Analyzing Power of $d(\bar{n} , n)d$

Elastic Scattering at Low Energy

Abusini Majid

Department of Physics
Al-Al bayt university
P.O. Box 928125
Al-Mafraq, Jordan
abusini@aabu.edu.jo

Abstract
An approach to studying three-body reactions $d(\bar{n} , n)d$ that takes consistently into account the single-collision mechanism is discussed. Specific calculations are performed for $d(\bar{n} , n)d$ elastic scattering which give a good description of cross section and polarization observable ($A_Y$) at low energy. The calculations based on modern realistic $2N$ interactions are compared with the experimental data.

Keywords: Impulse approximation, Analyzing power, Faddeev formalism

1 Introduction

The study of spin observable have become very important in nuclear physics. The calculated cross-sections and vector analyzing powers $A_Y$ for the system of $(n − d)$ using the $2N$ forces such as Bonn-CD [ R. Machleidt, 1996 ], Nijmegen II [V.G.J. Stokes, 1994] and Bonn-B [ R. Machleidt, 1989 ] without $3N$- forces pointed out that the $(n − d)$ system turn into a $A_Y$ puzzle and may be cured by $3N$- forces. The three-nucleon analyzing power puzzle was discovered by the Bochum-Krak´ow group when they compared their rigorous neutron-deuteron calculations based on Paris nucleon-nucleon ($NN$) potential [H. Witala,1987] with high precision analyzing power data for neutron energies below $E_n = 15$ Mev [W. Tornow et al.,1983]. Ever since that time the discrepancy between measurements and calculations has been the subject of intensive theoretical and experimental studies [ Y. Koike, 1987 ;C.R. Howell et al.,1987;A. Kievsky, 1995]. With the large amount of accurate $(p − d)$ and $(d − p)$ data available and with the use of the theoretical phase shifts of the Pisa group as starting values, it was possible to perform accurate phase-shift analysis. The main difference between the theoretical and experimental
phase shifts was observed for the $^3P_J$ phase shifts, which are directly related to the $NN$ interactions. This result triggered speculation that the low-energy $NN$ phase shifts, which cannot be determined accurately from $NN$ data which may be responsible for the $A_Y$ puzzle. Straight forward application of the Faddeev equations [L.D. Faddeev, 1960] to three-body problems involves technical difficulties. For this reason, theorists usually resort to various approximate schemes. In particular, unitary schemes were proposed in [R. T. Cahill, 1972; K. L. Kowalski, 1972; T. Sasakawa, 1973]. However, these schemes did not become popular because they do not simplify calculations substantially. The method of straightforwardly summing the truncated Watson–Faddeev iteration series [J. M. Wallac, 1973] also proved to be inefficient. Recently two formally different unitary approaches to studying three-body problem were proposed—the cutoff three-body impulse approximation and the unitarized three-body impulse approximation [J. V. Mebonia and T. I. Kvarackheliya, 1980]. Either approach is based on consistently taking into account the single-collision mechanism, but the specific implementations of this were different. The unitarized three-body impulse approximation based on approximately solving the Faddeev equations in the $K$–matrix formalism, then the differential elastic scattering cross section calculated within the unitarized three-body impulse approximation compares with the well-known Glauber–Sitenko formula. The cutoff three-body impulse approximation is based on the statement that, Faddeev integral equations can be solved in the $T$–matrix formalism by retaining only first order terms which correspond to true single collision, where the projectile particle does not hit simultaneously all particles of the bound state. The objective of the present study is to test the mentioned unitary approach ‘the three body impulse approximation with cut-off’ to calculate the differential cross section for elastic $nd$ scattering at low energy, since the differential cross-section (angular distribution) is not too sensitive to the particle spin and it can only provide a rough overview of the quality of the calculation, therefore we employed the same approach to the much more sensitive to the particle spin ‘vector analyzing power’ ($A_Y$) for polarized neutron deuteron elastic scattering $d(\vec{n}', n)d$ at incident neutron energy of 1.2 and 1.9 Mev.

2 Three-body impulse approximation with cut-off.

The Faddeev equations can be solved in the $T$–matrix formalism by retaining only first-order terms. However, such terms would correspond to a single collision proper only if the incident particle (say, particle 1) does not interact simultaneously with the two particles forming the bound state (particles 2 and
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3). So at the moment of the collision of some pair of the particles the third particle should be far from the event, otherwise the picture will be distorted at small distance. The realization of such requirement is accomplished by cutoff the wave function of the bounded state in coordinate space. So the Fourier transform of the wave function $F(\vec{q})$ in momentum space $\vec{q}$ which is defined by the equation $F(\vec{q}) = (2\pi)^{-\frac{\beta}{2}} \int_0^\infty \exp(-i\vec{q}\vec{r}) \varphi(r) dr$ should be replaced by the cutoff Fourier transform of the radial wave function $\varphi(r)$ in coordinate space for the bound state, that is by making the substitution: $F(\vec{q}) \rightarrow F(\vec{q}, R)$,

Where

$$F(\vec{q}, R) = \sqrt{\frac{2}{\pi}} \int_R^\infty r^2 dr \varphi(r) \frac{\sin(qr)}{qr}$$ (1)

Here $F(\vec{q}, R)$ is the Fourier transform of the radial wave function of bound state, $\varphi(r)$ - the radial wave function of bound state (deuteron) in coordinate space and $R$ - is the cutoff radius. The cutoff radius $R$ must be longer than the de Broglie wavelength $\bar{\lambda}$ ($\bar{\lambda} = \frac{\hbar}{k} = \frac{1}{2\pi}$) where $k$ is the wave number of the relative motion of colliding particles associated with the motion of particle 1 with respect to the bound state $2,3$ particles system, and it is related to the momentum of colliding particles by the following equation:

$$R = \frac{\mu |\vec{\chi}|}{\bar{\lambda}}$$ (2)

where $\vec{\chi}$ is a relative momentum of colliding particles 1 and the system of bound state $2,3$; $\mu$ is a constant that ensures fulfillment of the condition $\bar{\lambda} \ll R$.

It should be mentioned that cutoff procedure was declared as one of the methods of the consistent allowance made for the single collision. It is clear from (2) that $R$ depends on the collision energy, moreover, $R$ should increase as the energy decreases, since the wavelength of the incident particle increases with the decrease of energy and, hence, it can “knock against” the both particles of the target. The amplitude for three-body scattering $T_{fi}$ within the three body impulse approximation with cut-off then assumes the form

$$T_{fi} = \hat{A}(\phi_f | \sum_{2,3} t_{1i} | \phi_i)$$ (3)

where $t_{1i}$ - is the two-body scattering matrix for particles 2 and 3 ($123 = 123, 231, 312$) $\phi_i, \phi_f$ are the asymptotic functions associated with the initial and the final state, respectively; $\hat{A}$ is the operator of antisymmetrization.
with respect to the identical particles. The three body impulse approximation with cut-off approach appeared to be efficient in analyzing various three-body processes \[ J. V. Mebonia, 2003; O. L. Bartaya and Dzh, 1981; V. Sh. Jikia and J. V. Mebonia, 1995; J. V. Mebonia and M. A. Abusini, 2000. \] In view of the simplicity of this approach, it is therefore advisable to make an attempt at generalizing the three body impulse approximation with cut-off to processes more complicated than three-particle ones. In the present study, we analyze neutron-deuteron \( (nd) \) elastic scattering in the center mass frame \( (c.m) \). The problem can be solved in a closed form for any realistic nucleon–nucleon \( (NN) \) potential. These potentials leading to the same results for the on energy-shell amplitudes of nucleon–nucleon scattering. In this respect, available information comes not only from the differential cross sections \( \frac{d\sigma}{d\Omega} \) but also from the vector of analyzing power \( A_Y(\theta) \). Such studies make it possible to test various approximate methods for solving three-body problems in order to extend them to more complicated cases. Formula \( (3) \) implies that the differential cross section for elastic \( nd \) scattering in the \( c.m \) frame can be represented in the form:

\[
\frac{d\sigma}{d\Omega} = (2\pi)^4 \frac{2m^2}{27} \sum_{\text{spins}} |T|^2
\]

where

the \( T \)-matrix is defined as:

\[
T = \hat{A} \sum_{2,3} \Psi^*_{23}(\vec{k}_{23}) t_{12}(\vec{k}_{12}, \vec{k}_{12}, \varepsilon_{12}) \Psi_{23}(\vec{k}_{23}, R) d\vec{k}_3
\]

Where

\( \Psi_{23} \) is the total deuteron wave function in momentum space, \( t_{12} \) is the radial part of the two nucleon-scattering matrix element \( (1 \text{ and } 2) \). Summation in formula \( (4) \) is performed over the neutron and deuteron-spin projections prior to and after a collision event; explicitly, these spin projections, as well as the functions \( F(\vec{q}) \) and \( F(\vec{q}, R) \), appear after expanding \( nd \) and \( t_{12} \) in partial waves.

Using the momentum and energy conservation laws for \( (nd) \) neutron-deuteron elastic scattering:

\[
\vec{k}_1 + \vec{k}_{23} = \vec{k}_1' + \vec{k}_{23}', \quad E_1 + E_{23} = E_1' + E_{23}'
\]

where
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\(\vec{k}_{23}(\vec{k}_{23})\) is the momentum of the relative motion of particles 2 and 3 (deuteron) in the initial (final) state, \(\vec{k}_1(\vec{k}_1)\) is the momentum of the incident neutron in the initial (final) state, \(E_{23}(E'_{23})\) is the energy of relative motion of particles 2 and 3 (deuteron) in the initial (final) state, \(E_1(E'_1)\) is the energy of the incident neutron in the initial (final) state. We obtain the following expressions for the momentum and energy of the relative motion of particles in the initial and final states, respectively:

\[
\vec{k}_{23} = -\frac{1}{2} \vec{k}_1 - \vec{k}_3
\]

\(7\)

\[
\vec{k}'_{23} = -\frac{1}{2} \vec{k}'_1 - \vec{k}_3
\]

\(8\)

\[
\vec{k}_{12} = \vec{k}_1 - \frac{1}{2} \vec{k}_3
\]

\(9\)

\[
\vec{k}'_{12} = \vec{k}'_1 + \frac{1}{2} \vec{k}_3
\]

\(10\)

\[
\varepsilon_{12} = \frac{3}{4m}(k_1^2 - k_3^2) - Q
\]

\(11\)

where \(\vec{k}_{12}(\vec{k}'_{12})\) is the momentum of the relative motion of particles 1 and 2 in the initial (final) state, \(\varepsilon_{12}\) is the relative energy of particles 1 and 2, \(m\) is the nucleon mass, \(Q\) is the deuteron binding energy (\(Q = 2.23\) MeV).

3 Analyzing power

In the study of the fundamental strong interaction, spin observables have become very important in nuclear physics, especially since technologies now exist for producing polarized targets. Measurements from this experiment test our understanding of the spin dependent parts of the strong interaction. In general, vector analyzing powers \(A_Y\) are defined as a difference between cross sections with different orientations of incoming particles normalized to unpolarized cross sections. The expression for the differential cross section obtained with a polarized beam can be written as:

\[
\sigma(\theta, \varphi) = \sigma_0(\theta, \varphi)[1 \pm \sum P_Y A_Y(\theta)]
\]

\(12\)
Where $\sigma_0(\theta, \varphi)$ the differential cross section of unpolarized beam, $P_Y$ is the polarization vector. Combining the two equations, the product of $P_Y A_Y$ is equal to the up-down ($\uparrow \downarrow$) asymmetry of the differential cross section $(\sigma_{\uparrow}, \sigma_{\downarrow})$, then the differential cross section are given by

$$\sigma_{\uparrow}(\theta, \varphi) = \sigma_0(\theta, \varphi)[1 + \sum P_Y A_Y(\theta)]$$

$$\sigma_{\downarrow}(\theta, \varphi) = \sigma_0(\theta, \varphi)[1 - \sum P_Y A_Y(\theta)]$$

Combining the two equations (13) and (14), we get

$$P_Y A_Y(\theta) = \frac{\sigma_{\uparrow} - \sigma_{\downarrow}}{\sigma_{\uparrow} + \sigma_{\downarrow}}$$

The analyzing power is then calculated using the expression:

$$A_Y(\theta) = \frac{1}{P_Y} \frac{\sigma_{\uparrow} - \sigma_{\downarrow}}{\sigma_{\uparrow} + \sigma_{\downarrow}}$$

4 Results and Discussions:

We applied our theoretical approach (three body impulse approximation with cut-off) to calculate the differential cross section for elastic $nd$ scattering as a function of the scattering angle by using the equation (4) in the c.m frame. We used the system of units where $\hbar = c = 1$. In our calculation, the two-nucleon off-energy-shell $T-$matrix and the radial part of the deuteron wave function $\varphi(r)$ were constructed using the nonlocal separable Mongan potential [Th. Mongan,1968]. The results of our calculation are displayed in (Fig.1,2) along with relevant experimental data at energies 1.2 and 1.9 MeV in the center mass frame. The solid curve show the results obtained within the three body impulse approximation with cut-off, whereas the dotted curve correspond to similar calculations without a cutoff ($R = 0$). Also we compare our theoretical results for the differential cross-section with the theoretical calculations based on another realistic potential CD-Bonn potential (dash-dotted curve) at incident energies 1.2 and 1.9 MeV, where the experimental data were taken from [W.Tornow, et al., 2003]. In the calculations, we took into account the following two-nucleon states $^1S_0, ^1P_1, ^1D_2, ^3S_1 + ^3D_1, ^3P_0, ^3P_1, ^3P_2 + ^3F_2$ and $^3D_2$. However, the eventual results are dominated by the contribution of zero partial-wave amplitudes, since we restrict our consideration to sufficiently low energies scattering. As can easily be seen, the results of the cutoff-free impulse approximation ($R = 0$),
which is usually associated with the single-collision mechanism, yields for the
differential cross section differ sizably from experimental data in an order of
magnitude. On the other hand, as simple as the application of cutoff \((R)\) to
the bound-state of radial wave function \(\varphi(r)\) improves considerably the agree-
ment between the theoretical results and the experimental data, where the
cutoff procedure \((R \gg 0)\) eliminates the small region where the three particles
would be interact closely (the third particle should be far from the event). We
note that some degree of arbitrariness in choosing the cutoff parameter \(\mu \gg 1\)
was used to normalize the theoretical plots to the experimental data, it turned
out that, the parameter \(\mu\) changed within 5\% \((\mu = 1.1 \pm 0.06)\) for the interval
of energy 1.2 -1.9 Mev of incident neutron eq.(2). Hence, the approximate
method for calculating the cross-sections of \(nd\)-scattering with simple models
of \(NN\)- interaction allows a satisfactory agreement with the obtained exper-
imental data. It should be noted that with increasing energy of the incident
particle, the role of the cutoff becomes less pronounced. This must have been
expected because, as the energy is increased, the wavelengths of the colliding
particles decrease, and according to equations (2) and (2), the contribution
associated with the discarded part of the bound-state wave function decreases.
On the other hand, besides the Mongan model (see ref.22), we examine other
realistic two nucleon (\(NN\)) potential – CD-Bonn potential , the results of our
calculations based on the nonlocal separable Mongan potential are in good
agreement with calculations based on CD-Bonn potential (see Fig1,2), that
has been expected since theses potentials leading to the same results for the
on-shell energy amplitudes of nucleon–nucleon scattering. Since the different-
al cross-section (angular distribution) is not too sensitive to the particle spin
and it can only provide a rough overview of the quality of the calculation,
therefore we employed the same approach three body impulse approximation
with cut-off to the much more sensitive \(A_Y\) for polarized neutron deuteron
elastic scattering\(d(\overrightarrow{n},n)d\) at incident neutron energy 1.2 and 1.9 Mev. The
calculated vector analyzing-powers \(A_Y\) using equations (4) and (16) are dis-
played in ( Fig. 3,4 ) with experimental data taken from [W.Tornow, et al.,
2003]. A comparison between the experiment and the theory for the vector
analyzing power \(A_Y\) values show that the shapes of the analyzing powers are
very similar as has been expected from \(A_Y(\theta)\) data in this energy range (low
energy) , however our calculations disagree with the experimental data in the
angular region of \(A_Y(\theta)\) maximum about 10-15\%. As can be seen in ( Fig.
3,4 ) neither calculation (our approach , CD-Bonn calculation ) satisfactorily
describes the \(A_Y(\theta)\) in this angular region near the maximum. It seems to be
that the discrepancies between the experimental data and the calculations (our
approach , CD-Bonn calculation ) are related to the modern \(NN\) potential
which applied in the calculations . Actually, the discrepancy may show that
there is a room for improvement of modern \(NN\) potentials, it was pointed out
that changes in $^3P_j$ $NN$ forces or the spin-orbital component of the potential cause a dramatic increase in $A_Y$ values [H. Witala and W. Glockle, 1991; T. Takemiya, 1991; W. Tornow, H. Witala, 1998; P. Doleschall, 1998], however constraint from $NN$ observable made it difficult to obtain reasonable changes in the $NN$ potential to resolve the $A_Y(\theta)$ puzzle. Another possibility for resolving the $A_Y(\theta)$ puzzle is the introduction of a three-nucleon force $-3NF$ which based on the two pions exchange among three nucleons $(2pE - 3NF)$ can explain the needed attraction, so far, several $2pE - 3NF$ models have been proposed for $3N$ calculations, Tucson-Melbourne [S. A. Coon and W. Glockle, 1981], Brazil -the earlier version BR8 [H. T. Coelho, 1983] and the latter version BR [M. R. Robilotta and H. T. Coelho, 1986]. It should be noted that, the $3NF$ is not an experimental observable, so we can only learn about it by comparing an experiment with a $2N$ model that tells us how nature would behave without a $3NF$. Obviously, discrepancies between an experiment and the corresponding $2N$ calculation could be due to the missing $3N$ potential. However, to be sure we would have to demonstrate that addition of a $3NF$ reduces or removes these discrepancies, unfortunately our present model for low energy scattering is too small to make a statement to this result. But at higher energies it turns clearly out that not only the $^3P_j$ $NN$ forces or the spin-orbital component of the potential has to be corrected to reproduce analyzing-powers and cross-sections, but also the addition of a $3NF$. Therefore since we restrict our consideration to sufficiently low energies scattering, the introduction of a three-nucleon force $-3NF$ which based on the two pions exchange among three nucleons for the process $d(n, n)d$ at higher energies well be suited in further studies.

5 Conclusion

The above results reveal that the single-collision mechanism (cut-off procedure), when consistently taken into account improves considerably the agreement between the theoretical results and the experimental data for cross section of $nd$ elastic scattering at low energy, although the cut-off procedure improves the analyzing power magnitude, but our calculations based on the single-collision mechanism indicate the we still have discrepancies in the angular region of the maximum in the $A_Y(\theta)$ angular distribution, this indicates either an insufficient description of $^3P_j$ components of $NN$ force or a lack of a more fundamental ingredient such as a three-nucleon force $-3NF$ into the nuclear Hamiltonian is needed.
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Figure 1: Differential cross section for elastic $nd$ scattering as a function of the scattering angle $\theta_{c.m}$. Presented in the figure are the results of the calculations performed (solid curve) with and (dotted curve) without a cutoff. Theoretical predictions based on CD-Bonn (dash-dotted curve). The experimental data were taken from [23].
Figure 2: The same as in Fig1, but for En=1.9Mev.
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Figure 3: The calculated analyzing power of $A_\ell$, at incident-neutron energy of $E_n=1.2$ MeV, the results of the calculations performed (solid curve) with and (dotted curve) without a cutoff. Theoretical predictions based on CD-Bonn (dash-dotted curve). The experimental data were taken from [24].
Figure 4: The same as in Fig3, but for $E_n=1.9$Mev
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References


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