On the Problem of Sensor Placement

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Abstract

In this paper we consider an approach to solve the problem of sensor placement. This approach is based on constructing logical models for the problem.

PACS: 42.30.Tz

Keywords: sensor placement, logical models, SAT, 3SAT

Different formalizations of the problem of sensor placement received a lot of attention recently (see e.g. [1, 2]). For instance, sensor placement is extensively used for improved robotic navigation (see e.g. [3, 4]). In particular, visual landmarks problems are extensively studied in contemporary robotics (see e.g. [5, 6, 7]). In this paper we consider SP problem (see [2]). Let

\[ [c]_2 \iff c(1)c(2)\ldots c(\lceil \log_2 m \rceil) \]

where

\[ c = \sum_{i=1}^{\lceil \log_2 m \rceil} 2^{\lceil \log_2 m \rceil-i} i \]

\[ c \leq m \]. Let

\[ \delta_i = \land_{1 \leq c \leq m} (\land_{1 \leq j \leq \lceil \log_2 m \rceil} y_{i,j} = c(j)) \rightarrow \]
Thus, for any \( p \) view of \( T \),

\[
\varepsilon = \bigwedge_{1 \leq l \leq n} \big( \forall 1 \leq i \leq k \exists i,l \big),
\]

\[
\zeta_i = \neg \big( \bigwedge_{1 \leq c \leq m} \big( \forall 1 \leq j \leq \log_2 m \big) y_{i,j} \neq c(j) \big),
\]

\[
\psi = \big( \bigwedge_{1 \leq i \leq k} \delta_i \big) \land \varepsilon \land \big( \bigwedge_{1 \leq i \leq k} \zeta_i \big)
\]

where \( 1 \leq i \leq k \).

**Theorem.** There is \( T \subseteq S \) such that \( \cup_{x \in T} F(x) = N \) and \( |T| \leq k \) if and only if \( \psi \) is satisfiable.

**Proof.** Suppose that there is \( T \subseteq S \) such that \( \cup_{x \in T} F(x) = N \) and \( |T| \leq k \). Without loss of generality we can assume that \( |T| = k \).

Let \( T = \{ c_1, c_2, \ldots, c_k \} \). Let \( y_{i,j} = i(j) \) where \( 1 \leq i \leq k \), \( z_{i,l} = 1 \) for \( l \in \{ p \mid b_p \in F(c_i) \} \), \( z_{i,l} = 0 \) for \( l \in \{ p \mid b_p \notin F(c_i) \} \). Satisfiability of \( \delta_i \) and \( \zeta_i \) follows directly from the choice of values of variables.

Since \( \cup_{x \in T} F(x) = N \), for any \( b_p \in N \) there is \( c_i \in T \) such that \( b_p \in F(c_i) \). Thus, for any \( p, 1 \leq p \leq n \), there is \( i \) such that \( z_{i,p} = 1 \). So, \( \varepsilon \) is satisfiable. Therefore, \( \psi \) is satisfiable.

Suppose now that \( \psi \) is satisfiable. Consider some assignment to the variables of \( \psi \) such that \( \psi \) is satisfiable. Since \( \psi \) is satisfiable, it is easy to see that \( \varepsilon \) is satisfiable. Thus, for any \( l, 1 \leq l \leq n \), there is \( i, 1 \leq i \leq k \), such that \( z_{i,l} = 1 \). Since \( \zeta_i \) is satisfiable, \( 1 \leq i \leq k \), it is easy to check that for any \( i \) and some \( c, 1 \leq c \leq m \), we have \( c(j) y_{i,j} \) where \( 1 \leq j \leq \log_2 m \). Thus, in view of \( z_{i,l} = 1 \), we have \( l \in \{ p \mid b_p \in F(c_i) \} \). Therefore, for any \( l, 1 \leq l \leq n \), there are values of \( y_{i,1}, y_{i,2}, \ldots, y_{i,\log_2 m} \) such that there is \( c, 1 \leq c \leq m \), such that \( c(j) y_{i,j} \) where \( 1 \leq j \leq \log_2 m \) and \( l \in \{ p \mid b_p \in F(c_i) \} \). By definition of \( \psi \), there are no more then \( k \) different assignments for \( y_{i,1}, y_{i,2}, \ldots, y_{i,\log_2 m} \). Therefore, there is \( T \subseteq S \) such that \( \cup_{x \in T} F(x) = N \) and \( |T| \leq k \).

It is easy to see that

\[
\zeta_i \leftrightarrow \zeta'_i = \bigwedge_{m < c \leq 2 \log_2 m} \big( \forall 1 \leq j \leq \log_2 m \big) y_{i,j} \neq c(j)
\]

It is clear that

\[
\delta_i \leftrightarrow \delta'_i = \bigwedge_{1 \leq c \leq m} \big( \forall 1 \leq j \leq \log_2 m \big) y_{i,j} \neq c(j) \big) \lor \big( \forall 1 \leq j \leq \log_2 m \big) y_{i,j} \neq c(j) \big)
\]

Therefore,

\[
\delta_i \leftrightarrow \delta'_i = \bigwedge_{1 \leq c \leq m} \big( \forall 1 \leq j \leq \log_2 m \big) y_{i,j} \neq c(j) \big) \lor \big( \forall 1 \leq j \leq \log_2 m \big) y_{i,j} \neq c(j) \big)
\]

Thus,

\[
\psi \leftrightarrow \psi' = \big( \bigwedge_{1 \leq i \leq k} \delta'_i \big) \land \varepsilon \land \big( \bigwedge_{1 \leq i \leq k} \zeta'_i \big)
\]
In view of
\[ x = 1 \iff x, \quad x \neq 1 \iff \neg x, \]
\[ x = 0 \iff \neg x, \quad x \neq 0 \iff x, \]
it is clear that \( \psi' \) is a CNF. It is easy to check that \( \psi' \) gives us an explicit reduction from SP to SAT.

By direct verification we can check that
\[
\alpha \iff (\alpha \lor \beta_1 \lor \beta_2) \land \\
(\alpha \lor \neg \beta_1 \lor \beta_2) \land \\
(\alpha \lor \beta_1 \lor \neg \beta_2) \land \\
(\alpha \lor \neg \beta_1 \lor \neg \beta_2),
\]
\[ \vee_{j=1}^{l} \alpha_j \iff (\alpha_1 \lor \alpha_2 \lor \beta_1) \land \\
(\land_{i=1}^{l} \neg \beta_i \lor \alpha_{i+2} \lor \beta_{i+1}) \land \\
(\neg \beta_{l-3} \lor \alpha_{l-1} \lor \alpha_l), \tag{1} \]
\[ \alpha_1 \lor \alpha_2 \iff (\alpha_1 \lor \alpha_2 \lor \beta) \land \\
(\alpha_1 \lor \alpha_2 \lor \neg \beta), \tag{2} \]
\[ \vee_{j=1}^{l} \alpha_j \iff (\alpha_1 \lor \alpha_2 \lor \beta_1) \land \\
(\neg \beta_1 \lor \alpha_3 \lor \alpha_4), \tag{3} \]
where \( l > 4 \). Using relations (1) – (4) we can easily obtain an explicit transformation \( \psi' \) into \( \psi'' \) such that \( \psi' \Leftrightarrow \psi'' \) and \( \psi'' \) is a 3-CNF. It is clear that \( \psi'' \) gives us an explicit reduction from SP to 3SAT.

In papers [8, 9, 10, 11, 12] the authors considered some algorithms to solve logical models (see also [13, 14, 15, 16]). Our computational experiments have shown that these algorithms can be used to solve logical models for SP.

References


Received: July, 2012