Excitation of Ion Cyclotron Instability in

Inhomogeneous Rotating Dusty Plasma

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Abstract

Concentrates on this studying on the excitation of ion cyclotron instability in inhomogeneous anisotropic dusty plasma. Plasma with different temperature regime are considered. The dispersion relation is derived and solved for 1-D cyclotron instability excited in such plasma. The threshold frequency and the growth rate are obtained. Electron to ion temperature ratio, degree of plasma inhomogeneity and dust grains thermal velocity are found to play a crucial role in the growth and damping of cyclotron ion instability.

Keywords: cyclotron ion instability, anisotropic plasma, dusty plasma.

Introduction

The cyclotron ion instability [1-9] is one of the current-driven plasma instabilities. The cyclotron ion instability takes place in a current-driven system where electron and ion beams drift at different velocities, if the plasma affected by electric field at plasma diameter and magnetic field at plasma axis, the particles plasma circulate by different angular velocities according to electromagnetic force, the result of different motion between electron and ion is a current velocity. If the plasma affected by electric and magnetic field perpendicular to each other the ionic waves will excite. When the plasma affected
by two fields perpendicular to each other. The ion frequency excited under the condition \( \ll V_{ri} \). From the experimental point of view, a rapidly growing of the frequency of the cyclotron ion instability at the electron cyclotron frequency equal to half the cyclotron ion frequency [10]. The excitation of cyclotron ion frequency at the resonance values of electric and magnetic fields, the resonance occurs at slab area at which the cyclotron ion frequency found, as a result, growth in the cyclotron ion instability accompany with heating of ions [11]. The cyclotron ion instability excites at weak and non-collision magnetized plasma when the electrons drifts through the field [12]. The increase of the electrode diameter at Q-machine the increase of the cyclotron ion instability [13]. The excitation of electrostatic cyclotron ion frequency at dusty plasma studied, the negative charge of dusty plasma leads to growing in the instability of cyclotron ion frequency [14]. The instability of cyclotron ion frequency appear at high ion density while stable cyclotron ion frequency at low ion density, i.e., the instability of cyclotron ion frequency strong dependent on the density of ion [15]. The growth rate of the cyclotron ion instability affected by the source of energy like inhomogeneous in the temperature, ac electric field, heat electron [16]. In recent years, numerous studies have been confined to dusty plasmas having electrons, ions, and charged dust grains of spherical [17-23]. When dust grains immersed in plasma, they become negatively because impinging electrons move faster than impinging ions. The negatively charged dust grains can be considered a third plasma species. So the plasma consists of electrons (negative), ions (positive), and dust grains (negative) [24]. Since charged dust in a laboratory plasmas are generally levitated by electric fields, ions which acquire drifts due to these fields can stream through the dust, leading to various kinds of streaming instabilities [25]. Degree of Plasma inhomogeneity and dust grains thermal velocity are found to play a crucial role in the growth and damping of Buneman instability [26]. We extend previous work [15, 16, 26] by:

(i) Including dust,
(ii) Considering different temperature regimes, as mechanism for growing or damping of instability,
(iii) Effect of plasma inhomogeneity and dust on the growing or damping of cyclotron ion frequency instability.

**Basic equations and Kinetic dispersion relation**

\[
\frac{\partial f_\alpha}{\partial \tau} + V \cdot \nabla f_\alpha + \frac{e_\alpha}{m_\alpha} \left( E + \frac{1}{c} (V \times H) \right) \frac{\partial f_\alpha}{\partial V} = 0, \quad \text{Vlasov's equation}
\]

In a dusty plasma, the quasi-neutrality condition should be adopted to include dust grains as

\[
\sum_{\alpha} e_\alpha n_{\alpha 0} = 0, \quad e_\alpha, \quad n_{\alpha 0}, \quad \text{is the charge and density of such plasma, } \alpha = e, i, d, \quad \text{for electron, ion, dust grain respectively, } c \quad \text{is the speed of light and } E, H \quad \text{are the electric and magnetic field respectively and they are obey the Maxwell's}.
\]
equations. Let us consider an inhomogeneous dust plasma immersed in a static magnetic field \( H_z = e_z H_0 \) with weakly magnetized dusts. Plasma density inhomogeneity is perpendicular to \( H_z \), i.e., directed along x-axis. For low\( \beta \) (the ratio of thermal to magnetic pressure) the kinetic dispersion relation of the system is

\[
1 + \sum_{\alpha}^{\text{phys}} X_{\alpha} = 0 , \quad (1)
\]

\[
X_{\alpha} = \frac{\omega_{\text{pe}}^2}{k_i v_T^2} \left[ 1 + i \sqrt{\pi} l^{-1}_{\alpha} Z_{\alpha} W(Z_{\alpha}) A(\mu_{\alpha}) \right],
\]

\[
l^{-1}_{\alpha} = 1 - \frac{k_i v_T^2}{(\omega - k_i u_\alpha) \omega_{\text{ca}}} \left[ \frac{d}{dx} \ln n_\alpha + \frac{d}{dx} \ln T_\alpha \right]
\]

\[
Z_{\alpha} = \frac{\omega - k_i u_\alpha}{\sqrt{2} k_i v_{T\alpha}} , \quad A(\mu_{\alpha}) = I_0(\mu_{\alpha}) e^{-\mu_{\alpha}} , \quad \mu_{\alpha} = k_i^2 \rho_\alpha , \quad \rho_\alpha = V_{T\alpha} / \omega_{\text{ca}} , \quad \omega_{\text{ca}} = e_a H_0 / m_a c, \quad V_{T\alpha} = \sqrt{\gamma / m_{\alpha}} , \quad \omega_{\text{pa}}^2 = 4 \pi e^2 n_a (x) / m_{\alpha}
\]

Where, \( X_{\alpha} \) are the plasma susceptibility, \( k_i \), \( k_l \) are the wave number parallel and perpendicular to magnetic field \( H_0 \), \( W(Z_{\alpha}) \) is the probability integral, \( A(\mu_{\alpha}) \) represents the magnetic field effect, \( \rho_\alpha \) is the Larimer radius, \( \omega_{\text{ca}} \) is the cyclotron frequency, \( V_{T\alpha} \) is the thermal velocity, \( I_0(\mu_{\alpha}) \) is the modified Bessel function of the 0th order, \( \omega_{\text{pa}} \) is the Langmuir frequency, \( l^{-1}_{\alpha} \) represent the effect of plasma inhomogeneity and temperature.

If \( k_i u_i \approx k_i u_d, u_i \approx u_d \) and we set \( \omega = \omega - k_i u_d = \omega - k_i u_i = \omega_k + \Delta \omega \), then \( Z_{\alpha} \) for different species are

\[
Z_{\alpha} = \frac{\omega - k_i u}{\sqrt{2} k_i v_{T\alpha}} \quad \text{and} \quad Z_{(i,d)} = \frac{\omega - k_i u}{\sqrt{2} k_i v_{T(i,d)}} , \quad \text{we consider dusty plasma consisting of fairly massive grains, i.e.,} \quad m_i / m_d \ll 1.
\]

**Threshold frequency and growth rate**

Under the specific condition mentioned above, the dispersion relation (1) reads

\[
1 + \omega_{\text{pe}}^2 \cos \theta = \frac{\omega_{\text{pe}}^2 \cos \theta}{\omega_{\text{ce}}} \left( 1 - \frac{2 \Delta \omega}{\omega_k} \right) + \frac{\omega_{\text{pl}}^2}{k^2 v_{T\text{li}}} \left( 1 + i \sqrt{2} \omega_{\text{pl}} / \sqrt{2} k^2 v_{T\text{li}} \right) l_{iZ_{0i}} W(Z_{0i}) + \frac{\omega_{\text{pd}}^2}{k^2 v_{T\text{d}}} \left( 1 + i \sqrt{2} \omega_{\text{pd}} / \sqrt{2} k^2 v_{T\text{d}} \right) Z_{0d} W(Z_{0d}) = 0 , \quad (2)
\]

From the real part at (2), threshold frequency reads as

\[
\omega_k = \sqrt{\omega_{\text{pe}}^2 \cos \theta \left( 1 + \frac{\omega_{\text{pe}}^2}{\omega_{\text{ce}}} \right)^2 \left( \frac{\omega_{\text{pl}}^2}{k^2 v_{T\text{li}}} \right)^2 \left( \frac{\omega_{\text{pd}}^2}{k^2 v_{T\text{d}}} \right)^2}
\]

which gives strong dependant on the dust density, and the increase of such plasma density leads to decrease the threshold frequency \( \omega_k \). For \( \frac{\omega_{\text{pl}}^2}{k^2 v_{T\text{li}}} \), \( \frac{\omega_{\text{pd}}^2}{k^2 v_{T\text{d}}} \ll 1 \), we have two cases concerning the degree of
Langmuir frequency and cyclotron frequency, i.e., low electron Langmuir frequency $\omega_{pe}/\omega_{ce} \ll 1$, high electron Langmuir frequency $\omega_{pe}/\omega_{ce} \gg 1$, Accordingly, we have

For low Langmuir frequency $\omega_k = \omega_{pe} \cos \theta$, and for high Langmuir frequency $\omega_k = \omega_{ce} \cos \theta$ which gives the ratio between threshold frequencies at this different cases $\omega_{pe}/\omega_{ce}$.

The hydrodynamic cyclotron ion frequency excites at $\sqrt{2} k V_{T_I} \ll |\omega - ku + \omega_{ci,d}| \ll \omega_{ci,d}$, equation (2) take the come formula,

$$\begin{align*}
\Delta \omega (2\omega_k + \Delta \omega)(ku + \omega_k - n\omega_{ci} - n\omega_{cd} + \Delta \omega) - \frac{n \omega_{ci} (\omega_k + \Delta \omega)(1 - \omega_{ni}/\omega_0)}{4 k \rho_i (1 + k^2 \rho^2)^{1/2} \omega_{LH_i}} - \\
\frac{n \omega_{ci} \omega_{cd} (\omega_k + \Delta \omega)(1 - \omega_{nd}/\omega_0)}{4 k \rho_i (1 + k^2 \rho^2)^{1/2} \omega_{LH_i} d} = 0 ,
\end{align*}$$

where lower hybrid frequency $\omega_{LH_i,d} = \frac{\omega_{ci,d}^2}{\sqrt{2 \omega_{pe} / \omega_{ce}}}$.

At $\rho_i \approx \rho_d \approx \rho, \Delta \omega \ll 2 |\omega_k|, \omega_k = ku - \omega_{ci} - \omega_{cd}, \omega_{LH_i} \approx \omega_{LH_i} \approx \omega_{LH}$, $\Delta \omega = i \gamma$, then the growth rate of hydrodynamic cyclotron frequency for inhomogeneous dusty plasma take the form,

$$\gamma = \frac{n (ku - n\omega_{ci} - n\omega_{cd}) [\omega_{ci}^2 (1 - \omega_{ni}/\omega_0) + \omega_{cd}^2 (1 - \omega_{nd}/\omega_0)]}{4 k \rho_i (1 + k^2 \rho^2)^{1/2} \omega_{LH}} \frac{1}{\sqrt{2}} , \quad (4)$$

At $\omega_{ni}, \omega_{nd} \ll \omega_0$, equation (4) take the form,

$$\gamma = \frac{n (ku - n\omega_{ci} - n\omega_{cd}) [\omega_{ci}^2 + \omega_{cd}^2]}{4 k \rho_i (1 + k^2 \rho^2)^{1/2} \omega_{LH}} \frac{1}{\sqrt{2}} , \quad (5)$$

At $\omega_{ni}, \omega_{nd} \approx \omega_0, \gamma = 0 , \quad (6)$

By equations (4),(6), it is easy to remark that, the increase in the inhomogeneity the decrease in the growth rate.

The excitation of cyclotron ion instability and cyclotron dust instability put the inhomogeneous dusty plasma into a strongly turbulent state. Then a strong turbulent heating of the inhomogeneous dusty plasma leads to a rapid increase of the threshold current $u_{cr}$ of the instability up to a value $u$. Therefore, it will be of great interest to investigate here this instability at current velocities close to the threshold values.

Accordingly, let us consider that the current velocity slightly exceeds the instability threshold, i.e., we set $u = u_{cr} + \Delta u$, where $\Delta u \ll u_{cr}$, as mentioned above, we consider as usual, a small perturbation in the frequencies and wave numbers. To find the threshold current velocity we put the growth rate equal zero, $\gamma = 0$ at (4), i.e., $u_{cr} \approx n (\omega_{ci} + \omega_{cd}) / k$, which show that the excitation of instability of cyclotron ion frequency at inhomogeneous dusty plasma strong dependant on the $\omega_{ci}, \omega_{cd}$. 

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At equation (4) when, $n \approx 1, k \rho \approx 1$ the growth rate of the instability take the form,

$$\gamma = \left[ \frac{(k-\omega_{ci}-\omega_{cd})}{4\sqrt{2}\omega_{LH}} \left[ \omega_{ci}^2 \left( \frac{\omega_{ni}}{\omega_{ni} \omega_{cj}} \right) + \omega_{cd}^2 \left( \frac{\omega_{nd}}{\omega_{cj}} \right) \right] \right]^{1/2}, \quad (7)$$

**Conclusion**

Both the temperature ratio of electrons to ions, in addition to dust, are found to affect strongly on the growing or damping of cyclotron ion instability in dusty plasma. Besides, the dust grains size and temperature are found to play an important role in controlling this instability, the cyclotron ion instability and is found to appear faster in dusty plasma with strong inhomogeneity than in weak inhomogeneity, while the increase in the inhomogeneity leads to a reduction of the growth rate of instability. This work match some agreement with [26]. We are check that the electron to ion temperature ratio, degree of plasma inhomogeneity, dust grains thermal velocity and cyclotron frequency of ion $\omega_{ci}$ and dust $\omega_{cd}$ are found to play a crucial role in the growth and damping of growing or damping of cyclotron ion instability.

**References**

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