The Effect of Electric Field on the Flow of a Compressible Ionized Fluid in a Cylindrical Tube

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Abstract

The flow of a compressible non-Newtonian dielectric fluid placed between two electrodes and subjected simultaneously to the injection of electric charges and to an application of a temperature gradient is analyzed numerically. We consider the case of strong injection (to consider only the Coulomb force and to neglect the dielectric strength) from an electrode placed on one of the two walls of the tube. The combined effect of electric and thermal fields on the flow behaviour when these two fields are in competition is studied. The numerical code presented can also be applied to other industrial applications.

Keywords: compressible fluid, electric field, numerical simulation, rigid wall, heat transfer

I. Introduction

Injection molding is the most widely used process for developing in series plastic parts. It involves injecting melt and warm polymers in cold mold. Thus, the polymer undergoes significant transformations thermomechanical that determine the quality of the finished product. Modelling the state of the polymer in the mold will provide considerable assistance to moulder during the determination of casting parameters.

An electric field and a thermal gradient simultaneously applied on a layer of a dielectric liquid in a rigid cylinder induce complex phenomena within the liquid. A movement in the liquid arises if the potential difference or (and) if the temperature difference is large enough and above a threshold value. In this paper, we study the behaviour of the flow induced by electrical and thermal fields acting simultaneously [1], [2].

Most authors who have worked so far on the convective flow of an electrical layer of a dielectric liquid subjected to unipolar charge injection and a temperature gradient mainly addressed the solution of this problem through an analysis of stability [3], [4].

In the present work, for the first time, we solve numerically the equations of an electro-hydrodynamic (EHD) problem coupled with those of heat and Navier Stokes for a compressible and non-Newtonian fluid.

II. Formulation of the Problem

We consider a cylinder of length $L$ and radius $R$ filled with a dielectric liquid. The two electrodes are placed at the level of the two walls of the tube. The emitting electrode can
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alternatively be the basis of the input or output. The fluid is subjected to a temperature difference, \( \Delta T = T_e - T_i \) and a potential difference, \( \Delta V = V_e - V_i \). The problem is formulated by considering a non-Newtonian fluid and compressible flow with an apparent viscosity \( \eta_a \), a thermal diffusivity \( \kappa \) and a coefficient of volumetric expansion due to temperature \( \beta \).

If we consider the situation of a strong injection, the dielectric strength can be considered small compared to the Coulomb force and therefore may be at first approximation, neglected \([16]\).

It is customary to work with dimensionless equations and for this we introduce the following dimensionless quantities:

\[
\hat{r} = \frac{r}{R_0}, \quad \hat{z} = \frac{z}{L_0}, \quad \hat{t} = \frac{t}{T_1}, \quad \hat{\omega} = \frac{\omega}{w_0}, \quad \hat{u} = \frac{uL_0}{w_0R_0}, \quad \hat{\rho} = \frac{\rho R_0^2}{\eta_0 w_0 L_0}, \quad \hat{V} = \frac{V}{L_0}, \quad \hat{\lambda} = T_0 \lambda_1, \quad \hat{T} = \frac{T}{T_0},
\]

\[
\hat{\lambda} = \rho \hat{\lambda}_2, \quad \hat{\lambda} = T_0 \lambda_2, \quad \hat{E} = \frac{E}{\Delta V R}, \quad \hat{\rho} = \frac{\rho_0}{\rho_e 0}
\]

This leads to defining the following dimensionless numbers:

\[
X_e = \frac{\Delta T}{\omega T_e} \quad \text{Where} \quad T_e = \frac{L_0}{w_0} \quad \text{the convection time in the axial direction}
\]

\[
R_e = \frac{\rho R_0 w_0}{\eta} \quad \text{Reynolds number}
\]

\[
\beta = R_0 \sqrt{\frac{\rho}{T \eta}} \quad \text{Womersley number}
\]

\[
G_e = \frac{\rho C_w w_0 R_0^3}{\lambda_1 L_0} \quad \text{Graytz number}
\]

\[
G_f = \frac{\eta w_0 \beta}{\lambda} \quad \text{Griffith number}
\]

\[
\beta = \frac{1}{T_0} \quad \text{Expansion coefficient of the fluid}
\]

\[
\sigma = \frac{\hat{\lambda}}{\hat{\lambda}_1} \quad \text{A dimensionless number}
\]

\[
X_e = \frac{gT_e}{w_0} \quad \text{A dimensionless number}
\]

\[
T_{CV} = \frac{\epsilon_0 \Delta V}{\rho_0 V \kappa_0} \quad \text{the ratio of the Coulomb force and the viscous forces}
\]

\[
C_{NI} = \frac{\rho_0 R_0^2}{\epsilon_0 \Delta V} \quad \text{A measure of the level of injection,}
\]

\[
M = \left( \frac{\epsilon_0}{\kappa_0 \rho_0} \right)^{\frac{1}{2}} \quad \text{Reflects the electric- hydrodynamic properties of the fluid.}
\]

The system of equations in dimensionless form governing this type of flow electro-thermal convection is:
\[ \frac{1}{X_t} \frac{\partial \hat{p}}{\partial \hat{r}} + \frac{1}{\hat{r}} \frac{\partial (\hat{r} \hat{p} \hat{u})}{\partial \hat{r}} + \frac{\partial (\hat{p} \hat{w})}{\partial \hat{z}} = 0 \]

\[ \beta^2 \frac{\partial (\hat{p} \hat{w})}{\partial \hat{r}} + \text{Re} \frac{1}{\hat{r}} \frac{\partial (\hat{r} \hat{p} \hat{u} \hat{w})}{\partial \hat{r}} + \text{Re} \frac{\partial (\hat{p} \hat{w} \hat{w})}{\partial \hat{z}} = -\frac{\partial \hat{p}}{\hat{r} \partial \hat{r}} + \frac{1}{\hat{r}} \frac{\partial}{\partial \hat{r}} \left[ \hat{\eta} \hat{r} \left( \frac{\partial \hat{u}}{\partial \hat{r}} \right) \right] + C_{NI} \frac{T_{CV}^2}{M^2} \hat{p} \hat{E} \hat{z} \]

\[ \frac{1}{X_t G_f} \frac{\partial (\hat{p} \hat{T})}{\partial \hat{r}} + \frac{G_e}{G_f} \frac{1}{\hat{r}} \frac{\partial}{\partial \hat{r}} (\hat{r} \hat{p} \hat{u} \hat{T}) + \frac{G_e}{G_f} \frac{\partial}{\partial \hat{z}} (\hat{p} \hat{w} \hat{T}) = \left[ \frac{1}{G_f} \frac{\hat{r}}{\hat{r}} \frac{\partial}{\partial \hat{r}} \left( \hat{r} \frac{\partial \hat{T}}{\partial \hat{r}} \right) \right] - T \left( \frac{\partial \hat{p}}{\partial \hat{r}} \right) \left[ \frac{1}{\hat{r}} \frac{\partial}{\partial \hat{r}} (\hat{r} \hat{u}) \right] + \hat{\eta} \left[ \frac{\partial \hat{w}}{\partial \hat{r}} \right]^2 \]

The axisymmetry of the flow imposes a condition on the homogeneous tube axis \((r = 0)\) [10], [11] and [12]:

\[ \frac{\partial w}{\partial r} = 0 \]

The condition of adhesion to the wall \((r = 1)\) results in:

\[ w(r=1) = 0 \]

The temperature obeys to the following boundary conditions on the wall \((r = 1)\) we have:

\[ T(r=1) = 1 \]

On the axis of the conduct \((r = 0)\) we have:

\[ \left. \frac{\partial T}{\partial r} \right|_{r=0} = 0 \]

The problem is formulated by considering a fluid:

- **compressible with [5]:**
  \[ \rho = M_o \frac{(1 - f_v)}{N_a V_0} \]
  \[ f_v = \left[ 1 - \exp \left( -\frac{E_0}{RT} \right) \right] \frac{3}{2} - 1 \]

- **non-Newtonian with [6]:**
  \[ \eta_u = \eta_0 \alpha_{ref} \exp \left( \frac{E}{RT} \left( \frac{1}{T_{ref}} - \frac{1}{T_{rev}} \right) \right) \]
  \[ \eta_0 = \eta_0 \left( 1 + \left( \frac{\partial w}{\partial r} \right)^2 \right) \]
The electric force is given by:
\[
\vec{f}_e = \rho_e \vec{E} - \frac{1}{2} \vec{E} \times \nabla \vec{E} + \frac{1}{2} \nabla \left( \rho \frac{\partial \vec{E}}{\partial \rho} \right)
\]

Maxwell's equations are written as:
\[
\nabla \times \vec{E} = 0
\]
\[
\frac{\partial \rho}{\partial t} + \nabla \left( \sigma \vec{E} + \rho_e \vec{V} \right) = 0
\]

### III. Numerical method

The obtained algebraic equations are solved using the method of double scanning [7], [9] and [13].

The calculations will be initiated by an initial profile that whatever may be provided that meets the boundary conditions. However, to reduce the computation time, we choose an initial profile that is fairly close to the actual profile. We adopt for this a profile corresponding to a steady flow of a Newtonian fluid in rigid cylindrical pipe. By solving the flow equation, we determine the axial velocity component. Using this component of the velocity and the temperature equation, we determine the temperature. The corrected velocity component can permit the iteration until convergence of the solution.

**Data Program** [5], [8], [14] and [15]:
- Newtonian Viscosity of reference, \( \eta_0 \) is 4000 Pa.s
- Thermal conductivity, \( \lambda_i \), is 0178 Wm-1K-1
- Heat capacity, \( c_p \), is 2650 Jkg-1K-1
- Pressure at the entrance of the pipe is 7 MPa
- Pressure at the outlet of the pipe is 3 MPa
- The ratio of the activation energy and the gas constant is 9.73 K\(^{-1}\)

**Convergence test**

The convergence test of the solution for the studied problem concerns the axial velocity. If \( m \) is the number of computes cycles and \( \varepsilon_1 \) the approximation fixed in advance, then we impose to the axial velocity to verify [17]:

\[
\sup \left| \frac{\hat{w}_{m+1} - \hat{w}_m}{\hat{w}_m} \right| \leq \varepsilon_1
\]
IV. Results and Discussion

Figure 1 shows the temporary evolution of dimensionless temperature as a function of dimensionless radius in two cases: 1) heating without injection 2) heating with injection of electric charges. One can notice that at the beginning of the simulation the temperature profiles in two cases are combined. The electrical and thermal fields compete each other and cause the appearance of secondary structures in the profile of the dimensionless temperature with injection of electric charges, and growing up when time increases. The same effect appears in the dimensionless axial velocity (Figure 3) and in the dimensionless apparent viscosity (Fig. 2). One can always remark the presence of secondary structures and especially at the main axis of the cylinder. These secondary structures are due to the instability of the group $\frac{G_c}{G_f}$, we obtain results qualitatively similar to those obtained by other authors [16].
Fig1: dimensionless temperature profile as function of dimensionless radius for different times.
Fig2: Profile of the dimensionless apparent viscosity as function of dimensionless radius for different times

Fig3: Profile of the dimensionless axial velocity as function of dimensionless radius for different times
V. Conclusion

Numerical simulations show the effect of the unipolar injection of electric charges on the heat transfer in a layer of a dielectric fluid. It was observed that the injection of unipolar charges changed dramatically the topology of the thermo-convective flow. The effect of electric field on the flow is very strong in this case since the structure of the flow is fundamentally different.

Acknowledgements

M. DRIOUIH is supported by the National Centre for Scientific and Technical Researcher of Morocco, Abdus Salam International Centre for Theoretical Physics and Reseau Universitaire de Mécanique RUMEC of Morocco.

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Received: February, 2012