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Friedmann Cosmological Model in the Time Dependent Quasi-Maxwell Formalism

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Abstract

In this paper, we have investigated the spatially homogeneous isotropic Friedmann cosmological model in the time dependent quasi-Maxwell[1] formalism.

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1 TQM equations

A stationary spacetime[2] $(M, \mathrm{g}_{\mu\nu})$ is a 4-dim Lorentzian manifold with a time-like Killing vector field η^μ. We consider the observers in this spacetime have the velocity components $w^\mu = \frac{\eta^\mu}{\eta}$ in η^μ direction, where $\eta = (\mathrm{g}_{\mu\nu}\eta^\mu\eta^\nu)^{\frac{1}{2}}$. In projection formalism, the metric is decomposed as, [1,2]:

$$ds^2 = (w_\mu dx^\mu)^2 + (\mathrm{g}_{\mu\nu} - w_\mu w_\nu)dx^\mu dx^\nu. \tag{1}$$

If we choose $\{\eta^\mu\} = (1, 0, 0, 0)$ and $\{w^\mu\} = (\frac{1}{\sqrt{h}}, 0, 0, 0)$, where h is a function of x^μ, then the metric takes the following form, [1-3]:

$$ds^2 = h(dt - \mathrm{g}_i dx^i)^2 - \gamma_{ij}dx^i dx^j, \tag{2}$$

where the components of metric are

$$\mathrm{g}_{00} = h\ ,\ \mathrm{g}_{0i} = -h\mathrm{g}_i\ ,\ \mathrm{g}_{ij} = -\gamma_{ij} + h\mathrm{g}_i\mathrm{g}_j. \tag{3}$$

[1] Hereinafter abbreviated as TQM.

[2] We use Einstein summation convention with indices $\alpha, \beta, \ldots = 0, 1, 2, 3$ and indices $i, j, \ldots = 1, 2, 3$.

It is interesting to rewrite the Einsteins field equations in terms of gravitoelectromagnetism fields[3] in γ-space[4] with time dependent metric γ_{ij}. Hence, the field equations can be written as the TQM equations[5] , [6, 7]:

$$^*\nabla\cdot{}^*\mathbf{E} = {}^*\mathrm{E}^2 + \frac{1}{2}{}^*\mathrm{B}^2 - \frac{^*\partial \mathrm{D}}{\partial t} - d - \frac{1}{2}(\zeta + \mathrm{U}), \tag{4}$$

$$^*\nabla\times{}^*\mathbf{B} = 2({}^*\mathbf{S} + {}^*\mathbf{M} - \mathbf{j}_m), \tag{5}$$

$$\begin{aligned} {}^*\mathrm{K}_{ij} &= -{}^*\nabla_{(i}{}^*\mathrm{E}_{j)} + {}^*\mathrm{E}_i{}^*\mathrm{E}_j + \frac{1}{2}({}^*\mathrm{B}_i{}^*\mathrm{B}_j - \gamma_{ij}{}^*\mathrm{B}^2) + \\ &+ 2\mathrm{D}_{ik}\mathrm{D}^k_j - \mathrm{D}\mathrm{D}_{ij} + \sqrt{\gamma}\,\varepsilon_{nk(i}\mathrm{D}^n_{j)}{}^*\mathrm{B}^k - \frac{^*\partial \mathrm{D}_{ij}}{\partial t} + \mathrm{U}_{ij} + \frac{1}{2}\gamma_{ij}(\zeta - \mathrm{U}), \end{aligned} \tag{6}$$

where $\zeta = \dfrac{\mathrm{T}_{00}}{h}$ is density of the moving substance, $\mathrm{j}^i_m = \dfrac{\mathrm{T}^i_0}{\sqrt{h}}$ is the momentum density, $\mathrm{U}_{ij} = \mathrm{T}_{ij}$ is 3-dim kinematic stress tensors and $\mathrm{U} = \mathrm{U}^i_i$ while $\mathrm{T}_{\mu\nu}$ are energy-momentum tensors. Also, $\dfrac{^*\partial}{\partial t} = \dfrac{1}{\sqrt{h}}\dfrac{\partial}{\partial t}$, $\gamma = \det(\gamma_{ij})$ and $d = \mathrm{D}_{ij}\mathrm{D}^{ij}$ such that

$$\mathrm{D}_{ij} = \frac{1}{2}\frac{^*\partial\gamma_{ij}}{\partial t},\ \mathrm{D}^{ij} = -\frac{1}{2}\frac{^*\partial\gamma^{ij}}{\partial t},\ \mathrm{D} = \gamma^{ij}\mathrm{D}_{ij} = \frac{^*\partial\ln\sqrt{\gamma}}{\partial t}, \tag{7}$$

and time dependent gravitoelectromagnetism fields are defined in terms of gravoelectric potential $\psi = \ln\sqrt{h}$ and gravomagnetic vector potential $\mathbf{g} = (\mathrm{g}_1, \mathrm{g}_2, \mathrm{g}_3)$ as follows[6]

$$^*\mathbf{E} = -{}^*\nabla\psi - \frac{\partial \mathbf{g}}{\partial t}\ ;\ {}^*\mathrm{E}_i = -\psi_{*i} - \frac{\partial \mathrm{g}_i}{\partial t}, \tag{8}$$

$$\frac{^*\mathbf{B}}{\sqrt{h}} = {}^*\nabla\times\mathbf{g}\ ;\ \frac{^*\mathrm{B}^i}{\sqrt{h}} = \frac{\varepsilon^{ijk}}{2\sqrt{\gamma}}\,\mathrm{g}_{[k*j]}. \tag{9}$$

In equation (6), $^*\mathrm{K}_{ij}$ is 3-dim starry Ricci tensor constructed from 3-dim starry Christoffel symbols as $^*\mathrm{K}_{ij} = {}^*\lambda^k_{ij*k} - {}^*\lambda^k_{ik*j} + {}^*\lambda^n_{ij}{}^*\lambda^k_{kn} - {}^*\lambda^n_{ik}{}^*\lambda^k_{nj}$ where $^*\lambda^i_{jk} = \dfrac{1}{2}\gamma^{il}(\gamma_{jl*k} + \gamma_{kl*j} - \gamma_{jk*l})$ and also the starry covariant derivatives of an arbitrary 3-vector and a tensor are given by $^*\nabla_j\mathrm{A}_i = \mathrm{A}_{i*j} - {}^*\lambda^k_{ij}\mathrm{A}_k$ and $^*\nabla_k\mathrm{T}^{ij} =$

[3] See reference [4] for a discussion of this point.

[4] The quotient space obtained by quotienting spacetime by the action of the stationary isometry and it represents the collection of the orbits of the Killing vectors η^μ, [5].

[5] The symbols () and [] represent the commutation and anticommutation over indices, gravitational units with c=G=1 are used and the 3-dim Levi-Civita tensor ε_{ijk} is antisymmetric under interchange of any pair of indices such that $\varepsilon_{123} = \varepsilon^{123} = 1$, [1]. Also, we note that $^*\mathrm{E}^2_g = \gamma^{ij}\,{}^*\mathrm{E}_{gi}{}^*\mathrm{E}_{gj}$.

[6] Note that the divergence and curl of an arbitrary vector in γ-space are defined by $^*\nabla\cdot\mathbf{A} = \dfrac{1}{\sqrt{\gamma}}(\sqrt{\gamma}\,\mathrm{A}^i)_{*i}$ and $(^*\nabla\times\mathbf{A})^i = \dfrac{\varepsilon^{ijk}}{2\sqrt{\gamma}}\,\mathrm{A}_{[k*j]}$ while $_{*i} = {}^*\partial_i = \partial_i + \mathrm{g}_i\dfrac{\partial}{\partial t}$.

$\mathrm{T}^{ij}_{*k} + {}^*\lambda^i_{nk}\mathrm{T}^{jn} + {}^*\lambda^j_{nk}\mathrm{T}^{in}$. Finally, the vectors ${}^*\mathbf{S} = {}^*\mathbf{E} \times {}^*\mathbf{B}$ and $\mathbf{M}$ have components as ${}^*\mathrm{S}^i = \frac{\varepsilon^{ijk}}{\sqrt{\gamma}}\,{}^*\mathrm{E}_j{}^*\mathrm{B}_k$ and ${}^*\mathrm{M}^i = -{}^*\nabla_j\mathrm{D}^{ij} + {}^*\partial^i\mathrm{D}$.

2 Exact solution of the FRW metric via TQM equations

We consider the spatially homogeneous and isotropic Friedmann-Robertson-Walker (FRW) line element in the form

$$ds^2 = dt^2 - a^2(t)[\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2], \tag{10}$$

where k is the curvature parameter which takes the values -1, 0 and $+1$ for closed, flat and open models, respectively, and $a(t)$ is the scale factor of universe. As before, it is not difficult to check that all components of gravitoelectromagnetism fields are zero and also the nonzero starry Christoffel symbols are

$$\begin{aligned}
{}^*\lambda^1_{11} &= \frac{kr}{1-kr^2},\\
{}^*\lambda^1_{22} &= -r(1-kr^2),\\
{}^*\lambda^1_{33} &= -r(1-kr^2)\sin^2\theta,\\
{}^*\lambda^2_{12} &= \frac{1}{r},\\
{}^*\lambda^2_{33} &= -\frac{1}{2}\sin 2\theta,\\
{}^*\lambda^3_{13} &= \frac{1}{r},\\
{}^*\lambda^3_{23} &= \cot\theta.
\end{aligned} \tag{11}$$

In continuation, with applying these symbols, we can conclude

$$ {}^*\mathrm{K}_{ij} = \begin{cases} \frac{2k}{1-kr^2} & i,j=1,\\ 2kr^2 & i,j=2,\\ \sin^2\theta\,{}^*\mathrm{K}_{22} & i,j=3,\\ 0 & i\neq j. \end{cases} \tag{12}$$

We now assume that the source of the gravitational field is a perfect fluid, i.e.

$$\mathrm{T}_{\mu\nu} = (\rho + p)u_\mu u_\nu - p\mathrm{g}_{\mu\nu}, \tag{13}$$

where ρ, p and u_μ are, respectively, the energy density, isotropic pressure and 4-velocity vector of the matter distribution with co-moving coordinates as

$u_\alpha = (1, 0, 0, 0)$. Using equations (12) and (13), after some work, we find that the TQM equations reduce to[7]

$$6\mathrm{H}^2 + 6\dot{\mathrm{H}} + \rho + 3p = 0, \tag{14}$$

$$6\mathrm{H}^2 + 2\dot{\mathrm{H}} - \rho + p + \frac{4k}{a^2} = 0, \tag{15}$$

where $\mathrm{H} = \dfrac{\dot{a}}{a}$ is the Hubble parameter and the quantities ρ and p depend on t only. On the other hand, the equations of the law of energy are, [8]:

$$^*\nabla \cdot \mathbf{j}_m + \frac{^*\partial\zeta}{\partial t} + \mathrm{D}\zeta + \mathrm{U}_{ij}\mathrm{D}^{ij} - 2\mathrm{j}_m^k\, {}^*\mathrm{E}_k = 0, \tag{16}$$

$$\frac{^*\partial\, \mathbf{j}_m}{\partial t} + \mathrm{D}\mathbf{j}_m - \zeta\, {}^*\mathbf{E} - \mathbf{j}_m \times {}^*\mathbf{B} + \mathbf{\Pi} = 0, \tag{17}$$

here $\Pi^i = {}^*\nabla_k \mathrm{U}^{ik} - {}^*\mathrm{E}_k \mathrm{U}^{ik}$. A simple calculation shows that

$$\mathbf{\Pi} = {}^*\mathbf{M} = 0. \tag{18}$$

Therefore, equations (5) and (17) are trivial. Next, equation (16) changed to

$$\dot{\rho} + 3\mathrm{H}(\rho + p) = 0. \tag{19}$$

Finally, from equations (14), (15) and (19), we conclude that the exact solution of cosmological model via time dependent quasi-Maxwell equations is exactly equal to the Friedmann equations in the standard cosmology.

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[7] The overdot means differentiation with respect to the time.

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