Five Dimensional Strings Cosmological Models

with Bulk Viscous Fluid in Lyra Geometry

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Abstract. In this paper we have investigated Bianchi type –I cosmological models with bulk viscosity in Lyra geometry. Determinate solutions are obtained under four cases. Sub cases also considered. The various physical and kinematical features of the models are also discussed.

Keywords: Strings, Bianchi type –I, Five dimensional

Introduction

Early Universe was much S than today. The present four dimensional space-time of the Universe could have preceded by a higher dimensional space-time so the space -time of the early Universe is modeled as having more than four dimensions.

The higher dimensional cosmological models play an important role in description of the Universe in its early stages of evolution. The study of higher dimensional cosmological models is motivated mainly by the possibility of geometrically unifying the fundamental interaction of the universe. Randjbar-Daemi et al. [1], Marciano [2] suggested that the experimental detection of time variation of fundamental constants could provide strong evidences for extra dimensions. Five dimensional space-time is more attractive because 10D
and 11D super gravities admit solutions which spontaneously reduced to 5D. In last few decades there has been a considerable interest in alternative theories of gravitation. The most important among them being a scalar tensor theories proposed by Lyra [3] and Brans-Dicke [4] Weyl proposed a modification of Riemannian manifold in order to unify gravitation and electromagnetism. This theory was never considered seriously due to the non-inerrability of length transfer. Lyra introduced a gauge function to remove the non inerrability of length of a vector under parallel transport. This introduction of gauge function modified the Riemannian geometry into the structure less manifold which bears a close resemblance with Weyl’s geometry. Sen [5] and Sen and Dunn [6] proposed a new scalar tensor theory of gravitation and constructed an analogue to the Einstein field equations based on Lyra’s geometry Jeavon’s et al. [7] pointed that the field equations proposed by Sen and Dunn are very useful. Halford [8] pointed that constant vector field $\phi$ in the Lyra’s geometry play similar role of cosmological constant $\Lambda$ in the general theory of relativity. The scalar –tensor theory of gravitation in Lyra geometry predicts the same effects /with in observational limits as in Einstein theory. Halford [9], Bharma [10], Singh and Singh [11,12], Singh et al.[13], Rahaman et al.[14,15,16,17], Casana et al. [18] and Mohanty et al. [19,20] are some authors who studied various aspects of the cosmological models in Lyra manifold. In this paper we have investigated some five dimensional bulk viscous string cosmological models in Lyra geometry as proposed by Sen and Sen and Dunn. Varies cases have been considered. Some physical and kinematical features of the models are also discussed.

The Field Equations

We consider the five dimensional line elements in the form

$$dS^2 = -dt^2 + A^2 dX^2 + B^2 (dY^2 + dZ^2) + C^2 dW^2$$

(1)

Where A, B, C are functions of cosmic time $t$ only.

The energy momentum tensor for a cloud of string dust with a bulk viscous fluid is given by Letelier (1979) and Landau and Lifshitz (1963) as
\[ T_i' = \rho u_i u' - \lambda x_i x' - \xi u^k\cdot (g_k' + u_i u') \]  \hspace{1cm} (2)

where \( \rho \) is the proper energy density for a cloud of strings with particles attached to them, \( \lambda \) is the string tension density, \( \xi \) is the bulk coefficient of viscosity, \( u' \) is the five velocity of the cloud of particles and \( x' \) is the direction of string i.e. direction of anisotropy satisfying the relation

\[ u_i' u_i = -x_i' x_i = -1 \] \hspace{1cm} (3)

The field equations in normal gauge for Lyra’s manifold as obtained by Sen and Sen and Dunn are given as

\[ R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2} \phi_i \phi_j - \frac{3}{4} g_{ij} \phi_i \phi^k = -\chi T_{ij} \] \hspace{1cm} (4)

Where \( \phi_i \) is the displacement vector given by \( \phi_i = (0,0,0,0,0,\beta(t)) \)

The field equation (5.2.5) for the line element (5.2.1) leads to

\[ \frac{2A_4 B_4}{AB} + \frac{2B_4 C_4}{BC} + \frac{A_4 C_4}{AC} + \frac{B_4^2}{B^2} - \frac{3}{4} \beta^2 = \chi \rho \] \hspace{1cm} (5)

\[ \frac{2B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4 C_4}{BC} + \frac{2B_4^2}{B_4^2} + \frac{3}{4} \beta^2 = \chi (\lambda + \xi \theta) \] \hspace{1cm} (6)

\[ \frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{C_4 A_4}{CA} + \frac{3}{4} \beta^2 = \chi \xi \theta \] \hspace{1cm} (7)

\[ \frac{A_{44}}{A} + \frac{2B_{44}}{B} + \frac{2A_4 B_4}{AB} + \frac{3}{4} \beta^2 = \chi \xi \theta \] \hspace{1cm} (8)

Where the suffix ‘4’ after A, B, C denote ordinary differentiation with respect to t.

**Solution of Field Equations and Physical Properties**

Here we have four independent field equations in six unknowns. Hence in order to get determinate solution, we consider the following relation between the scale factors:

\[ B = g(A(t)) \] \hspace{1cm} and \hspace{1cm} \[ C = A^n \] \hspace{1cm} (9)
From equations (7) and (8), we have
\[
\frac{B_{44}}{B} - \frac{C_{44}}{C} + A_4 \frac{B}{A B} - B_4 \frac{C}{B C} - C_4 \frac{A}{C A} + B_4^2 \frac{1}{B^2} = 0 \tag{10}
\]
Using (5), we have
\[
A_{44} \left( \frac{g''}{g} - \frac{n}{A} \right) + A_4^2 \left( \frac{g''}{g} + \frac{g'^2}{g^2} + (1-n) \frac{g'}{g A} - \frac{n^2}{A^2} \right) = 0 \tag{11}
\]
Here prime indicates the differentiation with respect to the argument.
Equation (11) is satisfied for the following cases:

Case I \( A_4 = 0 \)

Case II \( A_{44} = 0 \) and \( \left\{ \frac{g''}{g} + \frac{g'^2}{g^2} + (1-n) \frac{g'}{g A} - \frac{n^2}{A^2} \right\} = 0 \)

Case III \( A_4 = 0 \) and \( \frac{g'}{g} - \frac{n}{A} = 0 \)

Case IV \( \frac{g'}{g} - \frac{n}{A} = 0 \) and \( \left\{ \frac{g''}{g} + \frac{g'^2}{g^2} + (1-n) \frac{g'}{g A} - \frac{n^2}{A^2} \right\} = 0 \)

Case I When \( A_4 = 0 \)
This leads to \( A = K \) where \( K \neq 0 \) is a constant of integration, with the help of (9) we have \( B = K \) and \( C = K^n \)
Hence five dimensional string models in this case, after using transformations, reduces to the form
\[
dS^2 = -dT^2 + dX^2 + dY^2 + dZ^2 + dW^2 \tag{12}
\]
The energy density \( \rho \), tension density \( \lambda \) and bulk viscosity \( \xi \) for the model (12) are given by
\[
\rho = -\frac{3\beta^2}{4\chi} \\
\lambda = 0 \\
\xi = \frac{3\beta^2}{4\chi}
\]
Since \( \theta = 0 \), therefore we conclude that the Lyra geometry and cosmic strings do not survive in this case and space-time becomes Minkowaskian.

Case II When \( A_{44} = 0 \) and \( \left\{ \frac{g''}{g} + \frac{g'^2}{g^2} + (1-n) \frac{g'}{g A} - \frac{n^2}{A^2} \right\} = 0 \)
This leads to
\( A = (l_t + l_z) \) \tag{13}
Where \( l_t (\neq 0) \) and \( l_z \) are constants of integration.
This case leads to two sub cases

**Sub case II (a)** when $g' = 0$ and $n = 0$

Here we have $g = l_5$, $B = l_5$, and $C = l_4$, where $l_4 \neq 0$ is a constant

Hence the model (1) becomes

$$dS^2 = -dT^2 + T^2 dX^2 + dY^2 + dZ^2 + dW^2$$

(14)

The energy density and tension density for this model are given by

$$\rho = -\frac{3\beta^2}{4\chi}$$

$$\lambda = 0$$

Bulk viscosity is given by the relation

$$\xi \theta + \rho = 0$$

The scalar of expansion, spatial volume, deceleration parameter and shear for the model are given by

$$\theta = \frac{1}{T}, \quad V = T, \quad q = 2, \quad \sigma = \frac{1}{3} \sqrt{\frac{7}{2}} \frac{1}{T}$$

In this model string tension density vanishes but energy density is present and it is only due to the bulk viscosity. Lyra geometry survives only due to the presence of bulk viscosity. In the absence of bulk the model reduces to five dimensional vacuum model in Einstein’s theory. The model starts to expand with a big-bang and stops at $T = \infty$. Volume increases with time. Since $q > 0$, therefore the model is not inflationary. Shear decreases as time increases. Since $Lt \sigma \rightarrow \infty \neq 0$, therefore the model is anisotropic for all values of $T$.

**Sub case II (b)** When $\frac{g''}{g} + \frac{g'}{g} + \frac{(1-n)}{A} = 0$ and $n = 0$

Using (13) we have

$$g = \left[l_5 \log(l_5 + l_2) + l_6\right]^\frac{1}{3}$$

Where $l_5 \neq 0$ and $l_6$ are constants of integrations

Then, with the help of (9), we have

$$B = \left[l_5 \log(l_5 + l_2) + l_6\right]^\frac{1}{2}$$

and $C = 1$

(15)

Using (13) and (15) in (8) we have

$$\frac{3}{4} \beta^2 = \chi(\xi \theta) + \frac{l_5^2 l_2^2}{4(l_5 + l_2)^3} \left[l_5 \log(l_5 + l_2) + l_6\right]^\frac{1}{2}$$

Therefore, after using some transformations the five dimensional string cosmological model in this case reduces to

$$dS^2 = -dT^2 + T^2 dX^2 + \left[\log(l_5 T) + \frac{l_6}{l_5}\right] (dY^2 + dZ^2) + dW^2$$

(16)
The energy density $\rho$, tension density $\lambda$ and bulk viscosity for the model are given by,

$$
\rho = \left[ \chi T^2 \left\{ \log \left( lT \frac{l_s}{l_6} \right) \right\} \right]^{-1} - \xi \theta \\
\lambda = \left[ \chi T^2 \left\{ \log \left( lT \frac{l_s}{l_5} \right) \right\} \right]^{-1} \\
\xi \theta = \frac{3 \beta^2}{4 \chi} - \left[ 4 \chi T^2 \left\{ \log \left( lT \frac{l_s}{l_5} \right) \right\} \right]^{-1}
$$

The scalar of expansion $\theta$, spatial volume $V$, deceleration parameter $q$ and shear $\sigma$ for the model (16) are given by

$$
\theta = \frac{\log(lT) + \frac{l_6}{l_s} + 1}{T \log(lT) + \frac{l_6}{l_5}} \\
V = T \left\{ \log(lT) + \frac{l_6}{l_s} \right\}
$$

$$
q = 3 \left[ \log(lT) + \frac{l_6}{l_s} + 1 \right]^{-1} \left[ \left\{ \log(lT) \frac{l_6}{l_s} \right\}^2 + \left\{ \log(lT) + \frac{l_6}{l_s} \right\} + 1 \right] - 1
$$

$$
\sigma = \frac{1}{6T} \left[ 5 \left\{ \log \left( lT + \frac{l_6}{l_s} \right) \right\}^{-2} - 8 \left\{ \log(lT) + \frac{l_6}{l_s} \right\} + 14 \right]^{\frac{1}{2}}
$$

The model (16) starts with a big bang and stops at $T=\infty$. Volume increases as $T$ increases. As $T \to \infty$, The string energy density $\rho \to -\frac{3 \beta^2}{4 \chi}$ and string tension density $\lambda \to 0$. Here we can conclude that the energy density is only due to the Lyra geometry. Since $\frac{L T}{T \to \infty} \sigma \theta \neq 0$, therefore the model is anisotropic for large values of $T$.

**Case III** when $A_4 = 0$ and $\frac{g'}{g} - \frac{n}{A} = 0$

In this case the field equations admits the same solution as obtained in case – I.
Case IV when \( \frac{g'}{g} - \frac{n}{A} = 0 \) and \( \left\{ \frac{g''}{g} + \frac{g'^2}{g^2} + (1-n)\frac{g'}{gA} - \frac{n^2}{A^2} \right\} = 0 \)

This leads to
\[
\frac{g'}{g} + (1-n) \frac{g'}{A} = 0
\]

This again leads to the following sub cases:

**Sub case IV (a) when** \( g = 0 \)

Here we have \( g = m_1, B = m_1 \) and \( C = m_2 \)

Where \( m_1 \neq 0 \) is constant of integration and \( m_2 \neq 0 \) is a constant. For this case the physical parameters are as follows

\[
\rho = -\frac{3\beta^2}{4\chi}, \quad \lambda = \frac{A_{44}}{\chi A}, \quad \xi\theta = \frac{3\beta^2}{4\chi} + \frac{A_{44}}{\chi A}
\]

Here, we find that \( \rho, \lambda, \xi\theta \) depend on \( A \).

To see the picture more clear, first we consider \( A = \sin(at+b) \), then after making suitable transformations the model becomes

\[
dS^2 = -dT^2 + \sin^2(aT)dX^2 + dY^2 + dZ^2 + dW^2
\]

In this case we see that string energy density \( \rho \) in the model is only due to the Lyra geometry but string tension density depends on the metric coefficient in X-direction.

If we consider \( A = e^{(at+b)} \) then after suitable transformations we have the model in the form

\[
dS^2 = -dT^2 + e^{2at}dX^2 + dY^2 + dZ^2 + dW^2
\]

Here, as the time increases and \( a < 0 \) the model contracts along X-direction. The extra dimension contracts and become unobservable at infinity time. Finally the model reduces to 4-dimension flat space –time. String density is only due to the bulk viscosity. The model follows exponential expansion and deceleration parameter also supports this.

If we consider \( A = (at+b)^m, m \neq 0 \) then the model reduces to

\[
dS^2 = -dT^2 + T^{2m}dX^2 + dY^2 + dZ^2 + dW^2
\]

This model follows the power law inflation. Expansion in the model is only in X-direction.

**Sub case IV (b) when** \( \frac{g''}{g} + \frac{1-n}{A} = 0 \)

This leads to \( g = \frac{r_1A^n}{n} \) where \( r_1 \neq 0 \) is a constant of integration

With the help of (9) we have

\[
B = \frac{r_1A^n}{n} \quad \text{and} \quad C = A^n
\]

Using equations (20) in (5), (6), (7) and (8) we have
\[ \rho = -\xi \theta + (1 + 2n) \frac{A_{n+1}}{\chi A} + (3n + 6n^2) \frac{A_{n+2}}{\chi A^2} \]

\[ \lambda = (1 - n) \frac{A_{n+1}}{\chi A} + (3n^2 - 3n) \frac{A_{n+2}}{\chi A^2} \]

\[ \xi \theta = \frac{3 \beta^2}{4\chi} + (1 + 2n) \frac{A_{n+1}}{\chi A} + \frac{3n^2 A_{n+2}}{\chi A^2} \]

Here we find that \( \rho, \lambda \) and \( \xi \theta \) depend on \( A \).

To see the picture more clearly, consider \( A = \sin(pt + q) \), where \( p \neq 0, q \) are constants. Then after suitable transformations we have the metric in the form

\[ dS^2 = -dT^2 + \sin^2(pt) dX^2 + \sin^2(pt) dY^2 + dZ^2 + \sin^{2n}(pt) dW^2 \quad (21) \]

This model starts with a big bang at \( T = 0 \) and stops at \( T = \left( \frac{\Pi - q}{p} \right) \frac{1}{p} \) when \( 3n + 1 \neq 0 \).

If we consider \( A = e^{(pt+q)} \) where \( p \neq 0, q \) are constants. Then after some transformations metric reduces to the form

\[ dS^2 = -dT^2 + e^{2pt} dX^2 + e^{2npT} \left( dY^2 + dZ^2 \right) + e^{2npT} dW^2 \quad (22) \]

This model reduces to four dimensional space–time for \( p < 0 \) and \( n < 0 \) for large values of \( T \). The model becomes flat at initial epoch. As time increases with \( p < 0 \) and \( n < 0 \) the model expands in three spatial directions but contract along \( X \)-direction. The extra dimension contracts and become unobservable at \( t = \infty \). The model follows exponential expansion.

If we take \( A = (pt + q)^m, m \neq 0 \) where \( p \neq 0, q \) are constants, then model reduces to the form

\[ dS^2 = -dT^2 + T^{2m} dX^2 + T^{2mn} dY^2 + dZ^2 + T^{2mn} dW^2 \quad (23) \]

This model with \( m < 0 \) and \( n < 0 \) reduces to four dimension space–time continuum. The model follows power law inflation in this case and anisotropic for all epoch. Shear decreases as time increases.

References


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