, as given by

\[ R_0 \cong c (2\pi G \rho_d)^{-1/2} \]  

(11)

With the value of \( \rho_d \) this results in \( R_0 = 0.64 \times 10^{-26} \text{ m} \), being of the same order as that estimated from astronomical observations. The theory is thus reconcilable with cosmical dimensions. The reason for this is that the parameter \( C_0 = 1 \) of equation (6) includes the Planck length \( L_p \) which is the smallest one appearing in theoretical physics. Here it is simply due to the fact that the gravitational force is very small as compared to the pressure gradient force of the photon gas at laboratory dimensions. It therefore results in a radius \( r_c \) of cosmical dimensions.
It has further to be noticed that the total mass (9) vanishes at the origin \( r = 0 \). The energy density (7) diverges on the other hand at \( r = 0 \), which can be taken as a remnant of the earliest stage of a Big Bang.

The equilibrium solution (7) for \( C_0 = 1 \) finally corresponds to the local forces

\[
\begin{align*}
    f_p &= \frac{2}{3} u_c r_c^2 / r^3 \\
    f_g &= -\left(4\pi/c^4\right) u_c^2 r_c G / r^3
\end{align*}
\]

and their ratio

\[
-f_p / f_g = \left(6\pi G / c^4\right) u_c r_c^2
\]

3.2 Small Deviations from a Flat Equilibrium

Small deviations from the flat equilibrium solution (4) are now considered, first with respect to a corresponding acceleration or retardation, then to the stability of perturbations due to compressive or expansive changes. A crude estimate of the acceleration can be obtained in considering the forces

\[
(dF_p, dF_g) = 4\pi R_0^2 (f_p, f_g) \, dr
\]

acting on a volume element of thickness \( dr \) near the radius \( r = R_0 \). The net force

\[
dF^* = (\delta - 1) \, dF_p
\]

acting on this element is introduced, where \( dF_p \) is the pressure force at equilibrium defined by \( \delta = 1 \), and \( \delta \neq 1 \) corresponds to an accelerated or retarded expansion. From equations (12), (6) with \( C_0 = 1 \), and (9)

\[
dF_p = \left(4c^4 / 9GR_0\right) \, dr \quad dM = \left(2c^2 / 3G\right) \, dr
\]

and the acceleration becomes of the order of

\[
dF^* / dM = 2(\delta - 1) \, c^2 / 3R_0
\]

The present estimated distribution of mass-energy content of the accelerated expansion[6,7] would then correspond to \( \delta = 5/3 \). Even if this is not a small value of \( \delta - 1 \), it can in a first approximation be used to give an order of magnitude of the corresponding acceleration. It results in an acceleration of the order of \( 4 \times 10^{-10} \text{ m/s}^2 \).

This should be compared to the observed redshift and relative intensity of light of supernovae of type Ia in the present universe. The deviation of the redshift due to the acceleration corresponds to an additional increment \( \Delta v = (z_0 - z_a)c \) in velocity, where \( z_0 = 1 \) is the redshift near \( r = R_0 \) and
\( z_a \approx 0.8 \) is that due to accelerated expansion. The corresponding time interval is of the order of \( \Delta t \approx R_0/2c \). This yields an observed acceleration of about \( \Delta v/\Delta t \approx 4 \times 10^{-10} \text{ m/s}^2 \), being of the same order as that estimated from equation (18).

The second question concerns a simple stability analysis on small deviations of the characteristic radius \( r_c \) from equilibrium, i.e. where \( r_c \) is replaced by \((1 + \varepsilon)r_c\) with \(|\varepsilon| \ll 1\) and \( u_c \) being kept constant. Equation (14) then results in

\[
-f_g/f_p = 1 + 2\varepsilon
\] (19)

When \( \varepsilon > 0 \) the configuration then has a somewhat larger radius \( r_c \) and is thus expanded, but this results in an excess of gravitation force which tends to counteract the expansion. If instead \( \varepsilon < 0 \) the configuration is compressed, but this results in an excess of “antigravitational” pressure force which tends to counteract the compression. Consequently the equilibrium becomes stable at least to these types of perturbation.

4 Discussion and Conclusions

A spherically symmetric model has been elaborated for a flat expanding universe, within the range of its observable parts. There is then a balance between dark energy in the form of a pressure gradient force of a zero-point-energy photon gas on one hand, and dark matter in the form of the energy density of the same gas plus the density of normal matter on the other. A self-consistent solution is found for such a freely expanding cloud, in which the equivalent energy content per solid angle is uniformly distributed in the radial direction. The model becomes consistent both with the required cosmical scale and with the observed acceleration of the expansion at the present stage.

An important question can be raised, as being initiated by this obtained self-consistent but particular solution, being stable at least to expansive and compressive perturbations. Within the frame of the present theoretical restrictions, it is thus suggested that the expanding universe may possess an intrinsic mechanism which aims at flat geometry.

References


On a flat expanding universe


Received: November, 2012