Contribution to the Study of Composite Materials

Based Piezoelectric Fibers: Use Mori Tanaka Homogenization Model

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Abstract

In the present work, we study the behaviour of composite piezoelectric. In the first part, we contribute to the understanding of the phenomenon of piezoelectricity, ferroelectricity and the main interest of piezoelectric materials and their applications. In the second part, we give a prelude to the study of composite materials, also a bibliographical synthesis of homogenization techniques in the most classical linear elasticity, we are interested in the model of Mori Tanaka, the main interest of this model that allows to study a composite material containing a fairly high quantity of volume fraction of fiber. One of the most important problems that must be taken into consideration, is the cost of these piezoelectric materials, the interest of this study is to find the behaviour of piezoelectric composites using the homogenization model, which can replace the piezoelectric materials and reduce the cost of these materials. In the last part, we demonstrate the constitutive equation of piezoelectric composites, and simulate the equation obtained in a programming language to determine stress-strain behaviour during a compression test.

Keywords: Piezoelectricity, ferroelectricity, composite piezoelectric, homogenization, Mori Tanaka model, behaviour, fiber matrix
I. INTRODUCTION

In recent years, applications involving piezoelectric materials have developed considerably. The advantage of these materials is that they couple strongly the mechanical and electrical. The main advantage of piezoelectric materials is their ability to act on the mechanical condition of a structure by changing the applied electric field to the material. This has resulted in applications of vibration control and actuator positioning. By promoting the flow of charges in a piezoelectric material bonded to a structure, it is possible to extract low-power (milliwatt or microwaveable). At the end of the piezoelectric materials have found a particularly interesting field of application in the conversion of electrical energy: Piezoelectric transformers can advantageously replace the electromagnetic transformers especially for applications subject to miniaturization.

In general, their use tends to grow in the mobile and embedded electronics. Recalling a piezoelectric composite, consists of at least one piezoelectric material as the active phase called "fiber", with one or more non-piezoelectric phases called "Matrix".

II. PIEZOELECTRICITY

Piezoelectricity was discovered by Pierre and Jacques Curie in 1880. We distinguish the direct and inverse piezoelectric effects. The direct effect is a phenomenon that results in the appearance of an electric field when the material is subjected to mechanical stress.

And the opposite effect corresponds to the appearance of a mechanical deformation of the material when subjected to an electric field. For a piezoelectric crystal is, it must be noncentrosymmetric that is to say that the centroids of positive and negative charges do not coincide in the mesh, as shown in Figure 1 [3].

![Figure 1: Representation of the direct and inverse.](image)
The equations that describe the piezoelectricity are:

\[ D_k = k_{ij} E_j + d_{klm} \sigma_{lm} \]
\[ \epsilon_{ij} = d_{ijn} E_n + s_{ijkl} \sigma_{kl} \]

Where \( \epsilon_{ij}, \sigma_{kl}, E_k, D_i \) are the components of the total strain tensor, the local stress tensor, the electric field and electric displacement vectors. \( s_{ijkl} d_{kl}, k_{ij} \) are the components of the elastic compliance, the piezoelectric and dielectric permittivity tensors.

### III. MATHEMATICAL FORMULATION

#### III.1 Constitutive equation of a pure single crystal matrix

We consider a single crystal material (matrix) undergoes a displacement field \( u^M \), related to the strain tensor \( \epsilon^M \):

\[ \epsilon_{ij}^M = \frac{1}{2} \left( u_{j,i}^M + u_{i,j}^M \right) \]  

(1)

using the hypothesis of small strain framework. This total strain tensor is the summation of an elastic strain tensor \( \epsilon^{eM} \), a thermal expansion strain tensor \( \epsilon^{thM} \). As the general, the decomposition of strain is adopted:

\[ \epsilon_{ij}^M = \epsilon_{ij}^{eM} + \epsilon_{ij}^{thM} \]  

(2)

The elastic strain tensor is related to the stress tensor by Hooke's equation:

\[ \epsilon_{ij}^{e} = S_{ijkl}^{M} \sigma_{kl}^{M} \]  

(3)

Where: \( S_{ijkl}^{M} \) is the fourth order stiffness tensor.

The thermal strain tensor is related to the thermal expansion \( \alpha^M \) and to the temperature through the equation:

\[ \epsilon_{ij}^{th} = \alpha_{ij}^{M} T \]  

(4)

Multiply by: \( C_{ijkl}^{M} \) the fourth order elasticity tensor,

We have:

\[ S_{ijkl}^{M} \sigma_{kl}^{M} = \epsilon_{ij}^{M} - \alpha_{ij}^{M} T \]

We obtain:

\[ \sigma_{ij}^{M} = C_{ijkl}^{M} \epsilon_{ij}^{M} - C_{ijkl}^{M} \alpha_{kl}^{M} T \]  

(5)
III.2 Constitutive equation of piezoelectric composite with an elastic inclusion

The constitutive equation of piezoelectric material is written as follows:

$$\sigma_{ij}^l = l_{ijkl} \varepsilon_{kl}^l - m_{p ij} E_p$$  \hspace{1cm} (6)

Where:

$$l_{ijkl} = C_{ijkl} - \frac{1}{2} \sum_n \sum_m K_{nm} C_{ijpq} C_{rskl} (e_{pq} + d_{ipq} E_t)$$

$$m_{p ij} = \Gamma_{p ij} + \frac{1}{2} \sum_n \sum_m K_{nm} C_{ijkl} (P_{pn} - e_{rs} T_{pn} + d_{ijkl} E_g)$$

$$K_{nm} = \left[ H_{nm} + \frac{1}{2} e_{ij} e_{ijkl} (e_{sn} + d_{ijkl} E^0_p) \right]^{-1}$$

We consider a single crystal with an elastic inclusion with volume $V_i$ inside matrix with volume $V_M$. To behave in a single crystal, we use the model of Mori Tanaka homogenization.

The total strain tensor reads:

$$\varepsilon_{ij} = (1 - \rho^l)\varepsilon_{ij}^M + \rho^l \varepsilon_{ij}^l$$ \hspace{1cm} (7)

Where: $\rho^l = \frac{V_i}{V_i + V_M}$ is the total volume fraction of inclusion in the matrix.

We assume that the matrix and the inclusions have the same temperature. The stress tensor of the inclusion and stress tensor in the matrix is given by equation 8:

$$\begin{cases}
\sigma_{ij}^M = C_{ijkl}^M \varepsilon_{kl}^M - C_{ijkl}^M \alpha_{kl}^M T \\
\sigma_{ij}^l = l_{ijkl} \varepsilon_{kl}^l - m_{p ij} E_p
\end{cases}$$ \hspace{1cm} (8)

We use the Mori Tanaka homogenization model which gives the strain tensor in the inclusion through the total strain tensor by:

$$\varepsilon_{ij}^l = A_{ijkl}^{MT} \varepsilon_{kl}$$ \hspace{1cm} (9)

where the Mori–Tanaka strain concentration tensor $A_{ijkl}^{MT}$ is expressed through the dilute Eshelby strain concentration tensor $A^{ech}$ and the identity tensor I by:

$$A^{MT} = A^{ech} \left[ (1 - \rho^l) + \rho^l A^{ech} \right]^{-1}$$ \hspace{1cm} (10)
and where $A^{\text{esh}}$ is expressed through the modified Green tensor in the matrix by:

$$A^{\text{esh}} = \left[I - S^{\text{esh}} S^{\text{m}} (C_{M} - C_{T})\right]^{-1}$$  \hspace{1cm} (11)

Now, if we use the expression of the inclusions strain tensor given by (9) in Eq. (7), we can write the matrix strain tensor through the total strain tensor:

$$\varepsilon_{ij}^{M} = \frac{1}{1 - \rho'} \left[l_{ijkl} - \rho' A_{ijkl}^{MT}\right] \varepsilon_{kl}$$  \hspace{1cm} (12)

So if we use (9) and (12) in (8), we get:

$$\sigma_{ij}^{M} = \frac{1}{1 - \rho'} \left[l_{ijpq} - \rho' A_{ijpq}^{MT}\right] C_{pqkl}^{M} \varepsilon_{kl} - C_{ijkl}^{M} \alpha_{kl}^{M} T$$

$$\sigma_{ij}' = l_{ijpq} A_{pqkl}^{MT} \varepsilon_{kl} - m_{pqj} E_{p}$$

and using these equations in the expression of the total stress tensor given by:

$$\sigma_{ij} = \left(1 - \rho'\right) \sigma_{ij}^{M} + \rho' \sigma_{ij}'$$  \hspace{1cm} (14)

Where:

$$\sigma_{ij} = \left[C_{ijkl}^{M} + \rho' \left(l_{ijpq} - C_{ijpq}^{M}\right) A_{pqkl}^{MT}\right] \varepsilon_{kl} - \left(1 - \rho'\right) C_{ijkl}^{M} \alpha_{kl}^{M} T - \rho' m_{pqj} E_{p}$$

we obtain the single-crystal constitutive equation:

$$\sigma_{ij} = K_{ijkl} \varepsilon_{kl} - P_{ij} T - W_{ijpq} E_{p}$$  \hspace{1cm} (15)

We note:

$$K_{ijkl} = C_{ijkl}^{M} + \rho' \left(l_{ijpq} - C_{ijpq}^{M}\right) A_{pqkl}^{MT}$$

$$P_{ij} = \left(1 - \rho'\right) C_{ijkl}^{M} \alpha_{kl}^{M}$$

$$W_{ijpq} = \rho' m_{pqj}$$

IV. RESULTS

As an application, we assume the composite piezoelectric: copper / barium titanate (Cu/BaTiO3), the matrix is copper and the reinforcement is the Barium Titanate.
Copper (Cu) has the following properties:

- Young's modulus: 124000 MPa
- Poisson's ratio: 0.33
- Thermal expansion coefficient $\alpha$: $16.5 \times 10^{-6}$ (1 / K)

Barium Titinate (BaTiO3) has the properties following:

- Young's modulus: 33000 MPa
- Poisson's ratio: 0.3
- Longitudinal piezoelectric constant $d_{33}$: $1.52 \times 10^{-6}$ (mm / V)
- Deformation piezoelectric spontaneous DPS: 0.27
- Spontaneous polarization $P_s$: $2 \times 10^{-7}$ (C/mm²)

The figure 2 shows the evolution of strain during a compression test.

The mechanical depolarization is characterized by an increase in remanent polarization, the value of the polarization at zero field, with respect to the applied stress.

We note that the increase in fiber volume fraction of BaTiO3 has an effect on the properties of the composite.
When we increase the volume fraction of reinforcement, more the property of the composite increases. In the case of $f_i = 40\%$, this allows to lead a material with a performance nearly equal to the piezoelectric material.

**Figure 3:** Evolution of stress as a function of deformation at different temperature in volume fraction of reinforcement constant $F_i = 45\%$.

Figure 3 shows the evolution of stress as a function of the deformation of the composite piezoelectric (Cu/BaTiO3) with 45% of the volume fraction of reinforcement for different temperature: $T = 15^\circ C$ et $T = 35^\circ C$. We find that the temperature has an effect on the property of the composite; this effect is due to increased stiffness of the composite.

**V. CONSLUSION**

We have presented in this work the study of piezoelectric fiber composites using the homogenization model of Mori Tanaka, this model that was developed in the case of a material that contains a relatively high amount of inclusions.
The results provide information on the behavior of piezoelectric composites.

It appears clearly that an increase in high enough volume fraction of inclusions, can lead to a material whose performance is approximately equal to a piezoelectric material separately.

REFERENCES


Received: November, 2012