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CHSH and Local Hidden Causality

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Abstract

Mathematics equivalent to Bell's derivation of the inequalities, also allows a local hidden variables explanation for the correlation between distant measurements.

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1 Introduction

Bell inequalities [1] are a well studied subject. To many the experimental verification of the violation of inequalities e.g. [2], [3] is sufficient evidence for the completeness of quantum theory. Here, it will be demonstrated that Bell's form of local hidden correlation

$$P(\vec{a},\vec{b}) = \int_{\lambda\in\Lambda} \rho_\lambda A_\lambda(\vec{a}) B_\lambda(\vec{b}) d\lambda \tag{1}$$

can be transformed to violate Bell's inequality. We have, $\vec{a}$ and $\vec{b}$ for unitary parameter vectors of e.g. Stern-Gerlach magnets in an ortho-positronium decay experiment. λ represents the extra hidden parameters in a set Λ. The probability density ρ_λ is a classical density. The measurement functions $A_\lambda(\vec{a})$ and $B_\lambda(\vec{b})$ project in $\{-1,1\}$. Bell showed, using the expression below, that models with a classical probability density may not violate the inequality[1].

$$P(\vec{a},\vec{b}) - P(\vec{x},\vec{y}) = \int_{\lambda\in\Lambda} \rho_\lambda A_\lambda(\vec{a}) B_\lambda(\vec{b}) A_\lambda(\vec{x}) B_\lambda(\vec{y}) \left\{ A_\lambda(\vec{x}) B_\lambda(\vec{y}) - A_\lambda(\vec{a}) B_\lambda(\vec{b}) \right\} \tag{2}$$

[1]If there is no confusion the $d\lambda$ will be suppressed.

1.1 Singlet state Bell inequality

Bell expressed the singlet state of the electron and positron in the positronium as $\forall : \vec{a}(|\vec{a}| = 1)\forall : \lambda(\lambda \in \Lambda)\left\{A_\lambda(\vec{a}) + B_\lambda(\vec{a}) = 0\right\}$. The following steps are elementary. Let us take, $\vec{x} = \vec{b}$ and $\vec{y} = \vec{c}$. With the singlet, we see that equation (2) can be written as

$$P(\vec{a},\vec{b}) - P(\vec{b},\vec{c}) = \int_{\lambda\in\Lambda} \rho_\lambda \left\{A_\lambda(\vec{b})A_\lambda(\vec{c}) - A_\lambda(\vec{a})A_\lambda(\vec{b})\right\} \tag{3}$$

Or, noting $1 - A_\lambda(\vec{a})A_\lambda(\vec{c}) \geq 0$,

$$\left|P(\vec{a},\vec{b}) - P(\vec{b},\vec{c})\right| \leq \int_{\lambda\in\Lambda} \rho_\lambda \left|A_\lambda(\vec{c})A_\lambda(\vec{b})\right| \{1 - A_\lambda(\vec{a})A_\lambda(\vec{c})\} \tag{4}$$

Because, $\left|A_\lambda(\vec{c})A_\lambda(\vec{b})\right| = 1$ and ρ_λ classical, we have the Bell inequality

$$\left|P(\vec{a},\vec{b}) - P(\vec{b},\vec{c})\right| \leq 1 + P(\vec{a},\vec{c}) \tag{5}$$

The quantum correlation is: $P_{qm}(\vec{x},\vec{y}) = -(\vec{x}\cdot\vec{y})$. If in two-dimensions, $\vec{a} = \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, $\vec{b} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\vec{c} = (0,1)$, then, inequality is violated because, $\left|0 - \frac{-1}{\sqrt{2}}\right| \leq 1 - \frac{1}{\sqrt{2}}$ is false. Associated to this inequality in equation(5) a more general inequality, the CHSH inequality [4], exists. The principle is the same.

2 Sets and Integrals

Keeping an eye on equation (2), hidden parameters sets can be defined

$$\Omega_\pm = \left\{\lambda \in \Lambda | A_\lambda(\vec{a})B_\lambda(\vec{b}) = A_\lambda(\vec{x})B_\lambda(\vec{y}) = \pm 1\right\} \tag{6}$$

and

$$\Omega_0 = \left\{\lambda \in \Lambda | A_\lambda(\vec{a})B_\lambda(\vec{b}) = -A_\lambda(\vec{x})B_\lambda(\vec{y}) = \pm 1\right\} \tag{7}$$

Given, $\vec{a}$, $\vec{b}$, $\vec{x}$ and $\vec{y}$, either, $A_\lambda(\vec{a})B_\lambda(\vec{b}) = A_\lambda(\vec{x})B_\lambda(\vec{y})$ or $A_\lambda(\vec{a})B_\lambda(\vec{b}) = -A_\lambda(\vec{x})B_\lambda(\vec{y})$ for arbitrary, $\lambda \in \Lambda$. Moreover, $A_\lambda(\vec{a})B_\lambda(\vec{b}) = \pm 1$ for arbitrary, $\lambda \in \Lambda$. Hence, $\Lambda = \Omega_0 \cup \Omega_+ \cup \Omega_-$ and equation (2) is

$$P(\vec{a},\vec{b}) - P(\vec{x},\vec{y}) = \int_{\lambda\in\Omega_0} \rho_\lambda A_\lambda(\vec{a})B_\lambda(\vec{b})A_\lambda(\vec{x})B_\lambda(\vec{y})\left\{A_\lambda(\vec{x})B_\lambda(\vec{y}) - A_\lambda(\vec{a})B_\lambda(\vec{b})\right\} \tag{8}$$

From Ω_0 folows $A_\lambda(\vec{a})B_\lambda(\vec{b})A_\lambda(\vec{x})B_\lambda(\vec{y}) = -1$ and $\left\{A_\lambda(\vec{x})B_\lambda(\vec{y}) - A_\lambda(\vec{a})B_\lambda(\vec{b})\right\} = 2A_\lambda(\vec{x})B_\lambda(\vec{y})$. Hence,

$$P(\vec{a},\vec{b}) - P(\vec{x},\vec{y}) = -2\int_{\lambda\in\Omega_0} \rho_\lambda A_\lambda(\vec{x})B_\lambda(\vec{y}) \tag{9}$$

Suppose, $P(\vec{a},\vec{b}) = 0$, as 'starting position' in the experiment. This gives a reformulation of $P(\vec{x},\vec{y})$ where $\vec{x}$ and $\vec{y}$ are different form $\vec{a}$ and $\vec{b}$. Hence,

$$P(\vec{x},\vec{y}) = 2 \int_{\lambda\in\Omega_{0|P(\vec{a},\vec{b})=0}} \rho_\lambda A_\lambda(\vec{x}) B_\lambda(\vec{y}) \tag{10}$$

Note that according to equation (1) and the Ω sets we may write for $P(\vec{a},\vec{b}) = 0$

$$P(\vec{a},\vec{b}) = 0 = \int_{\lambda\in\Omega_{0|P(\vec{a},\vec{b})=0}} \rho_\lambda A_\lambda(\vec{a}) B_\lambda(\vec{b}) + \int_{\lambda\in\Omega_{+|P(\vec{a},\vec{b})=0}} \rho_\lambda - \int_{\lambda\in\Omega_{-|P(\vec{a},\vec{b})=0}} \rho_\lambda \tag{11}$$

Moreover, generally $P(\vec{x},\vec{y}) \neq P(\vec{a},\vec{b})$ which follows from comparing equation (10) with (11). Because, in Ω_0, we see for arbitary $\lambda \in \Omega_0$ that $A_\lambda(\vec{a})B_\lambda(\vec{b}) = -A_\lambda(\vec{x})B_\lambda(\vec{y}) = \pm 1$, it follows from equation (11) that we may rewrite $P(\vec{x},\vec{y})$ as

$$\frac{1}{2}P(\vec{x},\vec{y}) = \int_{\lambda\in\Omega_{+|P(\vec{a},\vec{b})=0}} \rho_\lambda - \int_{\lambda\in\Omega_{-|P(\vec{a},\vec{b})=0}} \rho_\lambda \tag{12}$$

Equations (6) and (7) show that the Ω sets depend on $\vec{a}$, $\vec{b}$, $\vec{x}$ and $\vec{y}$. Given $P(\vec{a},\vec{b}) = 0$, this fixes the $\vec{a}$ and $\vec{b}$. Hence, $\Omega_{\pm|P(\vec{a},\vec{b})=0} = \Omega_{\pm|P(\vec{a},\vec{b})=0}(\vec{x},\vec{y})$, implicit in equation(12). Start the experiment with two parameters $\vec{a}$ and $\vec{b}$ that produces the condition $P(\vec{a},\vec{b}) = 0$ and let $\vec{x}$ and $\vec{y}$ free[2]. $\vec{x}$ does not afect $B_\lambda(\vec{y})$ and vice versa, hence, no locality violation.

3 Violation CHSH

We will show that there is a classical probability density that allows violation of the CHSH $|D| \leq 2$, with,

$$D = P(1_A, 1_B) - P(1_A, 2_B) - P(2_A, 1_B) - P(2_A, 2_B) \tag{13}$$

Here, $1_{A(B)}$ and $2_{A(B)}$ are unitary vectors randomly selected by $A(B)$.

3.1 Probability density

We postulate a density for $(\lambda_1, \lambda_2) \in [\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}] \times [\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}] = \Lambda$ with $n = 1, 2$

$$\rho_{\lambda_n} = \begin{cases} \frac{1}{\sqrt{2}}, & \frac{-1}{\sqrt{2}} \leq \lambda_n \leq \frac{1}{\sqrt{2}} \\ 0, & elsewhere \end{cases} \tag{14}$$

This density is Kolmogorovian.

[2]see the discussion section

3.2 Selection of parameters

We establish the parameter vectors that the observers A and B will use. For A, $1_A = (1, 0)$ and $2_A = (0, 1)$. For B, $1_B = (\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}})$ and $2_B = (\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}})$. If we take the quantum correlation, it follows, $P_{qm}(1_A, 1_B) = \frac{-1}{\sqrt{2}}$, $P_{qm}(1_A, 2_B) = \frac{1}{\sqrt{2}}$, $P_{qm}(2_A, 1_B) = \frac{1}{\sqrt{2}}$ and $P_{qm}(2_A, 2_B) = \frac{1}{\sqrt{2}}$. Quantum mechanics violates $|D| \leq 2$, because $|D| = 2\sqrt{2}$ is found. Because, $\rho_{\lambda_1}\rho_{\lambda_2} = \frac{1}{2}$ for $(\lambda_1, \lambda_2) \in [\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}] \times [\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]$ and $\Omega_{\pm|P(\vec{a},\vec{b})=0}(\vec{x}, \vec{y}) \subset [\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}] \times [\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]$, we obtain from equation (12)

$$P(\vec{x}, \vec{y}) = \int_{\lambda \in \Omega_{+|P(\vec{a},\vec{b})=0}(\vec{x},\vec{y})} d\lambda_1 d\lambda_2 - \int_{\lambda \in \Omega_{-|P(\vec{a},\vec{b})=0}(\vec{x},\vec{y})} d\lambda_1 d\lambda_2 \tag{15}$$

If, subsequently, observer A selects 1_A, then the hidden parameter λ_1 is in $[\frac{-1}{\sqrt{2}}, 1 - \frac{1}{\sqrt{2}}] \subset [\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]$. If, A selects 2_A then λ_1 is in $[-1 + \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}] \subset [\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]$. Similarly, if B selects 1_B, then then λ_2 is in $[0, \frac{1}{\sqrt{2}}] \subset [\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]$. Finally, if B selects 2_B, then λ_2 is found in $[\frac{-1}{\sqrt{2}}, 0] \subset [\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]$. The intervals responding to settings do not violate locality: A settings are associated to λ_1 intervals, B settings to λ_2 intervals. Suppose A selects 1_A and B selects 1_B. We turn to $\Omega_{\pm|P(\vec{a},\vec{b})=0}(1_A, 1_B)$. If, $\Omega_{+|P(\vec{a},\vec{b})=0}(1_A, 1_B) = \emptyset$ and $\Omega_{-|P(\vec{a},\vec{b})=0}(1_A, 1_B) = [\frac{-1}{\sqrt{2}}, 1 - \frac{1}{\sqrt{2}}] \times [0, \frac{1}{\sqrt{2}}]$, from equation (15) it follows that $P(1_A, 1_B) = \frac{-1}{\sqrt{2}}$. Hence, a selection of $\Omega_{\pm|P(\vec{a},\vec{b})=0}(\vec{x}, \vec{y})$ is possible giving $|D| > 2$.

4 Conclusion and discussion

The result of violating $|D| \leq 2$ with proper $\Omega_{\pm|P(\vec{a},\vec{b})=0}(\vec{x}, \vec{y})$ and locality obeying interval selection rules, is surprising. The mathematics was similar to the one used by Bell [1]. Moreover, no violations of locality were introduced. In a random selection experiment there is a non-zero probability that, combined with the deterministic interval selection, a proper selection of $\Omega_{\pm|P(\vec{a},\vec{b})=0}(\vec{x}, \vec{y})$ is obtained. When Bell's reasoning is sound, no violation should be possible *at all* with the use of classical local hidden models given the employed parameters. Note that other violating instances can be treated similarly. If there can be no reasons given why locality and causality selections of $\Omega_{\pm|P(\vec{a},\vec{b})=0}(\vec{x}, \vec{y})$ are impossible, then a local hidden variable explanation of experiments cannot be excluded. The transformation of (1) is based on a single fixing of $\vec{a}$ and $\vec{b}$, *independent* of the $\vec{x}$ and $\vec{y}$. If one assumes that the functional form of $A_\lambda(\cdot)$ and $B_\lambda(\cdot)$ changes in time (see also [5] for the role of time in Bell's theorem) then the fixing of $P(\vec{a}, \vec{b}) = 0$ can take place at times different than the measurement parameters selection and the sets in equations (6) and (7) will always be possible.

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