Kaluza-Klein Type Cosmological Model

with Strange Quark Matter

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Abstract

In this paper we analyzed the Kaluza-Klein type Robertson Walker (RW) cosmological model by considering form of variable cosmological term $\Lambda : \Lambda \sim \frac{\dot{R}^2}{R^2}$ and $\Lambda \sim \rho$ in the presence of strange quark matter. The various physical aspects of the model are also discussed.
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1 INTRODUCTION

A great number of exact cosmological solutions of Einstein field equations with different equation of state and different symmetries, including or not a cosmological constant, has been found with 5D [Demarat and Hanquin (1985), Davidson and Vozmediano (1985a, 1985b)] and also with arbitrary number of dimension [Lorenz-Petzold (1984, 1985, 1986a, 1986b)]. Sahdev (1984), Emelyanov et al. (1986) and Chatterjee and Bhui (1993), have studied physics of the universe in higher-dimensional space-time.

The possibility that the world may have more than the four dimensions is due to Kaluza (1921) and Klein (1926), who used one extra dimension to unify gravity and electromagnetism in a theory which was essentially five dimensional general relativity. This idea has been worked by a large number of people, who have found models for various phenomenon in particle physics and cosmology using five (5D) or more dimensions [De Sabbatt and Schmutzer (1983), Lee (1984), Appelquist et al. (1987), Collins et al. (1989)]. Overuin and Wesson (1997) have presented an excellent review of Kaluza-Klein theory and higher-dimensional unified theories, in which the cosmological and astrophysical implications of extra-dimension have been discussed. Also, many authors have studied Kaluza-Klein cosmological models with different matters [Fukui (1993), Ponce de Leon (1988), Chi (1990), Liu and Wesson (1994), and Coley (1994)]. There is now extensive literature dealing with different aspect of higher dimensional cosmologist.

In this study, we will examine quark matter in the higher-dimensional space-time. It is well known that quark-gluon plasma existed during one of the phase transitions of the universe in the early time when the universe had higher dimensions than it has today and cosmic temperature was $T \sim 200MeV$. Recently, quark matter and the relations between the quark matter and domain walls and also strings have been studied by several authors.

The possibility of the existence of quark-matter dates back to the early seventies. Itoh (1970), Bodmer (1971) and Witten (1984) proposed two ways
of formation of quark matter: the quark-hadron phase transition in the early universe and conversion of neutron stars into strange ones at ultrahigh densities. In the theories of strong interaction quark bag models, suppose that breaking of physical vacuum takes place inside hadrons. As a result, vacuum energy densities inside and outside a hadron become essentially different, and the vacuum pressure on the bag wall equilibrates the pressure of quarks, thus stabilizing the system.

Typically, strange quark matter is modeled with an equation of state (EOS) based on the phenomenological bag model of quark matter, in which quark confinement is described by an energy term proportional to the volume. In this model, quarks are thought as degenerate Fermi gases, which exist only in a region of space endowed with a vacuum energy density $B_c$ (called as the bag model). Also, in the framework of this model, the quark matter is composed of massless $u, d$ quarks, massive $s$ quarks and electrons. In the simplified version of this model, on which our study is based, quarks are massless and non-interacting. Then we have quark pressure $\rho = \rho_q + B_c$ and the total pressure $p = p_q - B_c$. One therefore gets equation of the state for strange quark matter

$$p = \frac{1}{3}(\rho - 4B_c).$$

There are two possibilities for the occurrence of a phase transition between hadronic and strange quark matter (SQM) that are well known. The first could occur at very high temperatures and very low baryon density in the early universe, and the second, a suggested by Bodmer (1971), at densities of higher order than the nuclear density $n_0 \sim 0.16 fm^{-3}$. This phase transition would occur in the universe, every time that a massive scalar explodes as a supernova with its consequent remnant. If the Bodmer’s conjecture is true, SQM could be succeeded in the inner core of neutron stars when strange quarks would be produced through the weak processes with a dynamical chemical equilibrium among the constituents. It is also possible that after a supernova explosion its core forms directly a strange quark star.

Yilmaz et. al (2007) studied strange quark matter for Robertson Walker model in the context of general theory of relativity. Also Yilmaz and Yavuz (2006) have obtained higher dimensional Robertson Walker cosmological models in the presence of quark-gluon plasma in general theory of relativity.

The cosmological constant problem can be expressed as the discrepancies between negligible values $\Lambda$ has for the present universe and the values $10^{50}$
times larger expected by the Glashow Salam Weinberg model or by Grand Unified Theory (GUT) where it should be $10^{107}$ times larger. A current of thought holds the view that the cosmological term is not really constant, but its value decreases as the universe expands. Some of the discussions on cosmological problem and on cosmology with time varying cosmological constant, Ratra and Peebles (1988), Dolgov (1983, 1990, 1997) and Sahni and Starobinsky (2000) pointed out that in absence of any interaction with a matter or radiation the cosmological constant remains constant. [Rajeev (1983), Maia et. al. (1994), Moffat (1996), Hoyle et. al. (1997), John et. al (1997)] have derived analytical decay laws in other theories of gravitation from modified version of Einstein’s action. One of the motivations for introducing $\Lambda$ term is to reconcile the age parameter and the density parameter of the universe with recent observational data.

Vishwakarma (2001) has studied the magnitude-redshift relation for the type Ia supernovae data and the angular size-redshift relation for the updated compact radio sources data Gurvits (1999) by considering four variable $\Lambda$-models: $\Lambda \sim R^{-2}$, $\Lambda \sim H^{-2}$, $\Lambda \sim \rho$ and $\Lambda \sim t^{-2}$. Ray and Mukhopadhyay (2004) have solved Einstein’s equations for specific dynamical models of the cosmological term $\Lambda$ in the form: $\Lambda \sim \left(\frac{\dot{R}}{R}\right)^2$, $\Lambda \sim \left(\frac{\ddot{R}}{R}\right)$ and $\Lambda \sim \rho$ and shown that the models are equivalent in the framework of flat RW space time. In this context, the aim of the present work is based on recent available observational information. In this paper the implication of cosmological model with cosmological term of two different forms: $\Lambda = \alpha \left(\frac{\dot{R}}{R}\right)^2$, $\Lambda = \beta \rho$ are analyzed with strange quark matter within the framework of higher dimensional space time.

We study the strange quark matter with variable $\Lambda$ in Kaluza-Klein theory of gravitation. The paper is organized as follows: In section 2, we have obtained the field equations for Kaluza-Klein type cosmological model in the presence of strange quark matter. We solved the same field equations by assuming cosmological constant $\Lambda$ in the form of $\Lambda \sim \frac{\dot{R}^2}{R^2}$ and $\Lambda \sim \rho$ n section 3. In section 4, concluding remarks are given.
2 MODEL AND FIELD EQUATIONS

Let us consider the Kaluza-Klein type metric

$$ds^2 = dt^2 - R^2(t) \left[ \frac{dr^2}{(1 - kr^2)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1 - kr^2)d\psi^2 \right],$$

where $R(t)$ is the scale factor and $k = 0, -1, or +1$ is the curvature parameter for flat, open and closed universe, respectively. The universe is assumed to be filled with distribution of matter represented by energy-momentum tensor for quark matter

$$T_{\mu\nu} = (p + \rho)u_\mu u_\nu - pg_{\mu\nu},$$

where $\rho = \rho_q + B_c$ is the quark matter total energy density and $p = p_q - B_c$ is the quark matter total pressure and $u_\mu$ is the five-velocity vector such that $u_\mu u^\mu = 1$. Also $\rho$ and $p$ are related by the bag model equation of the state (1). The Einstein field equations with time-dependent cosmological is given by

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi GT_{\mu\nu} + \Lambda(t)g_{\mu\nu},$$

where $R_{\mu\nu}$ is the Ricci tensor, $G$ and $\Lambda(t)$ being the gravitational and variable cosmological constants.

The divergence of Eq. (3), taking into account the Bianchi identity, gives

$$(8\pi GT_{\mu\nu} + \Lambda g_{\mu\nu})^{\nu} = 0.$$  (5)

Using comoving coordinates

$$u_\mu = (1, 0, 0, 0, 0),$$

in Eq.(3) and with line element (2), Einstein’s field Eq.(4) yields

$$8\pi G\rho = \frac{6\dot{R}^2}{R^2} + \frac{6k}{R^2} - \Lambda(t),$$

$$8\pi Gp = -\frac{3\dot{R}}{R} - \frac{3\ddot{R}}{R^2} - \frac{3k}{R^2} + \Lambda(t),$$

where dot denotes derivative with respect to $t$ and we will use the geometrized unit so that $8\pi G = c = 1$. 
3 The Solutions of the Field Equations

In the above field equations the number of the unknowns is more than the equations, to be able to obtain an exact solutions of the field equations we assume the equations of the state of the strange quark matter and also the cosmological constant $\Lambda$ in the form of $\Lambda = \alpha H^2$ and $\Lambda = \beta \rho$.

3.1 For $\Lambda = \alpha H^2$

The Einstein’s field equations (7) and (8) for this value of the $\Lambda$ can be expressed as

$$\rho = \frac{(6 - \alpha)\dot{R}^2}{R^2} + \frac{6k}{R^2}, \quad (9)$$

$$-p = \frac{3\dot{R}}{R} + \frac{(3 - \alpha)\dot{R}^2}{R^2} - \frac{3k}{R^2}. \quad (10)$$

From the above equations by eliminating $\rho$ with the help of strange quark matter equations of the state described by Eq.(1), we get

$$\frac{3\dot{R}}{R} + \frac{(15 - 4\alpha)\dot{R}^2}{R^2} + \frac{5k}{R^2} = \frac{4B_c}{3} \quad (11)$$

After some mathematical manipulations, we get the first integral

$$\dot{R}^2 = A_0 R^2 - \frac{5k}{3b} + \frac{C_1}{R^2} \quad (12)$$

where $C_1$ is the constant of integration and $A_0 = \frac{4B_c}{9(b+1)}$ and $b = \frac{(15-4\alpha)}{9}$. It is very difficult to find the exact solution of Eq. (12). However, by setting $C_1 = 0$, without loss of generality we are able to get the solution of Eq. (12). Hence integrating of Eq. (12) for the particular value of $\alpha = \frac{3}{2}$ i.e. for $b = 1$ we get

$$R^2 = \sqrt{\frac{9C_1}{2B_c} - \left(\frac{15k}{4B_c}\right)^2} \sinh\left[\frac{2\sqrt{2}B_c}{3}(t - t_0)\right] + \frac{45k}{12B_c}, \quad (13)$$

where $C_1 > \frac{25}{8B_c}$ for $k = \pm 1$, $C_1 > 0$ for $k = 0$ and $t_0$ is an constant of integration.

From Eqs. (9), (10) and (13) we get the following physical quantities:

$$\rho = B_c \left[\frac{a_0 \cosh^2\theta + 40k^2 + 8k\sqrt{a_0}\sinh\theta}{(5k + \sqrt{a_0}\sinh\theta)^2}\right], \quad (14)$$
\[ p = \frac{B_c}{3} \left[ a_0 \cosh^2 \theta + 40k^2 + 8k\sqrt{a_0}\sinh \theta \right] \left(5k + \sqrt{a_0}\sinh \theta)^2\right] - 4 \], \quad (15)\]

where \( a_0 = 8B_cC_1 - 25k^2 \) and \( \theta = \frac{2\sqrt{2B_c}(t - t_0)}{3} \).

From the above equations we have the following expression for \( \rho \) and \( p \) depending on the universe type \((k = 0, \pm 1)\).

### 3.2 case (i)

For \( k = 0 \) we get

\[ \rho = B_c \coth^2 \theta, \]
\[ p = \frac{B_c}{3}(\coth^2 \theta - 4). \]

For \( k = 1 \) we get

\[ \rho = B_c \left[ a_0 \cosh^2 \theta + 8\sqrt{a_0}\sinh \theta + 40 \right] \left(5 + \sqrt{a_0}\sinh \theta)^2\right], \quad (18)\]
\[ p = \frac{B_c}{3} \left[ a_0 \cosh^2 \theta + 8\sqrt{a_0}\sinh \theta + 40 \right] \left(5 + \sqrt{a_0}\sinh \theta)^2\right] - 4 \]. \quad (19)\]

For \( k = -1 \) we get

\[ \rho = B_c \left[ a_0 \cosh^2 \theta + 8\sqrt{a_0}\sinh \theta + 40 \right] \left(-5 + \sqrt{a_0}\sinh \theta)^2\right], \quad (20)\]
\[ p = \frac{B_c}{3} \left[ a_0 \cosh^2 \theta + 8\sqrt{a_0}\sinh \theta + 40 \right] \left(-5 + \sqrt{a_0}\sinh \theta)^2\right] - 4 \]. \quad (21)\]

We get the solution for \( \Lambda = 0 \) case when \( \alpha = 0 \) in the equation (11).

### 3.3 For \( \Lambda = \beta \rho \)

In this case the field equations (7) and (8) can be expressed as

\[ \rho(1 + \beta) = \frac{6\dot{R}^2}{R^2} + \frac{6k}{R^2}, \]
\[ -p + \beta \rho = -\frac{3\ddot{R}}{R} - \frac{3\dot{R}^2}{R^2} - \frac{3k}{R^2} + \Lambda(t). \]

By again using equations of state (1) after eliminating \( \rho \) with some mathematical manipulation we get the first integral of the form

\[ \frac{\dot{R}^2}{9(A_2 + 1)}R^2 + \frac{C_2}{A_2R^2} - k \]
\[ \quad (24)\]
where $C_2$ is a constant of integration and $A_1 = \frac{6\beta - 2}{(\beta + 1)}$, $A_2 = \frac{3 - A_1}{3}$.

In general, it is very hard to solve Eq. (24) as depending on time. So, we will be interested in finding analytical solutions of $R$. However, setting $\beta = 1/3$, we are able to get solutions of Eq. (24).

Integrating Eq. (24) for a particular value of $\beta = \frac{1}{3}$, we get

$$R^2 = 3 \sqrt{\frac{C_2}{2B_c}} - \left( \frac{3k}{4B_c} \right)^2 \sinh \left[ \frac{2\sqrt{2B_c}}{3} (t - t_0) \right] + \frac{9k}{4B_c},$$ (25)

where $C_2 > \frac{9}{8B_c}$ for $k = \pm 1$, $C_2 > 0$ for $k = 0$ and $t_0$ is a constant of integration.

From Eqs. (22), (23) and (25) we get the following physical quantities:

$$\rho = \left[ \frac{8B_cA_3^2 \cosh^2 \theta + k}{2(A_3 \sinh \theta + A_4)^2} \right],$$ (26)

$$p = \frac{B_c}{3} \left[ \frac{8B_cA_3^2 \cosh^2 \theta + 1}{2(A_3 \sinh \theta + A_4)^2} - 4 \right],$$ (27)

where $A_3 = 3 \sqrt{\frac{C_2}{2B_c}} - \left( \frac{3k}{4B_c} \right)^2$, $\theta = \frac{2\sqrt{3B_c(t-t_0)}}{3}$, and $A_4 = \frac{9k}{4B_c}$.

Again we find out $\rho$ and $p$ for $k = 0, \pm 1$

### 3.4 case (i)

For $k = 0$ we get

$$\rho = 4B_c \coth^2 \theta,$$ (28)

$$p = \frac{4B_c}{3} (\coth^2 \theta - 1).$$ (29)

For $k = 1$ we get

$$\rho = \left[ \frac{8B_cA_{31}^2 \cosh^2 \theta + 1}{2(A_{31} \sinh \theta + A_{41})^2} \right],$$ (30)

$$p = \frac{B_c}{3} \left[ \frac{8B_cA_{31}^2 \cosh^2 \theta + 1}{2(A_{31} \sinh \theta + A_{41})^2} - 4 \right].$$ (31)

For $k = -1$ we get

$$\rho = \left[ \frac{8B_cA_{31}^2 \cosh^2 \theta - 1}{2(A_{31} \sinh \theta - A_{41})^2} \right],$$ (32)

$$p = \frac{B_c}{3} \left[ \frac{8B_cA_{31}^2 \cosh^2 \theta - 1}{2(A_{31} \sinh \theta - A_{41})^2} - 4 \right],$$ (33)

where $A_{31} = 3 \sqrt{\frac{C_2}{2B_c}} - \left( \frac{3}{4B_c} \right)^2$, and $A_{42} = \frac{9}{4B_c}$. 
4 Concluding Remark

This work has thus generalized to higher dimensional space time the well known results in the four dimensional space time. It is found that there may be significant difference in the principal at least, from the analogous situation in four dimensional space time. In this paper we have studied strange quark matter for Kaluza-Klein type Robertson-Walker cosmological model by assuming cosmological constant of the form $\Lambda = \alpha H^2$ and $\Lambda = \beta \rho$. For the physically realistic case of $\rho > 0$ and $p > 0$, we get $\ddot{R} < 0$ which via Eq.(12) and (24), yields $A_0 = \frac{4B_c}{9(6+1)} > 0$ and $A_2 = \frac{(3-A_1)}{3} > 0$, respectively. It is important to note that for $t >> \frac{3}{\sqrt{2} B_c}$ approaches closely the $k = 0$ (flat) solution for both cases of $\Lambda$, i.e.

$$R \sim \left[ \sinh\left(\frac{2\sqrt{2} B_c}{3}(t - t_0)\right) \right]^{1/2}.$$

We find out the solutions in both the form of $\Lambda$ for the values of $\alpha = \frac{2}{3}$ and $\beta = \frac{1}{3}$. We are also able to get the solution by setting $C_1 = 0$ and $C_2 = 0$ without loss of generality in the equations (12) and (24). For the case $k = 0$ in both the cases of $\Lambda$ we get the equivalent expression for $\rho$ and $p$. Also for the case $k = \pm 1$ we get the equivalent expression for quark energy density and quark pressure. It is also observed that in all the cases total pressure is negative for $t = t_0$ due to $B$ (bag constant or vacuum energy density). Because Milton (1980a,1980b) has pointed out that the normalized zero point energy of the confined fields in the MIT bag model is repulsive rather than attractive. So, vacuum energy density (bag constant) may have negative pressure. In this case quarks and gluons which are confined are moving freely. They are not moving collectively, i.e. as a perfect liquid.

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