Adv. Studies Theor. Phys., Vol. 4, 2010, no. 4, 165 - 172

Electromagnetic Wave Propagation in a Moving Chiral Medium

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Abstract

We analyze the propagation of an electromagnetic plane wave through a chiral medium moving with a constant velocity and endowed with the Post constitutive relations. The covariance under the Lorentz group of Maxwell's equations and of Post's relations is used to transform the electromagnetic fields in the moving frame into fields in the frame of a fixed observer. The physical consequences of the motion on the physical properties of the electromagnetic fields are discussed.

Keywords: Electromagnetic field, chiral medium, constitutive relations, Lorentz transform.

1. Introduction

The purpose of this work is to study the propagation of electromagnetic waves through a chiral medium, propagating with a constant velocity. Such a problem was previously analyzed by Youtsos [16] who, after a careful discussion of optically active media, choosed to proceed with the Tellegen constitutive relations [10]. We use here the Post constitutive relations [14] because, as Maxwell's equations, they are covariant under the proper Lorentz group which guarantees a consistent theory with , in addition, a simple mathematical tensorial formalism.

The electromagnetic field is defined by two antisymmetric tensors $F_{\mu\nu}$, $G^{\mu\nu}$ with components [9]

$$F_{ij} = \varepsilon_{ijk}B_k \quad , \quad F_{4j} = E_j/c \quad , \quad G^{ij} = \varepsilon^{ijk} H_k \quad , \quad G^{4j} = -\,cD^j \tag{1}$$

the greek (resp.latin) indices take the values 1,2,3,4, (resp.1,2,3,) with summation on repeated indices, ε_{ijk} is the antisymmetric Levi-Civita tensor equal to +1 if an even permutation of 1,2,3, −1 if odd, zero otherwise. The Post constitutive relations [14] are defined by a fourth rank tensor χ such as

$$G^{\mu\nu} = \tfrac{1}{2}\,\chi^{\mu\nu,\rho\sigma} F_{\rho\sigma} \tag{2}$$

with symmetries reducing to twenty the number of its independent components

$$\chi^{\mu\nu,\rho\sigma} = \chi^{\rho\sigma,\mu\nu} \quad , \quad \chi^{\mu\nu,\rho\sigma} = -\chi^{\nu\mu,\rho\sigma} \quad , \quad \chi^{\mu\nu,\rho\sigma} = -\chi^{\mu\nu,\sigma\rho} \tag{2a}$$

so that taking into acount (2a) we may write these relations in the form

$$G^{0j} = \varepsilon^{0j,0k} F_{0k} + 1/2\,\chi^{0j,kl} F_{kl}$$
$$G^{ij} = -\delta^{ij,kl} F_{0k} + 1/2\,\chi^{ij,kl} F_{kl} \tag{3}$$

in which δ is the conjugate tranposed of χ. The relations (1) and (3) define the constitutive relations between the fields (**D**, **H**) and (**E**, **B**) and, in a chiral isotropic medium, they reduce to

$$\mathbf{D} = \varepsilon\mathbf{E} + i\xi\mathbf{B} \quad , \quad \mathbf{H} = n^2\mathbf{B}/\mu + i\xi\mathbf{E} \quad , \quad i = \sqrt{-1} \tag{3a}$$

ε, μ, ξ, n are the permittivity, permeability, chirality and refractive index which satisfies the relation $\varepsilon\mu = n^2/c^2$ where c is the light velocity.

Remark:These fields have the dimension $[\mathbf{B}] = [L^{-1}T][\mathbf{E}]$, $[\mathbf{D}] = [L^{-2}T^2]\,[\mathbf{E}]$, $[\mathbf{H}] = [L^{-1}T][\mathbf{E}]$ and $[\varepsilon] = [L^{-2}T^2]$, $[n] = [\mu] = [L^0T^0]$, $[\xi] = [L^{-1}T]$.

We assume an electromagnetic plane wave propagating through this chiral medium moving with a constant velocity and we analyze successively the propagation in moving and fixed frames.

2. Electromagnetic fields in a moving Minkowskian frame

The tensor Maxwell equations [9] write

$$\partial_\sigma F_{\mu\nu} + \partial_\mu F_{\nu\sigma} + \partial_\nu F_{\sigma\mu} = 0 \quad , \quad \partial_\nu G^{\mu\nu} = 0 \tag{4}$$

$\partial_j = \partial/\partial x_j$, $\partial_4 = \partial/\partial t$ and, between covariant and contravariant components exist the relations $x_\mu = g_{\mu\nu}\, x^\nu$, with $g_{ij} = \delta_{ij}$, $g_{4\mu} = -\delta_{4\mu}$, where $\delta_{\mu\nu}$is the Kronecker tensor, and they have the (3+1)D-representation in terms of the $F_{\mu\nu}$ and $G^{\mu\nu}$ components

$$\nabla\wedge\underline{\mathbf{E}} + \partial_t\underline{\mathbf{B}} = 0 \quad , \quad \nabla.\underline{\mathbf{B}} = 0$$
$$\nabla\wedge\underline{\mathbf{H}} - \partial_t\underline{\mathbf{D}} = 0 \quad , \quad \nabla.\underline{\mathbf{D}} = 0 \tag{5}$$

We consider harmonic plane waves with amplitudes **E**, **B**, **D**, **H**

$$(\underline{\mathbf{E}}, \underline{\mathbf{B}}, \underline{\mathbf{D}}, \underline{\mathbf{H}})(\mathbf{x},t) = (\mathbf{E}, \mathbf{B}, \mathbf{D}, \mathbf{H})\,\psi(\mathbf{x},t) \tag{6}$$

$$\psi(\mathbf{x},t) = \exp(i\omega t + ik_x x + ik_z z) \tag{6a}$$

Substituting (6) into (5) gives the curl equations

$$-k_zE_y + \omega B_x = 0 \quad , \quad -k_zH_y - \omega D_x = 0$$
$$k_zE_x - k_xE_z + \omega B_y = 0 \quad , \quad k_zH_x - k_xH_z - \omega D_y = 0$$
$$k_xE_y + \omega B_z = 0 \quad (7a) \quad , \quad k_xH_y - \omega D_z = 0 \quad (7b)$$

with the divergence equations

$$k_xB_x + k_zB_z = 0 \quad , \quad k_xD_x + k_zD_z = 0 \tag{8}$$

Taking into account (8) a consequence of (3a) is the divergence equation

$$k_xE_x + k_zE_z = 0 \tag{9}$$

Substituting (3a) into (7b) gives

$$k_z(n^2B_y/\mu + i\xi E_y) + \omega\,(\varepsilon E_x + i\xi B_x) = 0$$
$$k_z(n^2B_x/\mu + i\xi E_x) - k_x(n^2B_z/\mu + i\xi E_z) - \omega\,(\varepsilon E_y + i\xi B_y) = 0$$
$$k_x(n^2B_y/\mu + i\xi E_y) - \omega\,(\varepsilon E_z + i\xi B_z) = 0 \tag{10}$$

and, taking into account (7a), these equations become

$$n^2(k_zB_y + \omega E_x/c^2) + 2i\omega\xi\mu\, B_x = 0$$
$$n^2(k_zB_x - k_xB_z - \omega E_y/c^2) - 2i\omega\xi\mu\, B_y = 0$$
$$n^2(k_xB_y - \omega E_z/c^2) - 2i\omega\xi\mu\, B_z = 0 \qquad (10a)$$

Then, eliminating **B** between (7a) and (10a) and using (9) give the homogeneous set of equations in which $\Delta = k_x^2+k_z^2-\omega^2c^{-2}$, $\alpha = 2\omega\mu\xi n^{-2}$

$$\Delta E_x - i\alpha k_zE_y = 0$$
$$\Delta E_y - i\alpha\,(k_xE_z - k_zE_x) = 0$$
$$\Delta E_x + i\alpha k_xE_y = 0 \qquad (11)$$

Nontrivial solutions exist when the determinant of this set is null and a simple calculation gives

$$\Delta[\Delta^2 - \alpha^2\,(k_x^2+k_z^2)] = 0 \qquad (12)$$

with the solutions

$$\Delta = 0 \quad , \quad \Delta = \alpha k \quad , \quad \Delta = -\alpha k \quad , \quad k = (k_x^2+k_z^2)^{1/2} \qquad (13)$$

$\Delta = 0$ imply $k = \omega c^{-1}$ while for $\Delta = \pm\,\alpha k$ we get

$$2k_\pm = \pm\,\alpha + (\alpha^2+4\omega^2c^{-2})^{1/2} \qquad (14)$$

Leaving aside $\Delta = 0$, we get from (8) and (3a) for $\Delta = \alpha k$, $2k_+ = \alpha + (\alpha^2+4\omega^2c^{-2})^{1/2}$

$$E_x = ik_zE_y/k_+ \;,\; E_z = -ik_xE_y/k_+ \;,\; B_x = k_zE_y/\omega, \;,\; B_y = -i\,k_+E_y/\omega \;,\; B_z = -\,k_xE_y/\omega \qquad (15a)$$

in which k_x, k_z, are written for k^+_x, k^+_z and substituting (15a) into (3a) :

$$D_x = ik_z(\varepsilon/k_+ + \xi/\omega)E_y \;,\; D_y = (\varepsilon + k_+\xi/\omega)E_y \;,\; D_z = -ik_x(\varepsilon/k_+ + \xi/\omega)E_y \qquad (15b)$$
$$H_x = k_z(n^2/\omega\mu - \xi/k_+)E_y \;,\; H_y = -i(k_+n^2/\omega\mu - \xi)E_y \;,\; H_x = -k_x(n^2/\omega\mu - \xi/k_+)E_y \qquad (15c)$$

Similarly with $\Delta = -\alpha k$ and $2k_- = \alpha - (\alpha^2+4\omega^2c^{-2})^{1/2}$, we have with $(k)_{x,z} \cong (k^-)_{x,z}$

$$E_x = -ik_zE_y/k_- \;,\; E_z = ik_xE_y/k_- \;,\; B_x = k_zE_y/\omega, \;,\; B_y = i\,k_-E_y/\omega \;,\; B_z = -\,k_xE_y/\omega \qquad (16a)$$
$$D_x = -ik_z(\varepsilon/k_- + \xi/\omega)E_y \;,\; D_y = (\varepsilon - k_-\xi/\omega)E_y \;,\; D_z = ik_x(\varepsilon/k_- + \xi/\omega)E_y \qquad (16b)$$
$$H_x = k_z(n^2/\omega\mu + \xi/k_-)E_y \;,\; H_y = i(k_-n^2/\omega\mu - \xi)E_y \;,\; H_x = -k_x(n^2/\omega\mu\xi/k_-)E_y \qquad (16c)$$

We observe that **E**, **B** do not depend explicitly on ξ but only through $k_\pm$. Substituting the amplitudes (15), (16) into (6) and using (14) in the phase (6a) which becomes

$$\psi_\pm(\mathbf{x},t) = \exp(i\omega t + ik^\pm_x x + ik^\pm_z z) \qquad (6a\pm)$$

achieves to determine the electromagnetic field. So, two modes can propagate with for each mode an arbitrary amplitude E_y.

3. Electromagnetic fields in the fixed frame

Let $L_{\mu\nu}$ be a transform of the proper Lorentz group

$$L_{\mu\nu}L^{\nu\rho} = \delta_\mu{}^\rho \qquad (17)$$

in which $\delta_\mu{}^\rho$ is the Kronecker tensor. As noticed in the introduction, the greek indices μ,ν,ρ,σ…take the values 1,2,3,4 for x,y,z,t (= x_4) and the latin indices i,j,k… the values 1,2,3 for x,y,z.The summation convention on repeated indices is used.

Applied to a four vector x^μ this transformation gives $x_\mu \Rightarrow x'_\mu$ with

$$x'_\mu = L_{\mu\nu}x^\nu \qquad (18)$$

and, in particular for the derivative operator $\partial_\mu \Rightarrow \partial'_\mu$

$$\partial'_\mu = L_{\mu\nu}\partial^\nu \tag{18a}$$

The field tensors $F_{\mu\nu}$, $G^{\mu\nu}$, transform according to

$$F'_{\mu\nu} = L_{\mu\rho}L_{\nu\sigma}F^{\rho\sigma} \quad , \quad G'_{\mu\nu} = L_{\mu\rho}L_{\nu\sigma}G^{\rho\sigma} \tag{19}$$

while for the Post tensor, we have

$$\chi'_{\mu\nu,\rho\sigma} = L_{\mu\alpha}L_{\nu\beta}L_{\rho\tau}L_{\sigma\zeta}\chi^{\alpha\beta,\tau\zeta} \tag{19a}$$

Taking into account (17), (18), (18a), (19), (19a), it is easy to prove the covariance of Maxwell's equations and of Post's constitutive relations.

Suppose now that x'_μ and x^ν are four vectors respectively in a fixed frame and in a frame moving with the constant velocity **v**, the relation (18) has the (3+1)D representation [9] (Jones that we follow here, except for this last notation, uses β instead of the most usual γ, β denoting generally |**v**|/c [8]).

$$x'_j = a_{jk}\, x_k - \gamma v_j\, t \quad , \quad ct' = \gamma(ct - \mathbf{v.x}/c) \tag{20}$$

$$a_{jk} = \delta_{jk} + (\gamma-1)v_j v_k/c^2 \quad , \quad \gamma = (1- v^2/c^2)^{-1/2} \tag{20a}$$

So, the harmonic plane waves (6) in the fixed frame are

$$(\underline{\mathbf{E}}', \underline{\mathbf{B}}', \underline{\mathbf{D}}', \underline{\mathbf{H}}')(\mathbf{x}',t') = (\mathbf{E}', \mathbf{B}', \mathbf{D}', \mathbf{H}')\, \psi(\mathbf{x}',t') \tag{21}$$

with according to (6a) and (20)

$$\psi_\pm(\mathbf{x}',t') = \exp[i\gamma\omega(t-\mathbf{v.x}/c) + k^\pm_x(a_{11}x+a_{13}z-\gamma v_x t) + k^\pm_z(a_{31}x+a_{33}z-\gamma v_z t] \tag{20a}$$

while, taking into account (1), (18), (19), (20) the amplitudes **E**', **B**', **D**', **H**' are transformed according to the following relations [4]

$$\mathbf{E}' = \gamma\mathbf{E} + (1-\gamma)(\mathbf{E.v})\mathbf{v}/v^2 + \gamma\mathbf{v}\wedge\mathbf{B}$$
$$\mathbf{D}' = \gamma\mathbf{D} + (1-\gamma)(\mathbf{D.v})\mathbf{v}/v^2 + \gamma\mathbf{v}\wedge\mathbf{H}/c^2 \tag{22a}$$

and

$$\mathbf{B}' = \gamma\mathbf{B} + (1-\gamma)(\mathbf{B.v})\mathbf{v}/v^2 - \gamma\mathbf{v}\wedge\mathbf{E}/c^2$$
$$\mathbf{H}' = \gamma\mathbf{H} + (1-\gamma)(\mathbf{H.v})\mathbf{v}/v^2 - \gamma\mathbf{v}\wedge\mathbf{D} \tag{22b}$$

These relations prove that the electric and magnetic fields **E**, **B** do not exist as separate entities: the resolution of the electromagnetic field into electric and magnetic components depends on the observer motion. Now, substituting the expressions (15a,b,c) and (16a,b,c) of the fields **E**.**B**.**D**.**H**. into (22a,b) achieves to determine the amplitudes of the plus and minus modes $\mathbf{E}'_\pm.\mathbf{B}'_\pm.\mathbf{D}'_\pm.\mathbf{H}'_\pm$ in the fixed frame.

Now, according to (3), (19a), (20), the Lorentz transform of the constitutive relations (3a), satisfied by the fields (22a,b), is

$$\mathbf{D}' = \varepsilon\, \mathbf{E}' + i\xi\, \mathbf{B}'_- \quad , \quad \mathbf{H}'_- = n^2/\mu\, \mathbf{B}'_- + i\xi\, \mathbf{E}' \tag{23}$$

in which $\mathbf{B}'_-$, $\mathbf{H}'_-$ are the expressions (22b) with **v** changed into **-v**

$$\mathbf{B}'_- = \gamma\mathbf{B} + (1-\gamma)(\mathbf{B.v})\mathbf{v}/v^2 + \gamma\mathbf{v}\wedge\mathbf{E}/c^2$$
$$\mathbf{H}'_- = \gamma\mathbf{H} + (1-\gamma)(\mathbf{H.v})\mathbf{v}/v^2 + \gamma\mathbf{v}\wedge\mathbf{D} \tag{23a}$$

As a simple example, we suppose **v** in the z-direction : $\mathbf{v} = (0,0,v_3)$, then we get from (22a,b)

$$\{E'_i, D'_i\} = A_{i3}\, \{E_i, D_i\} - \varepsilon_{ij3}\, \gamma v_3 \{B_j , H_j /c^2\}$$
$$\{B'_i\, H'_i\} = A_{i3}\, \{B_i, H_i\} + \varepsilon_{ij3}\, \gamma v_3 \{E_j/c^2, D_j\} \tag{24}$$

in which $A_{i3} = \gamma + (1-\gamma)\delta_{i3}$.

These relations in which δ_{i3} and ε_{ij3} are the Kronecker and Levi-Civita tensors with i, j = 1,2 give the amplitudes of the electromagnetic fields and, according to (21a), the phase function $\psi(\mathbf{x}',t')$ is

$$\psi_\pm(\mathbf{x}',t') = \exp[i\gamma\omega(t- vz/c^2) + k^\pm_x x + k^\pm_z(a_{33}z-\gamma vt)] \tag{25}$$

Finally, we get according to (23a) for the Post constitutive relations

$$\{B'_-, H'_-\}_i = A_{i3}\,\{B_i, H_i\} - \varepsilon_{ij3}\,\gamma v_3\{E_j/c^2, D_j\} \tag{26}$$

The substitution of (15a,b,c) and (16a,b,c) into (24) and (26) is performed in Appendix from which it is easily proved that the prime accented fields satisfy the Post constitutive relations (23).

4. Discussion

As previously stated, there is no independent existence for the electric and magnetic fields, their relations depend on the motion of the observer. It is also the case for the Doppler shift on the angular frequency and for the change in the direction of propagation. In addition, these ef fects differ for plus and minus modes: for instance, according to (14) for v in the z-direction

$$\omega_\pm' = \gamma(\omega_\pm - k^\pm_z v) \quad , \quad (k^\pm_x)' = \gamma\, k^\pm_x \quad , \quad (k^\pm_z)' = \gamma(k^\pm_z - \omega_\pm v/c^2) \tag{27}$$

Similarly, the fields $\underline{\mathbf{E}}^\pm$, $\underline{\mathbf{H}}^\pm$ with the amplitudes $\mathbf{E}^\pm$, $\mathbf{H}^\pm$ and the phase ($6a_\pm$) generate in the moving frame the Poynting vector

$$\begin{aligned} P &= \tfrac{1}{2}\,\mathrm{Re}\,[(\underline{\mathbf{E}}^+ + \underline{\mathbf{E}}^-) \wedge (\underline{\mathbf{H}}^+ + \underline{\mathbf{H}}^-)^*] \\ &= \tfrac{1}{2}\,\mathrm{Re}\,(\underline{\mathbf{E}}^+ \wedge \underline{\mathbf{H}}^{-*} + \underline{\mathbf{E}}^- \wedge \underline{\mathbf{H}}^{+*}) \end{aligned} \tag{28}$$

since according to (15), (16) :

$$\underline{\mathbf{E}}^+ \wedge \underline{\mathbf{H}}^{+*} + \underline{\mathbf{E}}^- \wedge \underline{\mathbf{H}}^{-*} = 0 \tag{28a}$$

the asterisk denoting the complex conjugation.

Then, taking into account (A.1) - (A.5) the Poynting vector in the fixed frame is with prime accented fields

$$\mathbf{P}' = \tfrac{1}{2}\,\mathrm{Re}\,(\underline{\mathbf{E}}^+ \wedge \underline{\mathbf{H}}^{-*} + \underline{\mathbf{E}}^- \wedge \underline{\mathbf{H}}^{+*})' \tag{29}$$

So, substituting (22a,b) into (29) shows that the observers in fixed and moving frames do not see the energy flow in the same direction, with a direct application to Cherenkov radiation.

The chiral medium in this work is implicitly assumed to be infinite so that the plus and minus modes propagate independently. The situation is different in a bounded chiral medium because the boundary conditions couple the two modes. This coupling may be illustrated on the reflection and refraction of harmonic plane waves on a moving chiral slab. This problem has already been discussed for a stationary chiral slab [1] and since the authors in this paper also use the Post constitutive relations, one has just to apply the Lorentz transform (22a,b) to their results valid for an observer in the moving frame to get the reflected and refracted fields in the frame of the fixed observer. Calculations are easy although a bit long.

It has been assumed here that the source of the harmonic plane wave is in the moving frame S but, the results obtained on the electromagnetic field would be also valid for the dual situation of an harmonic plane wave launched in S' to track a chiral moving target S, just changing the roles of primed and unprimed fields. Similar results are also obtained for a metachiral moving medium just with ε, μ becoming $-|\varepsilon|$ and $-|\mu|$.

Youtsos uses of course the Lorentz transform between the electromagnetic fields in moving and fixed frames together with a relativistic-like extension of Tellegen's constitutive relations, proposed by Lakhtakia et Al. [11] for a rigid optically active dielectric moving with a constant translational velocity (see [13] for a recent discussion). But these relations are not covariant, so Youtsos had to use a perturbation technique with manageable expressions only to the $0(v^2/c^2)$ order. As a consequence, his work is limited to the propagation of a plane monochromatic wave in a slab moving in the direction of wave propagation.

The comparison of the present work with that of Youtsos confirms the importance of the Post constitutive relations since their covariance under the Lorentz group allows to solve exactly the problem of electromagnetic wave propagation in media moving with a constant velocity, a result supporting the statement that mathematical expressions with a physical meaning have to be covariant. In addition, they are the only constitutive relations consistent with the relativistic electromagnetism, needed for instance, to analyze wave propagation in the Universe [3,5].

Remark. Electromagnetic differential forms [4,6,7,12,15] may supply further numerical solutions. Let us consider the two-forms defined with the $F_{\mu\nu}$ and $G^{\mu\nu}$ fields

$$\mathsf{F} = \tfrac{1}{2} F_{\mu\nu}\, dx^\mu \wedge dx^\nu \quad , \quad \mathsf{G} = \tfrac{1}{2} G_{\mu\nu}\, dx^\mu \wedge dx^\nu \tag{30}$$

with

$$G_{\mu\nu} = \varepsilon_{\mu\nu\alpha\beta} G^{\alpha\beta} \tag{30a}$$

in which $\varepsilon_{\mu\nu\alpha\beta}$ is the Levi-Civita tensor.

Let $d = dx^\mu\, \partial_\mu$ be the Cartan exterior derivative, then applying d to (30) gives the three-forms

$$d\,\mathsf{F} = \tfrac{1}{2}\, \partial_\rho F_{\mu\nu}\, dx^\rho \wedge dx^\mu \wedge dx^\nu \quad , \quad d\,\mathsf{G} = \tfrac{1}{2}\, \partial_\rho G_{\mu\nu}\, dx^\rho \wedge dx^\mu \wedge dx^\nu \tag{31}$$

In absence of charge and current, the electromagnetic field satisfies the equations

$$d\,\mathsf{F} = 0 \quad , \quad d\,\mathsf{G} = 0 \tag{32}$$

Then, making null the coefficients ot the three-forms in (31) gives, as strong solutions of (32) the Maxwell equations (4). It is evident for $d\,\mathsf{F} = 0$ while for $d\,\mathsf{G} = 0$, taking into account (30a)

$$\partial_\rho G_{\mu\nu} + \partial_\mu G_{\nu\rho} + \partial_\nu G_{\rho\mu} = 0 \quad \Rightarrow \quad \partial_\nu G^{\mu\nu} = 0 \tag{33}$$

So, according to the Stokes theorem $\int_M d\,\mathsf{F} = \int_{\partial M} \mathsf{F}$, the equations (32), in addition to the strong solutions, have weak numerical solutions. It has been proved [2] that the finite element technique, largely used in the numerical simulation of partial differential equations may be applied to differential forms and an illustration can be found in [17]. With this technique, boundary conditions are efficiently managed.

To take into account the constitutive relations, $d\,\mathsf{G} = 0$ is changed into $d\,\mathsf{F}^* = 0$ with

$$\mathsf{F}^* = \tfrac{1}{2} F_{\mu\nu}\, \chi^{\mu\nu,\rho\sigma}\, dx_\rho \wedge dx_\sigma \tag{34}$$

in which $\chi^{\mu\nu,\rho\sigma}$ is the Post tensor.

An important numerical work has still to be to make differential forms easily available.

Appendix

We prove in this Appendix that the primed fields obtained by substituting (15) and (16) into (24) and (26) satisfy the Post constitutive relations (23). To simplify, we only consider the plus mode (15), calculations being similar for the minus mode (16). In addition, only the fields $\{\mathbf{E}, \mathbf{D}\}'$, $\{\mathbf{H}_-, \mathbf{B}_-\}'$ are useful to check (23).

Then, substituting (15a,b,c) into (24) and using the relation $\varepsilon\mu = n^2/c^2$, we get with v written for v_3 and k_x, k_z for k^+_x, k^+_z wile k_+ is given by (14)

$$E'_x = i\gamma(k_z/k_+ + k_+ v/\omega)\, E_y \quad , \quad E'_y = \gamma(1+k_z v/\omega)\, E_y \quad , \quad E'_z = -i(k_x/k_+)\, E_y \tag{A.1}$$

and

$$\begin{aligned} D'_x &= \gamma\, [i\varepsilon(k_z/k_+ + k_+ v/\omega) + i\xi(k_z/\omega - v/c^2)]\, E_y \\ D'_y &= \gamma\, [\varepsilon(1+k_z v/\omega) + \xi(k_+/\omega - k_z v/k_+ c^2)]\, E_y \\ D'_z &= -ik_x(\varepsilon/k_+ + \xi/\omega)\, E_y \end{aligned} \tag{A.2}$$

while substituting (15) into (26) gives

$$(B_-')_x = \gamma(k_z/\omega - v/c^2)\ E_y \quad , \quad (B_-')_y = -i\gamma(k_+/\omega - k_z v/_+kc^2)\ E_y \quad , \quad (B_-')_z = -(k_x/\omega)\ E_y \qquad (A.3)$$

$$(H_-')_x = \gamma[n^2/\mu(k_z/\omega - v/c^2) - \xi\ (k_z/k_+ + k_+ v/\omega)]\ E_y$$

$$(H_-')_y = \gamma[-in^2/\mu(k_+/\omega - k_z v/k_+ c^2) + i\xi((1 + k_z v/\omega)]\ E_y$$

$$(H_-')_z = -k_x\ (n^2/\omega\mu - \xi/k_+)\ E_y \qquad (A.4)$$

One checks at once the relations

$$D'_x = \varepsilon\ E'_x + i\xi\ (B_-')_x, \quad , \quad H_-'_x = n^2/\mu\ (B_-')_x + i\xi\ E'_x \qquad (A.5)$$

which is (23) with a similar result for the y, z components.

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Received: October, 2009