

# Extended Entanglement to Quantum Networks

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## Abstract

We suggest two schemes to generate bipartite entangled states by means of a quantum measurement at a third party. The two parties to be entangled have separate entangled states with the third party in modes  $C_1$  and  $C_2$ . In our first scheme we generate entanglement between the two remote parties by considering the modes  $C_1$  and  $C_2$  indistinguishable. However, in the second scheme we generate entangled states by considering the two modes to be distinguishable. We discuss that the first scheme of remote entanglement generation can be extended to any  $N$  number of parties. On making a quantum measurement on this system, we develop quantum networks, based on W-states and other multipartite symmetric entangled states.

## 1 Introduction

Quantum networking is an art of communication between many parties based on quantum principles. The presence of quantum channels in quantum networking provides an advantage on its classical counterpart. Entanglement provides a successful mean to develop quantum channels, and, hence serves as an essential element in quantum communication [1, 2, 3], quantum cryptography [4, 5, 6, 7], and quantum computation [8, 9]. In this paper we suggest techniques to generate entanglement between remote parties, far apart from each other and without any means to generate entanglement directly. Furthermore, we show that generalization of our scheme leads us to develop quantum networks.

The generation of bipartite entanglement has been performed successfully between two electromagnetic cavities [10, 11, 12, 13], multimodes of a single cavity [14], Bose-Einstein condensates [15, 16, 17, 18], internal states of atoms [19, 20, 21, 22, 23, 24, 25], and in ions [26]. In order to generate these entanglements, we use an entanglement generator which may be an excited atom [10, 11, 14] or a cavity field [19, 20, 21, 22, 23, 24, 25, 26, 27, 28]. In

our scheme quantum measurement process plays the role of quantum entanglement generator. A fascinating question is the development of multipartite entanglement [29, 30, 31, 32], which may lead us to quantum networks. It has been discussed [33, 34, 35] that a finite system of  $N$  qubits, in which every pair out of  $N(N - 1)/2$  pairs is entangled, makes a quantum network. Physical realization of such systems in laboratory experiments sets an interesting goal for the experimentalists as well. In this paper, we discuss that our system acts as an interesting playground to develop quantum networks accessible in laboratory.

In our scheme we have two remote parties, Alice and Bob, which have electromagnetic cavities separately entangled with a third cavity of Charles. With the help of this system: (i) We develop bipartite entanglement between the two remote cavities of Alice and Bob in modes  $C_1$  and  $C_2$ , respectively, by means of quantum measurements at the cavity of Charles. In this scheme we consider the modes  $C_1$  and  $C_2$  as indistinguishable. This provides an information transfer capability between Alice and Bob; (ii) We develop tripartite entanglement of the entangled states, namely –super entanglement, via quantum measurement at Charles, by considering separate entangled states of Alice and Bob with Charles as distinguishable. We show that generalization of remote entanglement from two party system to  $N$  party system leads us to develop quantum networks between the  $N$  parties. The so-developed quantum network fabrication technique makes it possible to realize quantum networks corresponding to maximally entangled totally symmetric W-states and to other multipartite totally symmetric entangled states.

We have organized our paper, so that, in Sec. II, we present our system. In Sec. III, we develop entanglement between two remote parties for indistinguishable modes, whereas, in Sec. IV, we develop a three party super-entanglement, using distinguishable modes. In Sec. V, we discuss quantum entanglement which we may obtain as a generalization of our scheme of remote entanglement from two cavities to many cavities and leading to quantum networks.

## 2 The Model

We consider a three party system of Alice, Bob and Charles, such that, Alice-Charles and Bob-Charles systems are in entangled states. We may engineer such entangled state by following the scheme suggested in Ref. [10]. We consider a two level atom, initially prepared in the excited state, with transition frequency  $\nu_1$  between the two levels. Moreover we take the atom in resonance with the cavity mode  $C_1$ , as shown in Fig. 1. The atom propagates through the cavities  $A$  and  $C$ , of Alice and Charles, respectively. After its propagation through the two cavities we measure the atom in ground state. Therefore, it contributes a photon of frequency  $\nu_1$ , in either of the two cavities which leads

to a maximal entangled state between  $A$  and  $C$ , viz.,

$$|A C_1\rangle = \frac{1}{\sqrt{2}} [|1 0\rangle + |0 1\rangle]. \quad (1)$$

In order to generate the entangled states we use identical high-Q cavities of few centimeters in length. Moreover, we use the atoms with the two atomic levels as circular Rydberg levels [10, 14, 20], which have large radiative decay times and are strongly coupled to microwaves. The interaction time of atom moving with velocity 400m/s is of the order of few microseconds. Hence cavity with life time of the order of few milliseconds ensures that atom does not undergo radiative decay during its propagation through the cavity. We may follow the same procedure to develop entanglement between cavities  $B$  and  $C$ , by using another atom in excited state with transition frequency  $\nu_2$  which is in resonance with the cavity mode  $C_2$ . This provides the entangled state as

$$|B C_2\rangle = \frac{1}{\sqrt{2}} [|1 0\rangle + |0 1\rangle]. \quad (2)$$

Thus for the complete system of Alice, Bob and Charles, we may express the combined state as

$$\begin{aligned} |A B C_1 C_2\rangle = \frac{1}{2} & [|1 1 0_{C_1} 0_{C_2}\rangle + |0 0 1_{C_1} 1_{C_2}\rangle \\ & + |0 1 1_{C_1} 0_{C_2}\rangle + |1 0 0_{C_1} 1_{C_2}\rangle]. \end{aligned} \quad (3)$$

These four different terms describe that in cavity  $C$ , there may exist (i) no photon of either of the two modes; (ii) both the photons of the two modes; (iii) one photon in mode  $C_1$  with no photon in mode  $C_2$ ; and (iv) no photon in mode  $C_1$  and one photon in mode  $C_2$ , with the same probability.

### 3 Remote Entanglement generation

We consider that the cavities  $A$  and  $B$  are far apart so that there is no possibility to generate entanglement between them directly. However, to develop remote entanglement between cavities  $A$  and  $B$ , of Alice and Bob, respectively, we may use the three party system discussed in Sec. II. We consider that the cavities  $A$  and  $B$  are separately entangled with the cavity  $C$ , in modes,  $C_1$  and  $C_2$  which are identical as  $\nu_1 = \nu_2$ . Under the condition of indistinguishability of photons in cavity  $C$ , we may express the complete state of the system, as

$$|A B C\rangle = \frac{1}{2} [|1 1 0\rangle + |0 0 2\rangle + |0 1 1\rangle + |1 0 1\rangle]. \quad (4)$$

Measurement of the electromagnetic field by Charles in the cavity  $C$  would lead to the collapse of the combined state of the system. As a result of this

measurement, Charles may find one of the following cases: no photon in cavity  $C$  of Charles leaving the cavities  $A$  and  $B$  in definite state, that is,  $|A B\rangle = \frac{1}{2}|1 1\rangle$ ; two photons in the cavity  $C$ , leaving the states of cavities  $A$  and  $B$  in vacuum state, that is,  $|A B\rangle = \frac{1}{2}|0 0\rangle$ ; one photon in the cavity  $C$  which gives partial information about the states of cavities  $A$  and  $B$ , as the detected photon may belong to either of the two cavities. In the last case, measurement of one photon in cavity  $C$  leaves two possibilities: either the cavity  $A$  possesses the remaining one photon and the cavity  $B$  is in vacuum state, or, the cavity  $A$  is in vacuum state and cavity  $B$  possesses one photon. Hence, we may express the cavity field state as

$$|A B\rangle = \langle 1|A B C\rangle = \frac{1}{2} [|1 0\rangle + |0 1\rangle]. \quad (5)$$

Therefore, we find that the measurement process has disentangled the cavity  $C$  from the cavities  $A$  and  $B$ . In addition it has led to entangle the cavity  $A$  and the cavity  $B$  with a probability of  $1/2$ . This enables Alice and Bob to develop quantum channel between them and to share any information independently.

## 4 Super-Entanglement Generation

The three party system discussed in Sec. II, may provide a possibility to generate entanglement of the already entangled states, that is, super-entanglement. The super-entanglement is based on quantum measurement by Charles of cavity  $C$  when the modes  $C_1$  and  $C_2$  are distinguishable. The initial state of the complete system is a product state expressed in Eq. (3).

In order to generate three party super-entanglement Charles uses a three level atom in  $\Lambda$  configuration. The atom is initially prepared in the superposition state of lower two levels, that is levels  $|1\rangle$  and  $|2\rangle$ . The atomic transition from level  $|1\rangle$  to  $|3\rangle$  is in resonance with the field mode  $C_1$  and from atomic level  $|2\rangle$  to  $|3\rangle$  is in resonance with the field mode  $C_2$  of the cavity  $C$ . After its interaction with the cavity  $C$ , Charles measures the atom in excited state. This measurement process leaves the three party system with super entanglement. The measurement process,  $\frac{1}{\sqrt{2}}(\langle 1_{C_1}| + \langle 1_{C_2}|) |A B C_1 C_2\rangle$  leads us to the super-entangled state, expressed as,

$$\begin{aligned} |\Psi\rangle &= \frac{1}{\sqrt{2}} (\langle 1_{C_1}| + \langle 1_{C_2}|) |A B C_1 C_2\rangle \\ &= \frac{1}{2} [|0\rangle|B C_2\rangle + |A C_1\rangle|0\rangle] \\ &= \frac{1}{2} [|0\rangle|\phi\rangle + |\phi\rangle|0\rangle], \end{aligned} \quad (6)$$

where,  $|\phi\rangle$  is the already entangled state  $|B C_2\rangle$  or  $|A C_1\rangle$ , given in Eqs. (1) and (2), respectively.

## 5 Quantum Networks: Generalization of Remote Entanglement

We generalize bipartite remote entanglement generation scheme, discussed in Sec. III, to any  $N$  number of electromagnetic cavities. This helps us to develop quantum system where each of the  $N$  cavities is entangled with the cavity  $C$ , as shown in Fig. 2(a). This leads us to generate quantum networks based on any symmetric state. For the purpose we perform quantum measurement of the cavity field state in cavity  $C$ .

The given experimental procedure enables us to generate all possible quantum networks based on totally symmetric entangled states. These are the states which are invariant under the interchange of any two cavities in the network. These also include  $N$ -party  $W$ -states which have maximum bipartite degree of entanglement. In totally symmetric states each cavity of the network is equally entangled with every other cavity in the system. As discussed in Sec. III, we note that remote entanglement can be generated in laboratory, making our system experimentally accessible.

In order to generalize remote entanglement, we consider a system of  $N$  cavities, namely  $A_1, A_2, \dots, A_N$ . We generate separate entangled state of each of the  $N$  cavities with a cavity  $C$ , by following the scheme, given in Sec II. All  $N$  cavities are entangled in the same mode, so that photons belonging to different cavities are indistinguishable from each other. Moreover, each of the cavities  $A_1, A_2, \dots, A_N$ , may have either *zero* or *one* photon which makes them act as a qubit. We may represent the combined state of the complete system as,

$$|\Phi\rangle = \frac{1}{\sqrt{2^N}} \sum_{i=0}^N |\Psi_i\rangle |N-i\rangle. \quad (7)$$

Here,  $|\Psi_i\rangle$  represents the  $i$ th state of  $N$  cavities together, whereas,  $|N-i\rangle$  represents the corresponding state of the cavity  $C$  in the system. We may express  $|\Psi_i\rangle$ , as

$$|\Psi_i\rangle = |i, N-i\rangle, \quad (8)$$

where,  $|i, N-i\rangle$  indicates a totally symmetric entangled state including  $i$  zeros and  $N-i$  ones in all permutations. For example, for  $N=3$  and  $i=1$ , we may express  $|\Psi_1\rangle$  as

$$|\Psi_1\rangle = [|1\ 1\ 0\rangle + |1\ 0\ 1\rangle + |0\ 1\ 1\rangle],$$

which remains invariant by interchanging any two of the cavities. Each  $|\Psi_i\rangle$  contains  $N!/i!(N-i)!$  permutations which act like quantum registers [35] of

size  $N$  each. All the  $N$  cavities have separate identical entangled state with cavity  $C$  initially, however, no entanglement exists between them. Therefore, the cavity  $C$  acts like a hub in the system, as shown in Fig. 2(a).

A measurement on cavity  $C$  of the cavity field photon number will generate entanglement between these remote cavities. The measurement process at cavity  $C$ , may find photons, from 0 to  $N$  in number. In addition, it leads to disentangle cavity  $C$  from the rest of the cavities, and to develop entanglement between the  $N$  cavities.

A quantum network relies on multipartite entanglement such that any two of the parties can share entangled state even when the information about the remaining  $N - 2$  parties is lost. In a system of  $N$  qubits it is not possible to get such bipartite entanglement with degree of entanglement as one [36]. The maximum possible bipartite entanglement is obtained when the  $N$  party system is in totally symmetric Werner state [37, 38], that is, when all except one qubit are either in state  $|0\rangle$  or  $|1\rangle$  in all permutations [33, 34, 35, 38]. For such states each cavity is equally entangled with every other cavity in the network as shown in Fig. 2(b). We may define the degree of bipartite entanglement for the state as a ratio between the number of entanglements associated with each party and the total number of bipartite entanglements present in the system. Since in an  $N$  party W-state, there are  $N - 1$  bipartite entangled pairs associated with each party and there are  $N(N - 1)/2$ , total entangled pairs in the system, therefore, we find the degree of bipartite entanglement in the system as  $2/N$ . This result has also been discussed in Ref. [33, 34, 38]. Hence, we see that in W-state degree of entanglement is 1 for a bipartite system but decreases as the number of parties in the network increases.

In our system, we may obtain all possible totally symmetric entangled states, depending upon the number of photons detected in cavity  $C$ . We get W-state whenever we detect  $N - 1$  number of photons in the cavity  $C$ . For such detection, Eq. (7) leads us to

$$|W\rangle = \langle N - 1 | \Phi \rangle, \quad (9)$$

$$= \frac{1}{\sqrt{2^N}} |N - 1, 1\rangle. \quad (10)$$

In the above state all parties are equally entangled and it gives maximum bipartite entanglement. The probability of getting such state in our system is  $N/2^N$ . We may develop another quantum network based on W-state by detecting 1 photon in cavity  $C$ . This leads to the state where all except one cavity are in state  $|0\rangle$ , so that, we find the N-partite system as

$$|W\rangle = \langle 1 | \Phi \rangle, \quad (11)$$

$$= \frac{1}{\sqrt{2^N}} |1, N - 1\rangle, \quad (12)$$

which leads to maximum bipartite entanglement in the network. The probability of developing such network is the same as earlier.

## 6 Discussion

In this paper we have suggested techniques to establish entanglement between remote parties by means of quantum measurements. In Sec. II, we discuss the entanglement generation via quantum measurement on a third party  $C$  which has separate entangled state with the two parties. This remote entanglement occurs under the condition that the third party is separately entangled with each of the two parties in the same optical mode. However, if the third party entanglement with the two parties occurs in different optical modes, we generate super-entanglement, that is, the entanglement between two already entangled states. The generation of this state requires that the measurement process detects only one photon without being able to differentiate between the two modes.

We have developed quantum networks between  $N$  parties by extending the remote entanglement scheme from two cavity system to  $N$  cavity system. Quantum measurement of the field photon number at the cavity  $C$ , which acts like a hub in the system, leads to develop quantum networks. Depending upon the number of photons, 1 or  $N - 1$ , detected in the cavity  $C$ , we may develop quantum networks in  $W$ -state. Such networks behave as optimum quantum networks as there exist maximum bipartite entanglement for any two communicating parties out of all. Detection of field photon number in between one and  $N - 1$ , leads to develop quantum networks based on other totally symmetric entangled states.

## References

- [1] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, *Phys. Rev. Lett.* **70**, 1895 (1993).
- [2] D. Boschi, S. Branca, F. De Martini, L. Hardy, and S. Popescu, *Phys. Rev. Lett.* **80**, 1121 (1998).
- [3] D. Bouwmeester, J. W. Pan, K. Mattle, M. Eible, H. Weinfurter, and A. Zeilinger, *Nature (London)* **390**, 575 (1997).
- [4] C. H. Bennet, and G. Brassard, *Int. Conf. Computers, Systems, and Signal Processing*, Bangalore, India, 175 (1984).
- [5] A. K. Ekert, *Phys. Rev. Lett.* **67**, 661 (1991).

- [6] A. K. Ekert, J. G. Rarity, P. R. Tapster, and G. Massimo Palma, *Phys. Rev. Lett.* **69**, 1293 (1992).
- [7] F. Saif, *Int. Conf. Physics in Industry, Karachi, Pakistan*, (2001).
- [8] P. W. Shor, *SIAM J. Comput.* **26**, 1484 (1997); L. K. Grover, *Phys. Rev. Lett.* **79**, 325 (1997).
- [9] L. K. Grover, *Phys. Rev. Lett.* **79**, 4709 (1997).
- [10] L. Davidovich, N. Zagury, M. Brune, J. M. Raimond, and S. Haroche, *Phys. Rev. A* **50**, R895 (1994).
- [11] M. Ikram, and F. Saif, accepted for publication in *Phys. Rev. A*; e-print arXiv: quantum-ph/0201044.
- [12] Abolfazl Bayat, Sougato Bose, and Pasquale Sodano, *Phys. Rev. Lett.* **105**, 187204 (2010).
- [13] Leonardo Banchi, Abolfazl Bayat, Paola Verrucchi, and Sougato Bose, *Phys. Rev. Lett.* **106**, 140501 (2011).
- [14] A. Rauschenbeutel, P. Bertet, S. Osnaghi, G. Nogues, M. Brune, J. M. Raimond, and S. Haroche, *Phys. Rev. A* **64**, 050301(R) (2001).
- [15] L.-M. Duan, J. I. Cirac, and P. Zoller, e-print arXiv: quantum-ph/0107055.
- [16] H. Pu, and P. Meystre, *Phys. Rev. Lett.* **85**, 3987 (2000).
- [17] K. Helmerson, and L. You, e-print arXiv: quantum-ph/0101089.
- [18] B. Deb, and G. S. Agarwal, e-print arXiv: quantum-ph/0112029.
- [19] J. I. Cirac, and P. Zoller, *Phys. Rev. A* **50**, R2799 (1994).
- [20] P. Bogar, and J. A. Bergou, *Phys. Rev. A* **53**, 49 (1996).
- [21] M. Freyberger, *Phys. Rev. A* **51**, 3347 (1995).
- [22] C. C. Gerry, *Phys. Rev. A* **53**, 4583 (1996).
- [23] C. Cabrillo, J. I. Cirac, P. Garcia-Fernandez, and P. Zoller, *Phys. Rev. A* **59**, 1025 (1999).
- [24] D. Jaksch, H. J. Briegel, J. I. Cirac, C. W. Gardiner, and P. Zoller, *Phys. Rev. Lett.* **82**, 1975 (1999).

- [25] A. Muthukrishnan, and C. R. Stroud Jr., e-print arXiv: quantum-ph/0111058.
- [26] B. Kneer, and C. K. Law, Phys. Rev. A **57**, 2096 (1998).
- [27] Jun-Gang Li, Jian Zou, and Bin Shao, Phys. Rev. A **82**, 42318 (2010).
- [28] K. Stannigel, P. Rabl, A. S. S rensen, P. Zoller, and M. D. Lukin, Phys. Rev. Lett. **105**, 220501 (2010).
- [29] C. H. Bennett, S. Popescu, D. Rohrlich, J. A. Smolin, and A. V. Thapliyalet, e-print arXiv: quantum-ph/9908073.
- [30] J. Kempe, Phys. Rev. A **60**, 910 (1999).
- [31] V. Coffman, J. Kundu, and W. K. Wootters, e-print arXiv: quantum-ph/0001114.
- [32] W. K. Wootters, e-print arXiv: quantum-ph/0001114.
- [33] W. Durr, e-print arXiv: quantum-ph/0006105 v1.
- [34] M. Koashi, V. Buzek, and N. Imoto, Phys. Rev. A **62**, 05032(R) (2000).
- [35] M. Sasura, and V. Buzek, Phys. Rev. A **64**, 012305 (2001); e-print arXiv: quantum-ph/0103067.
- [36] V. Coffman, J. Kunda, and W. K. Wootters, e-print arXiv: quantum-ph/9907047.
- [37] R. F. Werner, Phys. Rev. A **40**, 4277 (1989).
- [38] W. Durr, G. Vidal, and J. I. Cirac, e-print arXiv: quantum-ph/0005115.

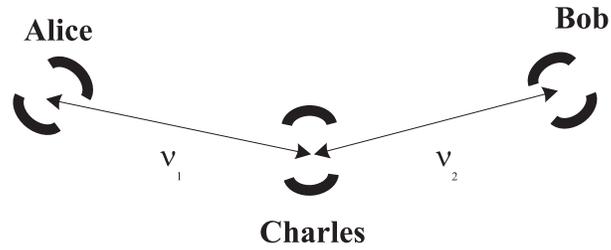


Fig. 1

Figure 1: The experimental setup: The two cavities of Alice and Bob have separate entanglement with the cavity of Charles, in modes  $C_1$  and  $C_2$  with frequency  $\nu_1$  and  $\nu_2$ , respectively. In order to develop remote entanglement we make measurement of the cavity field at the the cavity of Charles, keeping the two modes indistinguishable. The distinguishability of the two modes generates super-entanglement.

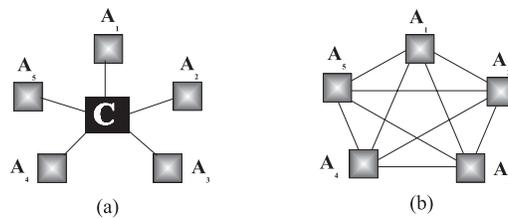


Fig. 2

Figure 2: Quantum networks: Generalization of the remote entanglement scheme from two cavity system to  $N$  cavity system leads to generate quantum network. In case we measure *one* photon or  $N - 1$  photons at cavity  $C$  of Charles we develop networks based on W-state as shown in (b).

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