The Problem of Sensor Placement

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Abstract

In this paper we describe an approach to solve the problem of sensor placement. Our approach is based on constructing logical models for considered problem.

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Placement problems for sensors received a lot of attention recently (see e.g. [1]). For instance, sensor placement is extensively used for improved robotic navigation. Note that for many formalizations determining the set of sensors to deploy is a difficult problem. Therefore, it is natural to consider intelligent algorithms to solve this problem.

In this paper we consider a formalization for the problem of sensor placement. For our formalization we describe an approach to create a solver for the problem of sensor placement that based on constructing a logical model for the problem. In particular, we give explicit polynomial reductions from the decision version of the problem to MAXSAT.

Let \( \mathbb{Z} \) be the set of integers. Let \( R \subseteq \mathbb{Z}^2 \) be a discretized workspace. Let \( N \subseteq R \) be a set of target locations. Respectively, let \( S \subseteq R \) be a set of candidate sensor locations. Assume that a point \( y \in R \) is visible from another point \( x \) if \( y \in F(x) \) where \( F : S \to 2^R \).

**The problem of sensor placement (SP):**

**Instance:** Given a discretized workspace \( R \), visibility function \( F \), a set \( N \) of target locations, a set \( S \) of candidate sensor locations, and a positive integer \( k \).

**Question:** Is there a set \( T \subseteq S \) such that \( \bigcup_{x \in T} F(x) = N \) and \( |T| \leq k \)?

Let \( S = \{a_1, a_2, \ldots, a_m\} \), \( N = \{b_1, b_2, \ldots, b_n\} \).

Without loss of generality we can assume that for any \( i \) there is \( j \) such that \( b_i \in F(a_j) \). Let

\[
\alpha_{i,j} = (\lor_{l \in \{p | b_i \in F(a_p)\}} x_l) \lor s_j,
\]

\[
\beta_{i,j} = (\lor_{l \in \{p | b_i \in F(a_p)\}} x_l) \lor \neg s_j
\]

where \( 1 \leq i \leq n, 1 \leq j \leq m + 1 \). Let

\[
\varphi = (\land_{1 \leq i \leq n, 1 \leq j \leq m+1} (\alpha_{i,j} \land \beta_{i,j})) \land (\land_{1 \leq i \leq m} \neg x_i).
\]

**Theorem.** Let \( \{s_i^0, x_j^0 | 1 \leq i \leq m + 1, 1 \leq j \leq m\} \) be an assignment to the variables of \( \varphi \) such that a maximum number of clauses of \( \varphi \) is satisfied. Let \( T = \{a_j | a_j \in S, x_j^0 = 1\} \). Then

\[
|T| = \min_{P \subseteq S, \bigcup_{x \in P} F(x) = N} |P|.
\]

**Proof.** It is easy to see that we can suppose that \( x_i = 1 \) if and only if a sensor placed in \( i \)th candidate sensor location.

Note that the total number of clauses of \( \varphi \) is \( 2n(m + 1) + m \). Since for any \( i \) there is \( j \) such that \( b_i \in F(a_j) \), it is easy to see that \(|\{p | b_i \in F(a_p)\}| \geq 1 \).
for all $\alpha_{i,j}$ and $\beta_{i,j}$. Suppose that $x_l = 1$ for all $l$. It is clear that, in this case, at least $2n(m+1)$ clauses of $\varphi$ is satisfied.

Note that if $x_i = 0$ for some $i$ and for all $l \in \{p \mid b_i \in F(a_p)\}$, then either $\alpha_{i,j} = 0$ or $\beta_{i,j} = 0$ for all $j$. Therefore, in this case, no more than $2n(m+1) - 1$ clauses of $\varphi$ is satisfied. So, if $\{s^0_i, x^0_j \mid 1 \leq i \leq m+1, 1 \leq j \leq m\}$ is an assignment to the variables of $\varphi$ such that a maximum number of clauses of $\varphi$ is satisfied, then $\alpha_{i,j} = 1$ and $\beta_{i,j} = 1$ for all $i$ and $j$. Thus, for any assignment to the variables of $\varphi$ such that a maximum number of clauses of $\varphi$ is satisfied, we have $\bigcup_{x \in T} F(x) = \mathbb{N}$.

In view of $\bigwedge_{1 \leq t \leq m} x_t$, it is easy to check that if a maximum number of clauses of $\varphi$ is satisfied, then a minimum number of sensors is placed. \hfill \Box

Clearly, $\varphi$ is a CNF. It is easy to check that $\varphi$ give us an explicit reduction from SP to MAXSAT.

In papers [2, 3, 4, 5, 6, 7] the authors considered some algorithms to solve logical models. Our computational experiments have shown that these algorithms can be used to solve the logical model for SP.

References


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