
Varying Gravitational Constant in Five Dimensional Universal Extra Dimension with Nonminimal Derivative Coupling of Scalar Field

Agus Suroso and Freddy P. Zen

Theoretical Physics Laboratory, THEPI Division, and Indonesia Center for Theoretical and Mathematical Physics (ICTMP) Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung Jl. Ganesha 10 Bandung 40132, Indonesia agussuroso@fi.itb.ac.id

Abstract

We study a nonminimal derivative coupling (NMDC) of scalar field, where the scalar field is coupled to curvature tensor in the five dimensional universal extra dimension model. We apply the Einstein equation and find its solution to study time variation of the gravitational constant.

Keywords: universal extra dimension, nonminimal derivative coupling, gravitational constant, cosmological constant

1 Introduction

Most of physical theories involve fundamental constants. The fundamental constant of a theory is a parameter that cannot be explained using the theory [1]. One of the fundamental constant that plays an important role in cosmology is the gravitational constant $G$. It is believed that the gravitational constant, together with the speed of light $c$ and Planck constant $h$, determine the the condition of our universe in its very early stage. A question then arise, ”Is the gravitational constant constant?” This question was first addressed by Dirac in 1937. After proposing the Large Number hypothesis, Dirac concluded that the gravitational constant is not constant, but evolve in time as $G \propto t^{-1}$. Recent observational results show that the gravitational constant is slightly evolve in time. The Lunar Laser Experiment show that the gravitational constant was evolves with the rate of $\dot{G}/G = (4 \pm 9) \times 10^{-13} \text{ yr}^{-1}$ [3]. From theoretical point of view, the varying gravitational constant can be achieved by treating
the gravitational constant as a dynamical fields of gravitational theory. For a review on experimental and theoretical study of varying physical constant especially gravitational constant, see [1].

Here, we study variation of gravitational constant based on the five dimensional (5D) universal extra dimension (UED) with non-minimal derivative coupling. The universal extra dimension (UED) suggest that there are one or more additional dimensions beyond the four dimensional spacetime. Differing the braneworld model, the UED model propose that the matter field is not confined in the brane but can move freely in the bulk [4]. This model provide a very good candidate for the cold dark matter (CDM) [5] and can be used to explained the stability of proton [6]. The cosmological aspect of this theory has been studied [7, 8]. The nonminimal derivative coupling was introduced by Amendola in [9]. This model also studied in the context of inflation [10], the dark energy [11], and the de Sitter behaviour of the universe where the coupling constants of the NMDC can be used to recover the cosmological constant [12]. The exact cosmological solution of NMDC has studied in [13]. We extend the model of nonminimal derivative coupling to five dimensional gravity model with flat 5D UED metric as a background in [14, 15, 16]. In [15] we show that the NMDC in 5D UED model have an accelerating universe solution, while in [16] we study de Sitter solution of the model and recover the cosmological constant from the constant parameters of NMDC model.

Our discussion will be organized as follow. After this brief introduction, we setup the field equations of the model. Then we solve the Einstein equations for a special cases of pure free scalar field and a general case of pure free scalar with pure NMDC. For each case, we discuss how the gravitational constant vary in time. Finally, we give the conclusions in the last section.

2 Field equations and its solution

Our model of the nonminimal derivative coupling in 5D UED described by the action

\[
S = \int d^5x \sqrt{-g} \left( \frac{R}{2\kappa^2} + \frac{1}{2} g_{AB} \partial^A \phi \partial^B \phi + \frac{\xi}{2} R g_{AB} \partial^A \phi \partial^B \phi + \frac{\eta}{2} R_{AB} \partial^A \phi \partial^B \phi \right). \tag{1}
\]

Here, the nonminimal derivative coupling is associated with two last terms of the action. The metric for five dimensional universal extradimensions (5D UED) model is [7],

\[
ds^2 = -dt^2 + a^2 \delta_{ij} dx^i dx^j + b^2 dz^2, \tag{2}
\]

where \(a = a(t)\) is the scale factor for three dimensional (3D) common space and \(b = b(t)\) is the scale factor for the extra dimension. Here, we choose different
scale factors $a$ and $b$ for the the 3D and extra dimension space for simplicity and we choose a flat metric for the 3D space to include recent observational results that our universe is flat [17]. We also consider the scalar depend on time only, $\phi = \phi(t)$ and impose the two scale factors related each other as $b = a^\gamma$ for $\gamma$ constant, as taken in [18]. This relation gives us $H_b = \gamma H_a \equiv \gamma H$. Taking a variation of the action, we obtain the field equations as

$$3 (1 + \gamma) H^2 = \left( \frac{1}{2} + \alpha_1 H^2 + \alpha_2 \dot{H} \right) \kappa^2 \ddot{\phi}^2 - \alpha_2 H \kappa^2 \dot{\phi} \ddot{\phi},$$

$$- \left( \beta_1 H^2 + \beta_2 \dot{H} \right) = \left[ \frac{1}{2} + (\xi + \eta) \left( \beta_1 H^2 + \beta_2 \dot{H} \right) \right] \kappa^2 \ddot{\phi}^2 + \beta_3 H \kappa^2 \dot{\phi} \ddot{\phi} + \beta_4 \kappa^2 \left( \dddot{\phi} + \ddot{\phi}^2 \right),$$

$$-3 \left( 2H^2 + \dot{H} \right) = \left[ \frac{1}{2} + 3 (\xi + \eta) \left( 2H^2 + \dot{H} \right) \right] \kappa^2 \ddot{\phi}^2 + 6 (\xi + \eta) H \kappa^2 \dot{\phi} \ddot{\phi} + \beta_3 \kappa^2 \left( \dddot{\phi} + \ddot{\phi}^2 \right),$$

where

$$\alpha_1 \equiv \gamma^2 (2\xi + \eta) + 3 (3\xi - \gamma \eta),$$
$$\alpha_2 \equiv (3 + \gamma) (2\xi + \eta),$$
$$\beta_1 \equiv (\gamma^2 + 2\gamma + 3),$$
$$\beta_2 \equiv (\gamma + 2),$$
$$\beta_3 \equiv 2 (\xi + \eta) (2 + \gamma),$$
$$\beta_4 \equiv 2\xi + \eta,$$

are constants. And the scalar field equation of motion reads,

$$\ddot{\phi} \left( 1 + \epsilon_1 H^2 + \epsilon_2 \dot{H} \right) + \dot{\phi} \left[ \epsilon_3 H + \epsilon_4 H^3 + \epsilon_5 H \dot{H} + \epsilon_2 \dot{H} \right] = 0,$$

where,

$$\epsilon_1 \equiv (\gamma^2 + 3) (2\xi + \eta) + 6\xi (\gamma + 1),$$
$$\epsilon_2 \equiv (\gamma + 3) (2\xi + \eta),$$
$$\epsilon_3 \equiv \gamma + 3,$$
$$\epsilon_4 \equiv (\gamma + 3) \epsilon_1,$$
$$\epsilon_5 \equiv 3 (2\xi + \eta) (\gamma^2 + 2\gamma + 5) + 12\xi (\gamma^2 + 1)$$

are constants.
In order to simplify the field equations, we choose a specific condition for \(\xi\) and \(\eta\) parameters that

\[2\xi + \eta = 0.\]  \hfill (9)

So we have \(\alpha_2 = \beta_4 = \epsilon_2 = 0\). We can also combine eqs. (4) and (5) as

\[-\left(\lambda_1 H^2 + \lambda_2 \dot{H}\right) = \left[1 - \xi \left(\lambda_1 H^2 + \lambda_2 \dot{H}\right)\right] \kappa^2 \dot{\phi}^2 + \lambda_3 H \kappa^2 \ddot{\phi},\]  \hfill (10)

where

\[\lambda_1 \equiv \gamma^2 + 2\gamma + 9,\]
\[\lambda_2 \equiv \gamma + 5,\]
\[\lambda_3 \equiv -\xi (2\gamma + 10).\]  \hfill (11)

As a special case, firstly we solve the field equations for \(\xi = 0\). For this case, the field equations reads

\[3 (1 + \gamma) H^2 = \frac{\kappa^2}{2} \dot{\phi}^2,\]  \hfill (12)

\[-\left(\lambda_1 H^2 + \lambda_2 \dot{H}\right) = \kappa^2 \ddot{\phi},\]  \hfill (13)

\[(\gamma + 3) H \dot{\phi} + \ddot{\phi} = 0.\]  \hfill (14)

From eqs. (12) and (14) we get

\[\phi = \phi_0 \ln (t - t_0),\]  \hfill (15)
\[a = a_0 (t - t_0)^{\frac{1}{1 + \gamma}},\]  \hfill (16)

where \(\phi_0, a_0,\) and \(t_0\) are constants. Here, for a static extradimension \((\gamma = 0)\), we have \(a = a_0 t^{1/3}\), which is the standard cosmological solution with stiff matter.

Substitution the value of \(a\) as in (16) to eq. (12) gives us

\[G = \frac{\kappa^2}{8\pi} = \frac{3 (1 + \gamma)}{4\pi (3 + \gamma)}.\]  \hfill (17)

Here we have a constant \(G\) which depends on the \(\gamma\) constant.

Now we are going to study a general case of \(\xi \neq 0\). For this case, eq. (3) gives us

\[H^2 = \frac{\kappa^2 \ddot{\phi}^2}{6 (1 + \gamma) - 2\alpha_1 \kappa^2 \dot{\phi}^2}.\]  \hfill (18)
or, equivalently
\[ \dot{\phi}^2 = \frac{6 (1 + \gamma) H^2}{\kappa^2 (1 + 2\alpha_1 H^2)}, \quad (19) \]

Thus, we have two conditions for \( H \) and \( \dot{\phi} \) as follow
\[ 3 (1 + \gamma) - \alpha_1 \kappa^2 \dot{\phi}^2 > 0, \quad (20) \]
\[ \frac{1 + 2\alpha_1 H^2}{1 + \gamma} > 0. \quad (21) \]

Resolving eqs. (10) and (7) for \( \dot{H} \) and substituting the value of \( \dot{\phi} \) from (19), we get
\[ \dot{H} = \frac{6 (1 + \gamma) \left[ 1 - \left( \xi \lambda_1 + \lambda_3 \epsilon_3 \right) H^2 \right] + \lambda_1 \left( 1 + 2\alpha_1 H^2 \right)}{\lambda_2 \left[ 1 + \left( 2\alpha_1 - 6 (1 + \gamma) \xi \right) H^2 \right]} \left( 1 + \epsilon_1 H^2 \right) + 6 (1 + \gamma) \lambda_3 \epsilon_5 H^4}. \quad (22) \]

We see here that the case of \( \dot{H} = 0 \) correspond to de Sitter solution \( e^{Ht} \) where
\[ H = (-\epsilon_1)^{-1/2} = 1/\sqrt{-6\xi (\gamma + 1)}. \quad (23) \]

For this solution, we will have
\[ G = \frac{1 + \gamma}{8\pi \xi (2 + \gamma) \dot{\phi}^2}. \quad (24) \]

Then we can write a relation between the gravitational constant and the cosmological constant (because in de Sitter solution, we have \( H^2 = \Lambda/3 \)), as
\[ \Lambda = -\frac{4\pi}{3} \frac{2 + \gamma}{(1 + \gamma)^2} \dot{\phi}^2 G. \quad (25) \]

Next, we are going to investigate two special cases of large and small Hubble constant. For a large Hubble constant, eq. (22); after substituting the value of \( \alpha_1, \epsilon \)'s and \( \lambda \)'s; gives us
\[ \dot{H} \approx -\frac{6 \left( 3\gamma^3 + 22\gamma^2 + 59\gamma + 48 \right)}{12 (\gamma + 1) (\gamma + 5) \left[ 3\xi (\gamma + 2) + (\gamma + 3) \right]} \left[ 1 + 6\xi (\gamma + 1) H^2 \right]. \quad (26) \]

Suppose that the Hubble constant decrease from large to a small one. For \( \xi > 0 \) and \( [1 + 6\xi (\gamma + 1) H^2] > 0 \), we found that \( \dot{H} < 0 \) is happened if \( \gamma < -5, -3 < \gamma < -2 \) (with \( \xi < -\frac{3(\gamma+2)}{(\gamma+3)} \)), or \( -2 \leq \gamma < -1.42 \). Then, after the Hubble constant reach a small value, eq. (22) reduced to
\[ \dot{H} = -\frac{\lambda_1 + 6 (1 + \gamma)}{\lambda_2 \left( 1 + 2\alpha_1 H^2 \right)} \frac{H^2}{1 + 2\alpha_1 H^2}. \quad (27) \]

The solution for the scale factor is
\[ a = a_0 (t - t_0)^{\frac{1}{3+\gamma}}, \quad (28) \]
which is equivalent with the case of pure free scalar case. Thus, the gravitational constant will take a constant value as in eq. (17).
3 Conclusion

We have studied the variation of the gravitational constant in five dimensional universal extra dimension with nonminimal derivative coupling (NMDC). For a very special case of pure kinetic scalar field without NMDC, we have a constant $G$ and the constant can be recovered from the $\gamma$ parameter of our model. Because the $\gamma$ parameter comes from the extra dimensional effect, we can conclude that the gravitational constant is depends on the extra dimensional effect. Considering the present value of the gravitational constant as $G = 6.67 \times 10^{-11}$ kg$^{-1}$m$^3$s$^{-2}$, we get the value of the $\gamma$ is close to $-0.999999994$. It means that the extra dimensional space shrink as the common three dimensional space expands.

In the case of de Sitter solution, the gravitational constant depends on dynamical variable $\phi$ as given in eq. (24). The evolution rate of the gravitational constant can be writen as

$$\dot{G}/G = -2\ddot{\phi}/\dot{\phi}. \quad (29)$$

The small value of $\dot{G}/G$ as given by experimental result suggest that the scalar field evolves nearly linear in time. According to eq. (24), for a nearly static $\phi$, i.e. $\dot{\phi} \rightarrow 0$, we need $\gamma \rightarrow -1$ to have a finite $G$. We can also get the relation between the cosmological constant $\Lambda$ and gravitational constant $G$ as given in eq. (25), and we see that the $\Lambda$ constant is linearly depend on the $G$ constant. We have also noticed that the both constants depend on the extradimensional constant $\gamma$ and the evolution rate of the scalar field $\phi$.

**ACKNOWLEDGEMENTS.** This work is supported by Excellent Doctoral Fellowship of Faculty of Mathematics and Natural Sciences from the I-MHERE Project of Institut Teknologi Bandung (ITB), Riset Peningkatan Kapasitas (RPK) ITB 2012, Riset dan Inovasi KK ITB 2012, and Hibah Desentralisasi Dikti 2012. This paper is tributed to Dr. Arianto who passed away on July 14, 2011. We acknowledged him for useful discussion in early stage of this work.

**References**


Received: September, 2012