Cause of Claimed Breach of Newton’s Third Law in Electromagnetism

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Abstract

It is shown that as Newton’s third law holds in static discussions of Electromagnetism it holds also in dynamic discussions of it, ie where the motion of electric charge in the magnetic field arising from the magnetic static (or magnetostatic) poles is under consideration. As we know, there is some force exerted on a current-carrying wire in the magnetic field of a magnet. It is shown that this magnet is in fact the same enlarged magnetic needle in Oersted’s experiment, and then there exists also some force exerted on the magnet due to the electric current in the wire. These two action and reaction forces are in opposite directions.

We consider point magnetic charges as the sources of the magnetostatic fields, like the point electric charges for the electrostatic fields. Forms of the mutual effects of electric and magnetic charges on themselves and on each other are presented in the forms of vectorial relations. Using these relations, incorrectness of a usual manner which eventually leads to the deviation from the classical physics and to the rejection of the Galilean transformations is shown.

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1 Introduction

When a high school student reads that the magnetic needle of a compass turns beside a wire carrying current and that a force is exerted on a hanging wire
carrying current in a fixed magnetic field, acceptance of this matter that this is the same law of action and reaction (and so there is also some force on the wire due to the compass needle and on the fixed magnet due to the wire) will be much more logical for him/her than ignoring the general validity of this law and attributing the force exerted on the compass needle directly to some field around the wire to which he/she should attribute more genuineness than to the agent causing this field. It is much more logical and desirable for him/her to visualize two electric and magnetic point charges moving toward each other on two different parallel lines and then to deduce the form of forces they exert on each other by comparing the situation with the mentioned forces exerted on the compass needle and on the hanging wire. This simple work has been done and at the end of this article has found its mathematical and in fact vectorial form. There, you can see the vectorial forms of force that two moving electric and magnetic charges exert on each other. What is certain is that surely many of the physicists will be glad if some way is found through which the electromagnetic theory [1-14] can be founded totally on the classical physics and in the frame of Galilean transformations, since the reason presented for the deviation from this physics and the choice of other transformations has been the inability to find just such a way. It seems that introduction of these vectorial forms of force and field can be such a way.

What are stated about the action-reaction law are the following matters:

1. The magnitudes of action and reaction forces are equal to each other. 2. Their directions are opposite to each other to the extent of 180°.

It is certain that Newton’s third law, as expressed through the above two matters, holds totally in Mechanics. To what extent do these matters hold in Electromagnetism? We try to answer this question section by section:

First question: Do they hold in Electrostatics? Answer: Yes. Coulomb’s law proves this.

Second question: Do they hold in Magnetostatics? Answer: Since, contrary to electric charges (or electric poles), we cannot isolate magnetic poles in the lab in order to do experiments over them to find their influences on each others, reply to this question is not as straightforward as the previous one. But, clearly, the influences of the magnetic dipoles (or, in fact, magnets) on each others in the lab indicates that some law similar to Coulomb’s law governs the form of forces between the magnetic poles and then Newton’s third law, as expressed through the above matters, holds among them.

Third question: Do they hold in Electrodynamics and Magnetodynamics? (The first is about the influences exerted on electric charges due to their motion in the magnetostatic fields (like the Lorentz force) and the second is about the influences exerted on magnetic poles due to the motion of electric charges in their magnetostatic field (like the force exerted on the poles of the magnet in
Oersted’s experiment.) Answer: The following reasoning shows that in this case too, the above two matters, ie the action-reaction law, hold.

2 Analysis

“We know that there is a force exerted on a wire carrying electric current in a magnetic field, which is inward in the experiment shown in Fig. 1. It seems that its reaction force must be a force exerted on the poles $N$ and $S$ of the magnet in this figure toward the direction opposite to the direction of the force exerted on the wire. We show existence of such a force.

With some attention we realize that this experiment is in fact enlargement of the same Oersted’s experiment shown in Fig. 2. In this experiment flowing of the electric current $I$ in the direction shown in Fig. 2 will exert a force in the direction $ON'$ on the pole $N$ of the magnetic needle and a force in the direction $OS'$ on the pole $S$ of this needle. Reaction of the first force is a force in the direction $OS'$ exerted on the wire carrying current $I$, and reaction of the second force is a force in the direction $ON'$ exerted on this wire, and these two reaction forces cancel each other out and the wire will remain motionless. (Of course, if eg the pole $N$ of the needle is toward the wire, since this pole will be in the region of intenser magnetodynamic field produced around the wire, resultant of forces exerted on the dipole $SN$ (ie the magnetic needle) will be in the direction $ON'$ and naturally its reaction will be a resultant force exerted on the wire in the direction $OS'$; it is evident that these two resultant forces are equal in magnitude and opposite in direction.)

Now consider Fig. 3 as an extended figure of the magnetic needle (or magnetic dipole). In this state, the force exerted on the pole $N$ is in the direction $ON'$ and the force exerted on the pole $S$ is in the direction $OW'$. Resultant of these two forces is a force exerted on the center of mass of the magnet toward the wire. (Reaction of the force exerted on $N$ will be a force in the direction $OS'$ exerted on the wire and reaction of the force exerted on $S$ will be a force in the direction $OE'$ exerted on the wire, and the resultant of them will be a force exerted on the wire toward the center of mass of the magnet.)

Now extend the magnetic dipole still more to obtain Fig. 4. In this state, the force exerted on $N$ is in the direction $ON'$ (and its reaction will be a force exerted on the wire toward $OS'$), and the force exerted on $S$ is also in the direction $ON'$ (and its reaction will be a force exerted on the wire still toward $OS'$). Fig. 4 is the same Fig. 1. In this manner, we have shown that reactions of the forces exerted on the wire in Fig. 1 (which their resultant is $F$) exist.”

The above questions and their presented answers are based on the supposi-
tion of existence of ((isolated) electric and (pole) magnetic) static charges and their influences on each other in which as we saw Newton’s third law is not violated. The reasoning leading to Biot and Savart law is that a first electric charge which is moving creates some magnetic field which exerts force on a second electric charge which is moving in this magnetic field; and, conversely, the second electric charge which is moving creates some magnetic field which exerts force on the first electric charge which is moving in this magnetic field. The form of these mutual forces that these two charges exert on each other, as can be seen in the textbooks of electromagnetism, is not symmetric. For instance at the beginning of the chapter 8 of [1] we can see the following expression: “If the charges $q$ and $q_1$ are uniformly moving, with velocities $v$ and $v_1$, respectively, there is a magnetic force $F_m$ exerted on $q$ by $q_1$: 

$$F_m = \frac{\mu_0 q q_1}{4\pi r^2} \mathbf{v} \times \left( \mathbf{v}_1 \times \frac{\mathbf{r}}{r} \right)$$

Since charges have no preference relative to each other, there is also a magnetic force $F_{m1}$ exerted on $q_1$ by $q$:

$$F_{m1} = \frac{\mu_0 q q_1}{4\pi r^2} \mathbf{v}_1 \times \left( \mathbf{v} \times \frac{\mathbf{r}}{r} \right)$$

It is natural that $F_m$ and $F_{m1}$ must be a couple of action-reaction forces. But, does we have really $F_m = -F_{m1}$? This is not the case since the expression

$$\mathbf{v} \times \left( \mathbf{v}_1 \times \hat{r} \right) = \mathbf{v}_1 \times \left( \mathbf{v} \times \hat{r} \right)$$

(in which $\mathbf{r}/r = \hat{r}$) is not an identity. Namely, the law of action-reaction is violated easily (even without any limit condition). Certainly this is not true to say that for magnetic forces between electric charges we must not consider two electric point charges (which are moving) (and we must only consider the whole charges in closed circuits), because this is not the case for electric forces between electric charges, and in principle the basis of obtaining Coulomb’s law is this same influences two point charges have on each other.

Now, considering that the situation related to the above-mentioned third question, as we saw, establishes Newton’s third law, we conclude that we must distinguish between the magnetic field arising from magnetic static poles (as the magnetic field due to the poles of a magnet) and a magnetic field which is not arising from the existence of any magnetic static poles (ie the magnetic field arising from the motion of the electric charge described above in the reasoning leading to Biot and Savart law), because in the first magnetic field, Newton’s third law holds, while, as the above equations show, in the second one, apparently, it does not hold. Such a distinguishing, as we will see, leads us to fundamental relations governing over all charges through which Newton’s third law always holds.
3 Fundamental relations

We can consider origin in the form of point magnetic charges for the magnetostatic fields as we consider origin in the form of point electric charges for the electrostatic fields. We show the magnetic point charge by “b” and we assume the signs + (referring to N pole) and − (referring to S pole) for the two kinds of magnetic charge. Now, with these definitions, if \( q \) and \( q' \) are two point electric charges and \( b \) and \( b' \) are two point magnetic charges and \( \hat{r} \) is the outward radial unit vector for each charge (see Fig. 5), then the following relations will be always true. In these relations the constant values \( k, k' \) and \( k'' \) are positive.

Electrostatic force arising from \( q \) exerted on \( q' \):
\[
F_{q'} = kqq'/r^2 = q'E_{q'}
\]
(1)

Electrostatic field arising from \( q \) in the place of \( q' \):
\[
E_q = F_{q'}/q' = kq\hat{r}_{q'/r^2}
\]
(2)

Magnetostatic force arising from \( b \) exerted on \( b' \):
\[
F_{b'} = k'bb'/r^2 = b'B_{b'}
\]
(3)

Magnetostatic field arising from \( b \) in the place of \( b' \):
\[
B_{b'} = F_{b'}/b' = k'\hat{b}_{b'/r^2}
\]
(4)

And when in an inertial reference, the point magnetic charge \( b \) has a relative velocity \( \mathbf{v}_b \) relative to the point electric charge \( q \), then we will have the following relations. (It is obvious that \( \mathbf{v}_q = -\mathbf{v}_b \); see Fig. 6 in which \( \mathbf{v}_q = \mathbf{v}_q' - \mathbf{v}_b' \) and \( \mathbf{v}_b = \mathbf{v}_b' - \mathbf{v}_q' \).)

Electrodynamic force arising from \( b \) exerted on \( q \):
\[
F_q^* = k''qb\mathbf{v}_q \times \hat{r}_q/r^2 = \frac{k''}{k'} q\mathbf{v}_q \times \mathbf{B}_q
\]
(5)

Electrodynamic field arising from \( b \) in the place of \( q \):
\[
E_q^* = F_q'/q = k''b\mathbf{v}_q \times \hat{r}_q/r^2 = \frac{k''}{k'} q\mathbf{v}_q \times \mathbf{B}_q = \frac{-k''}{k'} \mathbf{v}_b \times \mathbf{B}_q
\]
(6)

Magnetodynamic force arising from \( q \) exerted on \( b \):
\[
F_b^* = -k''qb\mathbf{v}_b \times \hat{r}_b/r^2 = -\frac{k''}{k} b\mathbf{v}_b \times \mathbf{E}_b
\]
(7)
Magnetodynamic field arising from $q$ in the place of $b$:

$$\mathbf{B}^*_q = \mathbf{F}^*_b / b = -k'' q \mathbf{v}_b \times \hat{r}_b / r^2 = -k'' \frac{q}{k} \mathbf{v}_b \times \mathbf{E}_b = \frac{k''}{k} \mathbf{v}_q \times \mathbf{E}_b$$

Relation (8) in the form of $\mathbf{B}^*_q = -k'' q \mathbf{v}_b \times \hat{r}_b / r^2$ is in fact the same final conclusion obtained from the experiences and experiments of Biot, Savart and Ampere[1]. On the basis of the above eight relations we can present the electromagnetic theory in a simple form by using the logic of the classical physics. Problem, complexity and deviation from the classical physics logic occur when we try to ignore the existent distinction between starred forces and fields (relations (5) to (8)) and nonstared ones (relations (1) to (4)). For example if, as at present is current, we claim that $\mathbf{B}_q = k'b \hat{r}_q / r^2$, which is the magnetostatic field arising from the magnetic charge $b$ in the place of the electric charge $q$, is in fact the same $\mathbf{B}^*_q = -k'' q_1 \mathbf{v}_q \times \hat{r}_q / r^2$, which is the magnetodynamic field arising from the electric charge $q_1$ in the place of the electric charge $q$, with this condition that the charge $q_1$ is in the same place of the charge $b$, and consequently we substitute $\mathbf{B}^*_q$ for $\mathbf{B}_q$ in the relation (5), ie $\mathbf{F}^*_q = (k'' / k') q \mathbf{v}_q \times \mathbf{B}_q$, then we will obtain $\mathbf{F}^*_q = k'' q_1 \mathbf{v}_q \times (\mathbf{v}_q \times \hat{r}_q)$, and on the other hand in a similar manner we will have $\mathbf{F}^*_q = k'' q_1 \mathbf{v}_q \times (\mathbf{v}_q \times \hat{r}_q)$ which is not identical with $-\mathbf{F}^*_q$, because considering that $\hat{r}_q = -\hat{r}_q$ it can be seen easily that the expression $\mathbf{v}_q \times (\mathbf{v}_q \times \hat{r}_q) = \mathbf{v}_q \times (\mathbf{v}_q \times \hat{r}_q)$ is not an identity; ie the law of action and reaction is breached easily; besides, in principle $\mathbf{B}_q$ and $\mathbf{B}^*_q$ cannot indicate only a single vector field, since otherwise it will be necessary that the vectors $\hat{r}_q$ and $\mathbf{v}_q \times \hat{r}_q$ be always parallel with each other which is not the case obviously.

References


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Figures 1-4
Figures 5-6