Right-Handed Sneutrinos as Self-Interacting Dark Matter in Supersymmetric Economical 3-3-1 Model

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Abstract

In this work we show that the supersymmetric economical $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (3-3-1) gauge model has a realistic candidate for self-interacting dark matter. In the model under consideration, the right-handed sneutrino is in bottom of the triplet, which is a singlet under the Standard Model $SU(2)_L$ group. By this property, the right-handed sneutrino does not interact with ordinary particles in the Standard Model except for its Higgs boson. In addition, the right-handed sneutrino is the lightest slepton, so they are stable without introduction of extra symmetry. From the Spergel-Steinhardt condition, the typical mass limit $\leq 10$ MeV is derived. With self-interacting coupling constant fixed by supersymmetry, this limit is deduced without any approximation. The smallness of the sneutrinos leads to $\tan \beta \approx 1$ in the high energy limit. The tachyon elimination in the Higgs sector leads to the mass degeneracy for left- and right-handed sneutrinos. From the condition for average density of cosmological cold dark matter, the ratios for the vacuum expectation values in the model are deduced.

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1 Permanent address
1 Introduction

One of the themes of the history of physics has been the discovery that the world familiar to us is only a tiny part of an enormous and multi-faceted Universe. Over the past ten years, astronomers have recognized that the matter fermion that we are made of accounts for only 4% of the total content of the Universe.

Until a few years ago, the more satisfactory cosmological scenarios were ones composed of ordinary matter, cold dark matter and a contribution associated with the cosmological constant. To be consistent with inflationary cosmology, the spectrum of density fluctuations would be nearly scale-invariant and adiabatic. However, in recent years it has been pointed out that the conventional models of collisionless cold dark matter (CCDM) lead to problems with regard to galactic structures. N-body simulations with CCDM indicate that galaxies should have singular halos [1] with large numbers of subhalos. The CCDM predictions for the Tully-Fisher relation and the stability of galactic bars in high surface brightness spiral galaxies are not in agreement with what is observed, indicating lower density galaxy cores than predicted by CCDM. A number of other inconsistencies, which we will not describe here, are discussed in [2].

In order to overcome the possible difficulties of CCDM, one suggestion has been that the cold dark matter particles have a non-dissipative self-interaction [3, 4], and it has been shown that such cold, non-dissipative self-interacting dark matter (SIDM) [5, 6] can be effective in alleviating the various problems of CCDM [8]. One should notice that self-interacting models lead to spherical halo centers in clusters, which is not in agreement with ellipsoidal centers indicated by strong gravitational lensing observations and by Chanda ones [9, 10]. However, SIDM models are self-motivated as alternative models. The key property of this kind of matter is that, although its annihilation cross-section is suppressed, its scattering cross section is enhanced.

Several authors have proposed models in which a specific scalar singlet that satisfies the SIDM properties is introduced in the Standard Model (SM) in an ad hoc way [5, 6, 7]. To be stable, this scalar cannot interact strongly with the SM particles and it is guaranteed by introduction of an extra symmetry (usually an $U(1)$).

The first gauge model for SIDM were found by Fregolente and Tonasse [11] in the minimal 3-3-1 model. The next version of SIDM is the 3-3-1 model with right-handed neutrinos [12] (For alternative direction in which the singlet Higgs fields are weakly interacting massive particles (WIMP), see Ref. [13]).

One of the main motivations to study the 3-3-1 models is an explanation in part of the generation number puzzle. In the 3-3-1 models, each generation is not anomaly free; and the model becomes anomaly free if one of quark families behaves
differently from other two [14, 15]. Consequently, the number of generations \( N_f \) is multiple of the color number. Combining with the QCD asymptotic freedom which is valid for \( N_f < 5 \), the generation number has to be three.

In one of the 3-3-1 models, the right-handed neutrinos are in bottom of the lepton triplets [16] and three Higgs triplets are required. It is worth noting that in the version with right-handed neutrinos, there are two Higgs triplets with neutral components in the top and bottom. In the earlier version, these triplets can have vacuum expectation value (VEV) either on the top or in the bottom, but not in both. Assuming that all neutral components in the triplet can have VEVs, we are able to reduce number of triplets in the model to be two [17, 18]. Such a scalar sector is minimal, therefore it has been called the economical 3-3-1 model [19]. In a series of papers, we have developed and proved that this non-supersymmetric version is consistent, realistic and very rich in physics [18, 19, 20, 21].

It is known that the economical (non-supersymmetric) 3-3-1 model does not furnish any candidate for self-interaction dark matter [6] with the condition given by Spergel and Steinhardt [3]. With a larger content of the scalar sector, the supersymmetric version is expected to have a candidate for the self-interaction dark matter. An supersymmetric version of the minimal version (without extra lepton) has been constructed in Ref. [22] and its scalar sector was studied in Ref. [23]. Lepton masses in framework of the above mentioned model was presented in Ref. [24], while potential discovery of supersymmetric particles was studied in [25]. In Ref. [26], the \( R \)-parity violating interaction was applied for instability of the proton.

The supersymmetric version of the 3-3-1 model with right-handed neutrinos has already been constructed in Ref. [27]. The scalar sector was considered in Ref. [28] and neutrino mass was studied in Ref. [29]. Note that there is three-family versions in which lepton families are treated differently [30] and their supersymmetric versions are presented in Ref. [31]. It is worth mentioning that in the previous papers on supersymmetric version of the 3-3-1 models, the main attention was given to the gauge boson, lepton mass and Higgs sectors. To our knowledge, there was no work concerning sfermions. An supersymmetric version of the economical 3-3-1 model has been constructed in Ref. [32]. Some interesting features such as Higgs bosons with masses equal to that of the gauge bosons – the \( W \) and the bileptons \( X \) and \( Y \), have been pointed out in Ref. [33].

In a supersymmetric extension of the (beyond) SM, each of the known fundamental particles must be in either a chiral or gauge supermultiplet and have a superpartner with spin differing by 1/2 unit. All of the matter fermions (the known quarks and leptons) have spin-0 partners called sfermions. Hence in supersymmetric models, besides scalar Higgs bosons, there are scalar sfermions.

In the other hands, supersymmetry [34] contains interesting Higgs physics [35], where Higgs masses are constrained by supersymmetry. While earlier one might have
viewed the Higgs fields as just one of many features of low energy supersymmetric models, the constraints on the Higgs mass are now problematic. In this paper, we show that despite Higgs sector does not provide the candidate for the SIDM, but the right-handed sneutrinos are good ones.

This paper is organized as follows. In Sec. 2 we recapitulate the necessary elements of the model under consideration. The couplings of SIDM are presented in Sec. 3, while in Sec. 4 we derive the lower mass limit for the SIDM. In Sec. 5 we get the condition for thermal generation of SIDM. Finally, the last section - Sec. 6 is devoted to our conclusions.

2 Basic elements

In this section we first recapitulate the basic elements of the model [32], which are related to our analysis below.

2.1 Particle content

The superfield content in this paper is defined in a standard way as follows

\[
\hat{F} = (\tilde{F}, F), \quad \hat{S} = (S, \tilde{S}), \quad \hat{V} = (\lambda, V),
\]

(2.1)

where the components \(F, S\) and \(V\) stand for the fermion, scalar and vector fields while their superpartners are denoted as \(\tilde{F}, \tilde{S}\) and \(\lambda\), respectively [34, 27].

The superfields for the leptons under the 3-3-1 gauge group transform as

\[
\hat{L}_{aL} = \left(\hat{\nu}_a, \hat{l}_a, \hat{\nu}^c_a\right)_L^T \sim (1, 3, -1/3), \quad \hat{\nu}^c_{aL} \sim (1, 1, 1),
\]

(2.2)

where \(\hat{\nu}^c_{aL} = (\hat{\nu}_R)^c\) and \(a = 1, 2, 3\) is a generation index. Here and in the following, the values in the parentheses denote quantum numbers based on the (SU(3)_C, SU(3)_L, U(1)_X) symmetry.

The superfields for the left-handed quarks of the first generation are in triplets

\[
\hat{Q}_{1L} = \left(\hat{u}_1, \hat{d}_1, \hat{u}'_1\right)_L^T \sim (3, 3, 1/3),
\]

(2.3)

where the right-handed singlet counterparts are given by

\[
\hat{u}_{1L}, \quad \hat{u}'_{1L} \sim (3^*, 1, -2/3), \quad \hat{d}^c_{1L} \sim (3^*, 1, 1/3).
\]

(2.4)

Conversely, the superfields for the last two generations transform as antitriplets

\[
\hat{Q}_{aL} = \left(\hat{d}_a, -\hat{u}_a, \hat{d}'_a\right)_L^T \sim (3, 3^*, 0), \quad \alpha = 2, 3,
\]
where the right-handed counterparts are in singlets
\[ \hat{u}^c_{aL} \sim (3^*, 1, -2/3), \quad \hat{d}^c_{aL}, \quad \hat{d}'^c_{aL} \sim (3^*, 1, 1/3). \] (2.5)

The primes superscript on usual quark types \((u', d')\) with the electric charge \(q_{u'} = 2/3\) and \(q_{d'} = -1/3\) indicate that those quarks are exotic ones. The mentioned fermion content, which belongs to that of the 3-3-1 model with right-handed neutrinos \([16, 18]\) is, of course, free from anomaly.

The two superfields \(\hat{\chi}\) and \(\hat{\rho}\) are at least introduced to span the scalar sector of the economical 3-3-1 model \([19]\):
\[ \hat{\chi} = \begin{pmatrix} \hat{\chi}_1^0, \hat{\chi}^-, \hat{\chi}_2^0 \end{pmatrix}^T \sim (1, 3, -1/3), \]
\[ \hat{\rho} = \begin{pmatrix} \hat{\rho}_1^+, \hat{\rho}_0^0, \hat{\rho}_2^- \end{pmatrix}^T \sim (1, 3, 2/3). \] (2.6)

To cancel the chiral anomalies of Higgsino sector, the two extra superfields \(\hat{\chi}'\) and \(\hat{\rho}'\) must be added as follows
\[ \hat{\chi}' = \begin{pmatrix} \hat{\chi}_1'^0, \hat{\chi}^{'+}, \hat{\chi}_2'^0 \end{pmatrix}^T \sim (1, 3^*, 1/3), \]
\[ \hat{\rho}' = \begin{pmatrix} \hat{\rho}_1'^-, \hat{\rho}_0'^0, \hat{\rho}_2'^- \end{pmatrix}^T \sim (1, 3^*, -2/3). \] (2.7)

In this model, the \(\text{SU}(3)_L \otimes \text{U}(1)_X\) gauge group is broken via two steps:
\[ \text{SU}(3)_L \otimes \text{U}(1)_X \xrightarrow{w, w'} \text{SU}(2)_L \otimes \text{U}(1)_Y \xrightarrow{v, v', u, u'} \text{U}(1)_Q, \] (2.8)

where the VEVs are defined by
\[ \sqrt{2}\langle \chi \rangle^T = (u, 0, w), \quad \sqrt{2}\langle \chi' \rangle^T = (u', 0, w'), \]
\[ \sqrt{2}\langle \rho \rangle^T = (0, v, 0), \quad \sqrt{2}\langle \rho' \rangle^T = (0, v', 0). \] (2.9)

The VEVs \(w, w'\) are responsible for the first step of the symmetry breaking while \(u, u'\) and \(v, v'\) are for the second one. Therefore, they have to satisfy the constraints:
\[ u, u', v, v' \ll w, w'. \] (2.10)

It is emphasized that the VEV structure in (2.9) is not only the key to reduce Higgs sector but also the reason for complicated mixing among gauge, Higgs bosons, etc. As it will be shown in the following, the mentioned VEV structure causes flavour violation in the \(D\)-term contributions.
2.2 Higgs sector

In the model under consideration, the supersymmetric Higgs potential takes the form [32, 33]

\[ V_{\text{susyeco}} \equiv \frac{\mu^2}{4} \left( \chi^\dagger \chi + \chi'^\dagger \chi' \right) + \frac{\mu'^2}{4} \left( \rho^\dagger \rho + \rho'^\dagger \rho' \right) + \frac{g^2}{12} \left( -\frac{1}{3} \chi^\dagger \chi + \frac{1}{3} \chi'^\dagger \chi' + 2 \frac{2}{3} \rho^\dagger \rho - \frac{2}{3} \rho'^\dagger \rho' \right)^2 + \frac{g^2}{8} \left( \chi^\dagger \chi - \chi'^\dagger \chi' \right) \left( \chi^\dagger \chi - \chi'^\dagger \chi' \right) + m^2 \rho^\dagger \rho + m^2 \chi^\dagger \chi + m^2 \rho'^\dagger \rho' + m^2 \chi'^\dagger \chi'. \] (2.11)

Assuming that the VEVs of neutral components \( u, u', v, v', w \) and \( w' \) are real, we expand the fields around the VEVs as follows

\[ \chi^T = \left( \frac{u + S_1 + iA_1}{\sqrt{2}}, \chi^-, \frac{w + S_1 + iA_1}{\sqrt{2}} \right), \quad \rho^T = \left( \rho_1^+, \frac{v + S_1 + iA_1}{\sqrt{2}}, \rho_2^+ \right), \]
\[ \chi'^T = \left( \frac{u' + S_3 + iA_3}{\sqrt{2}}, \chi'^+, \frac{w' + S_3 + iA_3}{\sqrt{2}} \right), \quad \rho'^T = \left( \rho_1^-, \frac{v' + S_6 + iA_6}{\sqrt{2}}, \rho_2^- \right). \] (2.12)

Requirement of vanishing the linear terms in fields, we get, at the tree-level approximation, the constraint equations (for details, see, [32]), and one of them

\[ (w^2 - u^2) u' w' = (w'^2 - u'^2) u w. \] (2.13)

From Eq. (2.13) it follows

\[ t_\theta \equiv \frac{u}{w} = \frac{u'}{w'}. \] (2.14)

Consequently, the model contains a pair of Higgs triplet \( \chi \) and antitriplet \( \chi' \) with the VEVs in top and bottom elements governed by the relation: \( u/w = u'/w' \).

The squared-mass matrix derived from (2.11) can be divided into three \((6 \times 6)\) matrices respective to the charged, scalar and pseudoscalar bosons. Note that there is no mixing among the scalar and pseudoscalar bosons.

Due to the fact that the self-interacting dark matter has to have unique interaction (direct) with the SM Higgs boson, therefore, it is important to find that one. Since the SM Higgs boson is electrically neutral, we are interested in only neutral Higgs boson sector.

The scalar sector contains (see Ref. [33])

1. Three massless fields: \( S_5', S_{1a}' \), and

\[ \varphi_{S_{2a}} = s_\beta S_2' + c_\beta S_4'. \] (2.15)

where

\[ t_\beta \equiv \frac{w}{w'}. \] (2.16)
2. Three massive fields corresponding to the masses:

\[
\phi_{S_{24}} = c_\beta S'_2 - s_\beta S'_4, \quad m^2_{\phi_{S_{24}}} = \frac{g^2}{4}(1 + t^2_\theta)(w^2 + w'^2) = m_X^2, \tag{2.17}
\]

\[
\varphi_{S_{36}} = s_\alpha S'_3 + c_\alpha S'_6, \quad m^2_{\varphi_{S_{36}}} = \frac{1}{2} \left[ m^2_{33a} + m^2_{66a} - \sqrt{(m^2_{33a} - m^2_{66a})^2 + 4m^4_{36a}} \right], \tag{2.18}
\]

\[
\phi_{S_{36}} = c_\alpha S'_3 - s_\alpha S'_6, \quad m^2_{\phi_{S_{36}}} = \frac{1}{2} \left[ m^2_{33a} + m^2_{66a} + \sqrt{(m^2_{33a} - m^2_{66a})^2 + 4m^4_{36a}} \right],
\]

where

\[
m^2_{33a} = \frac{18g^2 + g'^2}{54c_\theta^2}(w^2 + w'^2), \quad m^2_{66a} = \frac{9g^2 + 2g'^2}{27}(v^2 + v'^2),
\]

\[
m^2_{36a} = \frac{(9g^2 + 2g'^2)\sqrt{(v^2 + v'^2)(w^2 + w'^2)}}{54c_\theta}
\]

and

\[
t_{2\alpha} \equiv \frac{-2m^2_{36a}}{m^2_{66a} - m^2_{33a}} \propto \frac{v}{w}. \tag{2.19}
\]

From (2.18), we get

\[
m^2_{\varphi_{S_{36}}} \simeq h_1 h_2 - h_3^2 (v^2 + v'^2), \tag{2.20}
\]

where

\[
h_1 \equiv \frac{18g^2 + g'^2}{54c_\theta^2}, \quad h_2 \equiv \frac{9g^2 + 2g'^2}{27}, \quad h_3 \equiv \frac{9g^2 + 2g'^2}{54c_\theta^2}.
\]

Taking into account \(\alpha = \frac{\pi^2}{4\pi} = \frac{1}{12\pi}, \quad s^2_W = 0.2312, \quad t = \frac{g'}{g} = \frac{3\sqrt{2}s_W}{\sqrt{4c^2_W - 1}} \) [36] we have

\[
m^2_{\varphi_{S_{36}}} \simeq 91.4 \text{ GeV}. \tag{2.21}
\]

This value is very close to the lower limit of 89.8 GeV (95% CL) given in Ref. [37] p. 32. It is interesting to note that this mass is also close to the Z boson mass.

Pursuing interactions of the scalar Higgs bosons with the SM gauge ones [33], it was recognized that the physical field \(\varphi_{S_{36}}\) can be identified with the SM Higgs boson \(H: H \equiv \varphi_{S_{36}}\).

Thus, the weak eigenstates and physical eigenstates are related through the
following matrix [33]

\[
\begin{pmatrix}
S_1 \\
S_2 \\
S_3 \\
S_4 \\
S_5 \\
S_6
\end{pmatrix}
= \begin{pmatrix}
c_\beta s_\theta & -s_\beta c_\theta & -c_\beta c_\theta & -s_\alpha s_\beta s_\theta & -c_\alpha s_\beta s_\theta & 0 \\
c_\beta c_\theta & s_\beta s_\theta & c_\beta s_\theta & -s_\alpha s_\beta c_\theta & -c_\alpha s_\beta c_\theta & 0 \\
c_\beta s_\theta & -c_\beta c_\theta & s_\beta c_\theta & s_\alpha c_\beta s_\theta & c_\alpha c_\beta s_\theta & 0 \\
s_\beta c_\theta & c_\beta s_\theta & -s_\beta s_\theta & s_\alpha c_\beta c_\theta & c_\alpha c_\beta c_\theta & 0 \\
0 & 0 & 0 & -c_\alpha c_\gamma & s_\alpha c_\gamma & s_\gamma \\
0 & 0 & 0 & c_\alpha s_\gamma & -s_\alpha s_\gamma & c_\gamma
\end{pmatrix}
\begin{pmatrix}
S_{1a}' \\
\varphi_{S_{24}} \\
\phi_{S_{36}} \\
H
\end{pmatrix}
\tag{2.22}
\]

where the following notations were used:

\[
cot_\gamma \equiv \frac{v}{v'}.
\tag{2.23}
\]

It is to be noticed that in the MSSM \( \tan \beta = \frac{v'}{v} \). So, in our notation, the excluded region is \( 0.5 < \cot_\gamma < 2.4 \).

### 2.3 Right-handed sneutrinos - SIDM candidates

For the further analysis, it is convenience to introduce \( R \)-parity in the model. Following Ref. [29], \( R \)-parity can be expressed as follows

\[
R - parity = (-1)^{2S}(-1)^{3(B+L)}
\tag{2.24}
\]

where invariant charges \( L \) and \( B \) given in Ref. [39]).

With the help of the mentioned \( R \)-parity, we can separate the superpotential \( W \) and the soft-term for sfermion \( L_{SMMT} \) into the \( R \)-parity conserving \( (R) \) and violating \( (\bar{R}) \) part. Thus

\[
W = W_R + W_{\bar{R}}
\tag{2.25}
\]

where

\[
W_R = \frac{1}{2} \left( \mu_\chi \hat{\chi}' + \mu_R \hat{R}' \right) + \frac{1}{3} \left( \lambda_{\chiab} \hat{L}_{al} \hat{R}^c_{bl} + \lambda_{\rhoab} \hat{L}_{al} \hat{L}_{bl} \hat{\rho} \right) + \kappa' \hat{Q}_{1L} \hat{\chi}' u_L^c + \vartheta \hat{Q}_{1L} \hat{\rho}^c d_L^c + \pi_\alpha \hat{\varphi}_{\alpha L} \hat{\rho} u_L^c + \Pi_\alpha \hat{\varphi}_{\alpha L} \hat{d}_L^c \\
+ \kappa' \hat{Q}_{1L} \hat{\chi}' u_L^c + \vartheta' \hat{Q}_{1L} \hat{\rho}^c d_L^c + \pi_\alpha' \hat{\varphi}_{\alpha L} \hat{\rho} u_L^c + \Pi_\alpha' \hat{\varphi}_{\alpha L} \hat{d}_L^c \right)
\tag{2.26}
\]

and

\[
W_{\bar{R}} = \frac{1}{2} \mu_{\alpha L} \hat{L}_{al} \hat{\chi}' + \frac{1}{3} \left( \lambda_a \hat{L}_{al} \hat{\chi} \hat{\rho} + \epsilon f_{\alpha \beta \gamma} \hat{Q}_{\alpha L} \hat{Q}_{\beta L} \hat{Q}_{\gamma L} \right) + \xi_{1\beta j} \hat{d}_{jl}^c \hat{\chi}_{\rho L} + \xi_{2\beta j} \hat{d}_{jl}^c \hat{\chi}_{\rho L} + \xi_{3ijk} \hat{d}_{ij}^c \hat{\chi}_{\rho L} + \xi_{50\beta j} \hat{d}_{jl}^c \hat{\chi}_{\rho L} + \xi_{60\beta j} \hat{d}_{jl}^c \hat{\chi}_{\rho L} \\
+ \xi_{\alpha ij} \hat{L}_{al} \hat{Q}_{\alpha L} \hat{\chi}_{\rho L} + \xi_{\alpha ij} \hat{L}_{al} \hat{Q}_{\alpha L} \hat{\chi}_{\rho L} \right)
\tag{2.27}
\]
The $\mathcal{R}$ part contains odd number of matter superfields. For the soft terms, we have also
\[
\mathcal{L}_{SMT} = \mathcal{L}_{SMT}^R + \mathcal{L}_{SMT}^R,
\]
where
\[
-\mathcal{L}_{SMT}^R = M_\alpha^2 \bar{L}_3 L_\alpha + m_{ab} \bar{u}_L u_L + m_{Q1L} \bar{Q}_{1L} Q_{1L} + m_{Q2L} \bar{Q}_{2L} Q_{1L} L
\]
\[
+ m_{u_i} \bar{u}_i u_j + m_{d_j} \bar{d}_j d_j + m_{u_L} \bar{u}_L u_L + m_{d_L} \bar{d}_L d_L
\]
\[
+ \left\{ \eta_{ab} \bar{L}_{aL} \bar{L}_{bL} + \epsilon_{abc} \bar{L}_{aL} L_{cL} + p \bar{Q}_{1L} \chi L \right\}
\]
\[
+ p_{\alpha} \bar{Q}_{\alpha L} \rho \bar{c}_L h_i + h_{\alpha} \bar{Q}_{\alpha L} \chi d_L + h_{\alpha} \bar{Q}_{\alpha L} \chi d_L + r_{\alpha} \bar{Q}_{\alpha L} \rho \bar{c}_L + H.c. \}
\]
and
\[
-\mathcal{L}_{SMT}^R = M_\alpha^2 \chi^4 L_{aL} + \omega_{\alpha} \bar{L}_{aL} \chi \rho
\]
\[
+ p_{5\alpha} \bar{Q}_{\alpha L} \bar{Q}_{3L} Q_{3L} + \kappa_{ij} \bar{d}_{iL} \bar{d}_{jL} d^c_{ij} + \beta_{ij} \bar{d}_{iL} d^c_{iL} \bar{u}^c_{ij}
\]
\[
+ \pi_{ij} \bar{d}_{iL} d^c_{iL} \bar{d}_{jL} d^c_{jL} + \kappa_{ij} \bar{d}_{iL} d^c_{jL} \bar{d}_{jL} d^c_{ij}
\]
\[
+ \kappa_{ij} \bar{d}_{iL} d^c_{iL} \bar{d}_{jL} d^c_{jL} + \omega_{\alpha \alpha} \bar{L}_{aL} Q_{aL} \bar{Q}_{aL} Q_{aL} + H.c. \}
\]

The $\mathcal{R}$ soft terms consist of odd number of supersymmetric partners - sfermions. It is emphasized that $M_{ab}^2$ is coefficient of the soft term giving slepton masses.

Note that the last lines in (2.26) and (2.28) contain lepton-number violating terms (with $\Delta L = \pm 2$). Hence we have (see also [21])
\[
\kappa_i, \varphi_i', \pi_i, \Pi_i', p_i, r_i, h_i', h_{\alpha}, \psi_i \ll \kappa', \varphi, \psi, \Pi, p, p_i, h_i, h_{\alpha}. \quad (3.30)
\]

The scalar potential of the model is a result of summation over $F$ and $D$ terms:
\[
V = F^\phi \phi + \frac{1}{2} \sum_a D^a D_a, \quad (3.31)
\]
where [41]
\[
F_\phi = \frac{\partial W}{\partial \phi}, \quad W = W_2 + W_3,
\]
and
\[
D^a = -g \left( \sum_\phi \phi^a T^a \phi \right). \quad (3.32)
\]

The field $\phi$ stands for all the scalars or sfermions in the model.

The above potential yields the mass Lagrangian for sneutrinos. In the base $(\tilde{\nu}_{aL}, \tilde{\nu}_{bR})=(\tilde{\nu}_{1L}, \tilde{\nu}_{2L}, \tilde{\nu}_{3L}, \tilde{\nu}_{1R}, \tilde{\nu}_{2R}, \tilde{\nu}_{3R})$, the mass matrix is given by [40]
\[
\begin{pmatrix}
A_{ab} & E_{ab} \\
E_{ab} & G_{ab}
\end{pmatrix}, \quad (3.33)
\]
where

\[ A_{ab} = \frac{g^2}{2} \delta_{ab} \left( N_3 + \frac{1}{\sqrt{3}} N_8 - \frac{2t^2}{3} N_1 \right) + M_{ab}^2 + \frac{1}{4} \mu_0 a \mu_b \]  

(2.34)

\[ + \frac{1}{18} v^2 (\lambda_a \lambda_b + 4 \lambda'_a \lambda'_b) + \frac{1}{18} \lambda_a \lambda_b w^2, \]

\[ G_{ab} = -g^2 \delta_{ab} \left( \frac{1}{\sqrt{3}} N_8 + \frac{t^2}{3} N_1 \right) + M_{ab}^2 + \frac{1}{4} \mu_0 a \mu_b \]

\[ + \frac{1}{18} v^2 (\lambda_a \lambda_b + 4 \lambda'_a \lambda'_b) + \frac{1}{18} \lambda_a \lambda_b u^2, \]

\[ E_{ab} = -\sqrt{2} \left( \varepsilon_{ab} v + \frac{1}{6} \mu_0 \lambda'_a \lambda'_b v' \right), \]  

(2.35)

and [40]

\[ N_3 = -\frac{1}{4} \left( u^2 \cos 2\beta \frac{s^2}{s'^2} + v^2 \cos 2\gamma \frac{c^2}{c'^2} \right), \]

\[ N_8 = \frac{1}{4\sqrt{3}} \left[ v^2 \cos 2\gamma \frac{c^2}{c'^2} - (u^2 - 2w^2) \cos 2\beta \frac{s^2}{s'^2} \right], \]

\[ N_1 = \frac{1}{6} \left[ (u^2 + w^2) \cos 2\beta \frac{s^2}{s'^2} + 2v^2 \cos 2\gamma \frac{c^2}{c'^2} \right]. \]  

(2.36)

As usual, we assume that there is substantial mixing among \((\tilde{\tau}_L, \tilde{\tau}_R)\) only [41]. Then eigenstates and eigenmasses in this case are given in Table 1.

### Table 1: Masses and eigenstates of sneutrinos

<table>
<thead>
<tr>
<th>Eigenstate</th>
<th>(\tilde{\nu}_{1L})</th>
<th>(\tilde{\nu}_{2L})</th>
<th>(\tilde{\nu}_{3L})</th>
<th>(\tilde{\nu}_{1R})</th>
<th>(\tilde{\nu}_{2R})</th>
<th>(\tilde{\nu}_{3R})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Mass)(^2)</td>
<td>(A_{11})</td>
<td>(A_{22})</td>
<td>(A_{33})</td>
<td>(G_{11})</td>
<td>(G_{22})</td>
<td>(G_{33})</td>
</tr>
</tbody>
</table>

In the parameter space including: the Yukawa coupling Higg-lepton-lepton \((\gamma_{ab})\), the soft terms for lepton masses \((M_{ab}^2)\) and connecting with the D-terms \(H_i, i = 3, 8, 1\), we can outline the schematic spectrum of sleptons as in figure 1 (for details, see [40])

The mass splittings for the sleptons are governed by sum-rules [40]

\[ m_{\tilde{\tau}_{1L}}^2 - m_{\tilde{\tau}_{1L}}^2 = m_{\tilde{\tau}_{2L}}^2 - m_{\tilde{\tau}_{2L}}^2 = -g^2 T_3 = \frac{g^2}{4} \left( v^2 \cos 2\gamma \frac{c^2}{c'^2} + u^2 \cos 2\beta \frac{s^2}{s'^2} \right) \]
Figure 1: A schematic sample mass spectrum for sleptons, in which mass scales between generations are not taken into account and \( l = e, \mu \).

\[
\begin{align*}
\text{Mass[GeV]} & \quad \tilde{l}_L \quad \tilde{\nu}_L \quad \tilde{\nu}_R \quad \tilde{\mu}_R \quad \tilde{\tau}_R \quad \tilde{\tau}_L \\
400 & \quad \tilde{\nu}_L \quad \tilde{\nu}_R \quad \tilde{\nu}_L \quad \tilde{\nu}_R \quad \tilde{\nu}_L \quad \tilde{\nu}_R \\
200 & \\
0 & \quad \tilde{l}_L \\
\end{align*}
\]

\[m_{\tilde{\nu}^1_l} - m_{\tilde{\nu}^1_R} = m_{\tilde{\nu}^2_l} - m_{\tilde{\nu}^2_R} = \frac{g^2}{2} \left( T_3 + \sqrt{3} T_8 \right) = \frac{g^2}{4} \left( w^2 - u^2 \right) \cos 2\beta \left( s_\beta^2 \right), \quad (2.38)\]

In the limit \( u \approx 0 \), Eq. (2.37) is consistent with those in the Minimal Supersymmetric Standard Model. Assuming further \( \cos 2\beta > 0 \), we obtain: \( m_{\tilde{\nu}^1_l} > m_{\tilde{\nu}^2_R} \). Since no experimental data on supersymmetric partners, we have a right to assume that.

Let us discuss on mass scale in Fig.1. It is based on the limits given in [37], which are based on the MSSM. Let us take some of them

\[
m_{\tilde{\nu}^1_l} > 250 \text{ GeV} \quad [\tan \beta = 2], \quad (2.39)
m_{\tilde{\nu}^1_\ell} > 73 \text{ GeV}. \quad (2.40)
\]

As in (2.39), taking \( \tan \gamma = 2 \) and assuming that the second term in r.h. side of (2.37) is smaller, we get

\[
m_{\tilde{\nu}^1_l}^2 - m_{\tilde{\nu}^2_l}^2 \simeq m_W^2 \frac{3}{5} \approx 3876.57 \text{ GeV}^2. \quad (2.41)
\]

Combination of (2.40) and (2.41) gives

\[
m_{\tilde{\nu}^1_l}^2 \approx 38.11 \text{ GeV}^2. \quad (2.42)
\]
It is worth noting that since no convincing evidence for the production of super-symmetric particles has been found, the above mentioned limit should be considered rather symbolic than realistic.

Thus, we have shown that the right-handed sneutrinos are the lightest sfermions \cite{40}.

To finish this section, we note that the right-handed sneutrinos are the lightest sfermions (associatively with suggestion $\cos 2\beta > 0$). Hence they cannot decay, consequently are stable. In addition, since they are singlet under the SM $SU(2)_L$ gauge group, they do not interact with the ordinary particles of the SM. For some range of the parameters, they pose the right abundance for CDM (see below). Hence they are realistic candidate for DM. Concerning $\tilde{\nu}_{aL}$ stability, notice that they carry lepton number $L = -1$, so final state of their decay must be slepton and scalar Higgs boson. However, this is forbidden due to the smallness of their masses. In the model under consideration, the squarks are also singlet under the SM $SU(2)_L$. Due to the electric charge, however, these fields cannot be a candidate for dark matter.

For the short, let us call the right-handed sneutrinos as dark matter and denote $\tilde{\nu}_{aL}$ by $S$.

Let us consider a special case in which $w \approx w' \Rightarrow \tan \beta = 1, \cos 2\beta = 0$. If so, from Eq. (2.38) we get

$$m^2_{\tilde{\nu}_L} - m^2_{\tilde{\nu}_R} \simeq 0 \quad (2.43)$$

The mentioned case ($w \approx w'$) leads not only to the necessary mass degeneracy for the left and right-handed sneutrinos. but also to the tachyon elimination in the Higgs sector \cite{33}.

3 Interaction of the DM candidate

It is well-known that to be candidate for the SIDM, particles do not interact with particles of the SM except, with the Higgs boson. In the model under consideration, the couplings arise in both $F$- and $D$-term contributions. The scalar potential of the model is a result of summation over $F$ and $D$ terms \cite{41}:

$$V = F^\phi \phi + \frac{1}{2} \sum_a D^a D_a. \quad (3.1)$$

1. Coupling from $F$-terms

Here we display only the $F$-terms giving necessary interactions \cite{40}:

$$\mathcal{L}_F = \frac{1}{9} \lambda_a \lambda_b \left[ (\tilde{L}^*_{aL} \tilde{L}_{bL})(\rho^* \rho) - (\tilde{L}^*_{aL} \rho)(\rho^* \tilde{L}_{bL}) \right]$$

$$+ \frac{1}{9} \lambda_a \lambda_b \left[ (\tilde{L}^*_{aL} \tilde{L}_{bL})(\chi^* \chi) - (\tilde{L}^*_{aL} \chi)(\chi^* \tilde{L}_{bL}) \right]$$
Right-handed sneutrinos as self-interacting dark matter

\[ + \frac{4}{9} \lambda'_a \lambda'_{cb} [(\tilde{L}^*_a L)(\rho^* \rho) - (\tilde{L}^*_a L \rho)(\rho^* \tilde{L}^*_b)] \]

\[ + \frac{1}{9} \gamma_{ac} \gamma_{bc} (\tilde{L}^*_a L \rho')(\rho^* \tilde{L}^*_b \rho'). \quad (3.2) \]

Notations in this section is given in Ref. [40]. From (3.2), we get couplings of the right-handed sneutrinos with neutral scalar Higgs bosons:

\[ \mathcal{L}_{SSH}^F = \frac{1}{9} \lambda_a \lambda_b (\tilde{\nu}_c^* a L \tilde{\nu}_c b L)(\chi^*_0 1 \chi^*_0 1 + \chi^*_0 3 \chi^*_0 3 + \rho^*_0 \rho^*_0) \]

\[ + \frac{4}{9} \lambda'_a \lambda'_{cb} (\tilde{\nu}_c^* a L \tilde{\nu}_c b L)(\rho^*_0 \rho^*_0). \quad (3.3) \]

It is worth noting that \( \lambda_a \) is a coefficient of \( R \)-parity violating interactions (see [40]), hence they have to be very small. Therefore, the main contribution in (3.3) is the last term. It was known that the mentioned term provides mass for neutrinos, so it has to be much smaller as compared to \( \gamma_{ac} [21]: \lambda'_a \ll \gamma_{ac}. \)

2. Coupling from \( D \)-terms

As before, we display the terms giving necessary contribution only. It also exists in \( D \)-term forms:

\[ D^a = -g \left( \sum_{\text{sfermions}} \tilde{f}^\dagger T^a f + \sum_{\text{Higgs}} H^\dagger T^a H \right). \quad (3.4) \]

Since \( T_a = T^\dagger_a \), we have

\[ (D^a)^* D_a = \left( \sum_{\text{sfermions}} \tilde{f}^\dagger T^a f \right)^2 
\]

\[ + 2g^2 \left( \sum_{\text{sfermions}} \tilde{f}^\dagger T^a f \right) \left( \sum_{\text{Higgs}} H^\dagger T^a H \right) + \cdots, \quad (3.5) \]

where \( \cdots \) are the terms which do not contribute to sfermion masses. The first term gives sfermion self-interactions. The factor 2 in the second term in (3.5) is the Newton’s binomial coefficient. Since sneutrino masses and interactions are our interest, therefore, in the second factor at the last line of (3.5), only the diagonal \( T_8 \) satisfies this purpose. This factor is given by:

\[ H_8 \equiv \sum_{H=\chi, \chi', \rho, \rho'} <H^\dagger > T_8 <H> \]

\[ = \frac{1}{2\sqrt{3}} \left( \chi_1^0 \chi_1^0 - 2 \chi_3^0 \chi_3^0 - \chi_1^0 \chi_1^0 + 2 \chi_3^0 \chi_3^0 + \rho^0 \rho^0 - \rho^0 \rho^0 \right). \quad (3.6) \]

Here we have taken into account that for antitriplets, \( T_8 \) changes a sign. Let us consider the first factor of the about mentioned term in (3.5). Since the
singlet fields do not give contribution, hence for sleptons we have:

\[ SL_8 \equiv \tilde{l}^\dagger_{aL} T_8 \tilde{L}_{aL} = \frac{1}{\sqrt{3}} \left( \frac{1}{2} \tilde{\nu}^c_{aL} \tilde{\nu}_{aL} + \frac{1}{2} \tilde{\nu}_{aL} \tilde{\nu}^c_{aL} - \tilde{\nu}^c_{aL} \tilde{\nu}_{aL} \right). \tag{3.7} \]

Thus, the contribution from \( SU(3)_L \) subgroup is:

\[ g^2 SL_8 \times H_s. \tag{3.8} \]

So, sneutrino self-interaction arisen from \( SL_8 \) is given by:

\[ \frac{g^2}{6} (\tilde{\nu}^c_{aL} \tilde{\nu}^c_{aL})^2. \tag{3.9} \]

Now we are looking at \( U(1)_X \) subgroup:

First, for the Higgs part, we have

\[ H_1 = \sum_{H=\chi,\chi',\rho,\rho'} <H|X|H> \]

\[ = -\frac{1}{3} [(\chi_1^0 \chi_1^0 + \chi_3^0 \chi_3^0) - (\chi_1'^0 \chi_1'^0 + \chi_3'^0 \chi_3'^0) - 2(\rho^0 \rho^0 - \rho'^0 \rho'^0)]. \tag{3.10} \]

Similarly, for sleptons

\[ SL_1 \equiv -\frac{1}{3} (\tilde{\nu}^c_{aL} \tilde{\nu}_{aL} + \tilde{\nu}_{aL} \tilde{\nu}^c_{aL} + \tilde{\nu}^c_{aL} \tilde{\nu}^c_{aL}) + \tilde{\nu}^c_{aL} \tilde{\nu}^c_{aL}. \tag{3.11} \]

The contribution from subgroup \( U(1)_X \) is

\[ g^2 \times SL_1 \times H_1 = g^2 t^2 \times SL_1 \times H_1. \tag{3.12} \]

Again, sneutrino self-interaction is given by

\[ \frac{g^2 t^2}{18} (\tilde{\nu}^c_{aL} \tilde{\nu}^c_{aL})^2. \tag{3.13} \]

The total contribution is a result of summation over two above mentioned subgroup parts. Thus, the dark matter - Higgs boson interactions are given by

\[ \mathcal{L}^0_{SSH} \in (SL_8 H_s + SL_1 H_1) \]

\[ = -\frac{g^2}{6} (\tilde{\nu}^c_{aL} \tilde{\nu}^c_{aL})[(\chi_1^0 \chi_1^0 - 2\chi_3^0 \chi_3^0 - \chi_1'^0 \chi_1'^0 + 2\chi_3'^0 \chi_3'^0 + \rho^0 \rho^0 - \rho'^0 \rho'^0)

+ \frac{g^2 t^2}{9} (\tilde{\nu}^c_{aL} \tilde{\nu}^c_{aL})[(\chi_1^0 \chi_1^0 + \chi_3^0 \chi_3^0) - (\chi_1'^0 \chi_1'^0 + \chi_3'^0 \chi_3'^0) - 2(\rho^0 \rho^0 - \rho'^0 \rho'^0)]. \tag{3.14} \]
Hence the total DM-Higgs interaction Lagrangian is the following

\[ \mathcal{L}_{\text{int}} = \mathcal{L}_{\text{SSH}}^F + \mathcal{L}_{\text{SSH}}^D \]

\[ = \frac{1}{9} \lambda_a \lambda_b (\bar{\nu}_{aL}^c \nu_{bL}^c) \left( \chi_1^0 \chi_1^0 + \chi_3^0 \chi_3^0 + \rho^0 \rho^0 \right) + \frac{4}{9} \lambda_{ca} \lambda_{db} (\bar{\nu}_{aL}^c \nu_{bL}^c)(\rho^0 \rho^0) \]

\[ - \frac{g^2}{6} (\bar{\nu}_{aL}^c \nu_{bL}^c)(\chi_1^0 \chi_1^0 - 2 \chi_3^0 \chi_3^0 - \chi_1^0 \chi_1^0 + 2 \chi_3^0 \chi_3^0 + \rho^0 \rho^0 - \rho^0 \rho^0) \]

\[ + \frac{g^2 t_2^2}{9} (\bar{\nu}_{aL}^c \nu_{bL}^c)[(\chi_1^0 \chi_1^0 + \chi_3^0 \chi_3^0) - (\chi_1^0 \chi_1^0 + \chi_3^0 \chi_3^0) - 2(\rho^0 \rho^0 - \rho^0 \rho^0)]. \]

(3.15)

Substitution of (2.12) into (3.15) yields quartic couplings

\[ \mathcal{L}_{\text{SSH}} = \frac{1}{18} \lambda_a \lambda_b (\bar{\nu}_{aL}^c \nu_{bL}^c) \left( S_1^2 + S_2^2 + S_6^2 + A_1^2 + A_2^2 + A_5^2 \right) \]

\[ + \frac{4}{18} \lambda_{ca} \lambda_{db} (\bar{\nu}_{aL}^c \nu_{bL}^c)(S_6^2 + A_5^2) \]

\[ - \frac{g^2}{12} (\bar{\nu}_{aL}^c \nu_{bL}^c)(S_1^2 - 2S_2^2 - S_3^2 + 2S_4^2 + S_5^2 - S_6^2) \]

\[ + A_1^2 - 2A_2^2 - A_3^2 + 2A_4^2 + A_5^2 - A_6^2 \]

\[ + \frac{g^2 t_2^2}{18} (\bar{\nu}_{aL}^c \nu_{bL}^c)[(S_1^2 + S_2^2) - (S_3^2 + S_4^2) - 2(S_5^2 - S_6^2) \]

\[ + (A_1^2 + A_2^2) - (A_3^2 + A_4^2) - 2(A_5^2 - A_6^2)]. \]

(3.16)

We remind that \( A_5, A_6 \) are Goldstone bosons (massless) \cite{32} and three massless states are mixing of

\[ A_1' = s_\beta A_1 - c_\beta A_3, \]

\[ A_2' = s_\beta A_2 - c_\beta A_4, \]

\[ \varphi_A = s_\theta A_3 + c_\theta A_4, \]

(3.17)

where

\[ A_3' = c_\beta A_1 + s_\beta A_3, \quad A_4' = c_\beta A_2 + s_\beta A_4. \]

(3.18)

One massive eigenstate

\[ \phi_A = c_\beta A_3' - s_\beta A_4', \]

(3.19)

with mass equal to those of the \( X \) bilepton \cite{32}

\[ m_{\phi_A}^2 = \frac{g^2}{4} (1 + t_\beta^2)(w^2 + w'^2) = m_X^2. \]

(3.20)

Expressing \( S_i, A_i, i = 1, 2, ..., 6 \) through physical fields by (2.22), we will get quartic DM-DM-Higgs-Higgs interactions. However, we are just interested in the
coupling of the SM Higgs boson $H$. It reads

\[
\mathcal{L}_{SSHH} = \frac{1}{18} \lambda_a \lambda_b (\bar{\nu}_{aL} \bar{\nu}_{bL}) H^2 (s_\alpha^2 s_\beta^2 + c_\alpha^2 c_\gamma^2) + \frac{4}{18} \lambda'_c \lambda'_b (\bar{\nu}_{cL} \bar{\nu}_{bL}) H^2 (c_a^2 c_\gamma^2) \\
+ \frac{g^2}{12} (\bar{\nu}_{aL} \bar{\nu}_{bL}) H^2 [s_\alpha^2 (1 - 3 c_\beta^2)(1 - 2 c_\alpha^2) + c_a^2 (1 - 2 s_\gamma^2)] \\
- \frac{2 t^2}{3} (s_\alpha^2 c_{2\beta} + 2 c_\alpha^2 c_{2\gamma})].
\]

(3.21)

Expression in (3.21) can be rewritten in the form

\[
\mathcal{L}_{SSHH} = \frac{\lambda_{Sab}}{18} (\bar{\nu}_{aL} \bar{\nu}_{bL}) H^2,
\]

where

\[
\lambda_{Sab} = \lambda_a \lambda_b (s_\alpha^2 s_\beta^2 + c_\alpha^2 c_\gamma^2) + 4 \lambda'_c \lambda'_b (c_a^2 c_\gamma^2) \\
+ \frac{3 \delta_{ab} g^2}{2} [s_\alpha^2 (1 - 3 c_\beta^2)(1 - 2 c_\alpha^2) + c_a^2 (1 - 2 s_\gamma^2)] \\
- \frac{2 t^2}{3} (s_\alpha^2 c_{2\beta} + 2 c_\alpha^2 c_{2\gamma})].
\]

(3.23)

We turn now to the triple DM-DM Higgs boson interaction. Substitution (2.12) into (3.15) yields

\[
\mathcal{L}_{SSH} = \frac{1}{9} \lambda_a \lambda_b (\bar{\nu}_{aL} \bar{\nu}_{bL}) (u S_1 + w S_2 + v S_3) \\
+ \frac{4}{9} \lambda'_c \lambda'_b (\bar{\nu}_{cL} \bar{\nu}_{bL})(v S_5) \\
- \frac{g^2}{6} (\bar{\nu}_{aL} \bar{\nu}_{bL})(u S_1 - 2 w S_2 - u' S_3 + 2 u' S_4 + v S_5 - v' S_6) \\
+ \frac{g^2 t^2}{9} (\bar{\nu}_{aL} \bar{\nu}_{bL})(u S_1 + w S_2 - u' S_3 - w' S_4 + 2 v S_5 + 2 v' S_6). 
\]

(3.24)

Expressing $S_i, i = 1, 2, 3, ..., 6$ through physical Higgs fields by (2.12) yields the necessary couplings. Then, we can write triple DM-DM-Higgs couplings in the form:

\[
\mathcal{L}_{SSH} = \lambda_H (\bar{\nu}_{aL} \bar{\nu}_{bL}),
\]

(3.25)

where

\[
\lambda_H = \frac{-1}{9} [\lambda_a \lambda_b (u s_\alpha s_\beta s_\theta + w s_\alpha s_\beta c_\theta + v c_a c_\gamma) + 4 \lambda'_c \lambda'_b v c_a c_\gamma] \\
+ \frac{\delta_{ab} g^2}{6} \left[2 w \frac{s_\alpha c_\theta}{s_\beta} - v \frac{c_\alpha}{c_\gamma} - \frac{t^2}{3} \left( w \frac{s_\alpha c_\theta}{s_\beta} - 2 v \frac{c_\alpha}{c_\gamma}\right) \right].
\]

(3.26)
Here we have taken into account \( u = u' \) \[33\]. As mentioned above, both kinds of couplings constants in the \( F\)-terms are small. Thus, the main contribution in (3.26) is one from the \( D\)-terms.

The \( D\)-terms give also dark matter self-interaction. This kind of interaction exists only in \( D\)-terms. Summation over (3.9) and (3.13) yields quartic DM self-interaction

\[
\mathcal{L}_{SSSS} = \frac{g_3^2}{6} (\tilde{\nu}_{aL}^c \tilde{\nu}_{bL}^c) (\tilde{\nu}_{bL}^c \tilde{\nu}_{aL}^c) \left( 1 + \frac{t^2}{3} \right).
\]  

(3.27)

4 Limit on sneutrino mass

It has recently been noticed that dark matter cross section per unit mass of \( \sigma_{DM} \approx 10^{-23} - 10^{-24} \text{ cm}^2 \) is consistent with all current observational constraints (see, for example, Dave et al in Ref.\[2\]). In other words, we have \[3, 42, 43\]

\[
\frac{\sigma_{DM}}{m_{dm}} = 8 \times 10^{-25} - 1 \times 10^{-23} \text{ cm}^2 \text{ GeV}^{-1}
\]  

(4.1)

where \( \sigma_{DM} \) is the cross section for scattering between dark matter particles and \( m_{dm} \) is the mass of the dark matter particle.

Denoting left side of (4.1) by \( r_S \), the above equation can be rewritten in the form \[5\]

\[
r_S \equiv \frac{\sigma_{DM}}{m_{dm}} = (2.054 \times 10^3 \div 2.568 \times 10^4) \text{ GeV}^{-3}.
\]  

(4.2)

From (3.27), it follows that cross section \( \sigma_{DM} \), in our case, consists of two terms

\[
\sigma_{DM} = \sigma(SS^+ \to SS^+) + \sigma(SS \to SS) = \frac{3}{128\pi m_S^2} \left[ \frac{2g^2}{3} \left( 1 + \frac{t^2}{3} \right) \right]^2,
\]  

(4.3)

where \( m_S \) is mass of the right-handed sneutrino - the dark matter particle. Combination of (4.2) and (4.3) implies that \[5\]

\[
m_S = 35.8\alpha_\eta^{1/3} \left( \frac{2.054 \times 10^3 \text{ GeV}^{-3}}{r_S} \right)^{1/3} \text{ MeV}.
\]  

(4.4)

Here we have denoted \( \alpha_\eta \equiv \frac{\eta^2}{4\pi} \) with \( \eta \) is quartic self-coupling of dark matter. From (3.27), it follows

\[
\alpha_\eta = \frac{1}{9\pi} g^4 \left( 1 + \frac{t^2}{3} \right) = \frac{16m_W^4}{9\pi v^4} \left( 1 + \frac{t^2}{3} \right)^2 \simeq 0.027.
\]  

(4.5)
Here we have used $m_W = 80.388 \text{ GeV}$, $v = 246 \text{ GeV}$ [37]. Note that $\alpha_\eta$ in the model under consideration is quite fair for perturbative theory and this is in good agreement with estimation in Ref. [5]. Thus

$$m_S = \alpha_\eta^{1/3}(15.4 - 35.8) \text{ MeV} \simeq (9 \div 22) \text{ MeV}. \tag{4.6}$$

So sneutrino mass limit is in the Spergel-Steinhardt mass range $\sim 30$ MeV [5].

This is appropriate to mention that, in Ref. [38], the RH sneutrino was suggested to play the role of the curvaton. By this way, the author got the limit on RH neutrino $m_{\nu 1} \leq 10^{-3} \text{ eV}$, which is consistent with a solution on the cosmic problem of $D$-term inflation.

According Table 1, squared mass of the DM $S$ is given by

$$m_S^2 = G_{ii}, \ i = 1, 2 \tag{4.7}$$

This means that the last four terms in the expression of $G_{ii}$ are almost not different.

Keeping in mind that mass of the DM is very small and taking into account of (2.38) and (2.42) we get

$$\tan^2 \beta - 1 \simeq 1.37 \times 10^{-2} \left[ \frac{\omega}{1000 \text{ GeV}} \right]^{-2}. \tag{4.8}$$

Here we have used the fact that $\omega \gg u$. This means that at the high energy limit $\tan \beta \approx 1$.

5 Thermal generation of self-interacting dark matter

The determination of the average density of cosmological cold dark matter (CDM) [10]

$$\Omega_{CDM} h^2 = 0.111^{+0.011}_{-0.015} (2\sigma) \tag{5.1}$$

imposes a stringent constraint on any beyond the SM framework featuring a weakly interacting massive stable on cosmological time-scales.

There are two processes which can produce a density of $S$ scalars: $2 \leftrightarrow 2$ annihilation processes and the decay of a thermal equilibrium density of Higgs scalars to $SS^+$ pairs, $H \rightarrow SS^+ S$ [5]. Both these contributions have been calculated in Ref. [5]. For details we refer the reader to the above mentioned reference.

The cosmic density of light gauge singlet scalars has been calculated in Ref. [5] and is given by

$$\Omega_S = 2g(T)T_g^3 \frac{\sum_i m_i \Theta_i}{\rho_c g(T)} \tag{5.2}$$
with

$$\Theta_i \equiv \frac{n_i}{T^3} = \frac{\eta \Gamma_i^2}{4 \pi^3 K m_H^3}$$  \hspace{1cm} (5.3)$$

where $T_\gamma = 2.4 \times 10^{-4} \text{ eV}$ is the present photon temperature, $g(T_\gamma) = 2$ is the photon degree of freedom, $g(T) = g_B + \frac{7}{8} g_F$ ($g_B$ and $g_F$ are the relativistic boson and fermion degree of freedom, respectively), $\rho_c = 7.5 \times 10^{-47} \text{ eV}^4$ is the critical density of the Universe ($h \simeq 0.71$ is the Hubble constant in units of $100 \text{ km s}^{-1} \text{Mpc}^{-1}$), $\eta = 1.87$, $K^2 = 4 \pi^3 g(T)/45 m_{pl}^2$ and $m_{pl} = 1.2 \times 10^{19} \text{ GeV}$ is the Planck mass. For non-supersymmetric 3-3-1 model $g(T) \simeq 130$ \cite{11}, and for the supersymmetric one, following Ref. \cite{42}, we take $g(T) \approx 260$. We will take $T = m_S$, since most of the contribution to each $\Theta_i$ comes from $T \leq m_H \leq T_{ew}$ \cite{5}, where $T_{ew} \geq 1.5 \ m_H$ \cite{44}.

Decay rate for $H$ scalar with energy $E$ is

$$\Gamma_H = \frac{\lambda_H^2}{16 \pi E},$$  \hspace{1cm} (5.4)$$

where $\lambda_H$ is triple coupling constant (HSS) \cite{5}. For our purpose, we can assume that the decay is at rest; and in this case, $E = m_H$ – a mass of the Higgs boson.

Assuming that three right-handed sneutrinos are identical, we have

$$\Omega_S h^2 = 3 \frac{2 h^2 g(T_\gamma) T^3 T^3 m_S \eta \lambda^4_H}{\rho_c g(T) K (4 \pi m_H)^5}$$  \hspace{1cm} (5.5)$$

where $m_S$ is mass of the dark matter particle and factor 3 in (5.5) is due to the mentioned above identity. Putting the mentioned above numerical values into (5.5), we obtain

$$\Omega_S h^2 = 1.65 \times 10^{18} \frac{m_S \lambda^4_H}{g(T) m_H^5} \Rightarrow \lambda^4_H = 10^{-18} \Omega_S h^2 g(T)^2 m_H^5 \frac{1}{1.65 m_S}$$  \hspace{1cm} (5.6)$$

So for $m_H = 90 \text{ GeV}$, $m_S = 30 \text{ MeV}$, we get

$$\lambda^4_H = \frac{1}{2} \Omega_S h^2 \times 10^{-3} \text{ GeV}^4$$  \hspace{1cm} (5.7)$$

Assuming the resulting density of $S$ plus $S^\pm$ is the density of CDM, we finally obtain

$$\lambda^4_H = \frac{1}{2} \Omega_{CDM} h^2 \times 10^{-3} \text{ GeV}^4$$  \hspace{1cm} (5.8)$$

Eq.(5.8) is one of the conditions to fix the parameters of the model. Let us illustrate this statement by an example below. For simplicity, we keep only leading second term in expression of $\lambda_H$. Thus

$$\lambda_H = \frac{g^2}{6} \left[ 2 w \frac{s_c c_\theta}{s_\beta} - v \frac{c_\gamma}{c_\gamma} - \frac{t^2}{3} \left( w \frac{s_c c_\theta}{s_\beta} - 2 v \frac{c_\alpha}{c_\gamma} \right) \right]$$

$$= \frac{2 m_W^2}{3 v^2} \left[ 2 w \frac{s_c c_\theta}{s_\beta} - v \frac{c_\gamma}{c_\gamma} - \frac{t^2}{3} \left( w \frac{s_c c_\theta}{s_\beta} - 2 v \frac{c_\alpha}{c_\gamma} \right) \right].$$  \hspace{1cm} (5.9)$$
Remind that

\[ t = \frac{g'}{g} = \frac{3\sqrt{2}s_W}{\sqrt{4c_W^2 - 1}} \approx 2.0054 \text{ for } s_W^2 = 0.2132. \]  

(5.10)

Combining (3.26) and (5.8) yields

\[ -\frac{1}{9} \left[ \lambda_\alpha \lambda_\beta (u s_\alpha s_\beta s_\theta + w s_\alpha s_\beta c_\theta + v c_\alpha c_\gamma) + 4\lambda'_\alpha \lambda'_\beta v c_\alpha c_\gamma \right] \]

\[ + \frac{2\delta_{ab}m_W^2}{3v_{SM}^2} \left[ 2w \frac{s_\alpha c_\alpha}{s_\beta} - v \frac{c_\alpha}{c_\gamma} - \frac{t^2}{3} \left( w \frac{s_\alpha c_\alpha}{s_\beta} - 2v \frac{c_\alpha}{c_\gamma} \right) \right] \]

\[ = 1.89 \times 10^{-6} \left( \frac{g(T)}{260} \right)^{\frac{1}{3}} \left( \frac{m_H}{90 \text{ GeV}} \right)^{\frac{2}{3}} \left( \frac{m_S}{30 \text{ MeV}} \right)^{-\frac{1}{3}}. \]  

(5.11)

By suitable choice, the condition (5.11) for the SIDM in the model under consideration can be easily satisfied. Thus, a system of three equations in (5.11) is the constraint conditions to guarantee that the SIDM does not overpopulate the Universe.

### 6 Conclusion

In this paper we have shown that the supersymmetric economical 3-3-1 model has natural candidates for the SIDM. It is the light right-handed sneutrinos. The reason behind this choice relies on the fact that the right-handed sneutrinos are singlets under the SM \( SU(2)_L \) group and the lightest slepton. The first reason prevents interactions of the DM candidates with particles in the SM, except for the Higgs boson \( H \). The second one stabilizes the DM without imposition extra symmetry.

In difference with the previous SIDM candidates which are scalar Higgs bosons, the right-handed sneutrinos in this case are superpartners of leptons with \( L = -1 \). It is interesting to note that in Ref. [45], the right-handed neutrinos are a possible candidate of \textit{warm} dark matter - the fermion dark matter.

In order to be able to account for the properties of dark matter halos (the Spergel-Steinhardt condition), the right-handed sneutrinos have to be light with mass of ten MeV. It is emphasized that the DM self-interaction is fixed from \( D \)-terms, hence the above mentioned limit was obtained without any assumption. Meanwhile they do not overpopulate the Universe by satisfaction \( \Omega_S h^2 = 0.111 \). This dark matter arises naturally in the model without imposition of extra symmetry.

Due to the smallness of the RH sneutrinos mass, we have got \( \tan \beta \approx 1 \) in the high energy limit. Beside this limit we have also got a system of two equations for the DM mass, which will be useful constraints for parameters of the model.
In the special case in which \( w \approx w' \) there is mass degeneracy for the left and right-handed sneutrinos.

It is worth noting that the recent determination of the average density of cosmological cold dark matter provides very stringent constraint on any beyond the standard model. This result should be implied for other 3-3-1 model.

Finally, we would like to mention that the economical 3-3-1 model contains the minimal Higgs sector (economical) with very rich phenomenology, specially in neutrino sector. Its supersymmetric version contains very interesting phenomenology in Higgs sector, namely, many scalar Higgs bosons have masses equal to that of the gauge bosons. In addition, in this supersymmetric version, the candidates for self-interacting dark matter exist naturally.

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