Special Relativity and Inertia in Curved Spacetime

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Abstract

A proposed theory explains the origin of Inertia without violating Einstein's two postulates that form the basis for Special Relativity. The new model agrees with observational aspects of Special Relativity and is compatible with General Relativity. The relativistic momentum becomes a property of curved spacetime during acceleration, and Newton's second law of motion is derived from a line-element in General Relativity. The new model unambiguously resolves the Twin Paradox, since aging always progresses at the same pace, and it admits an absolute temporal reference.

1. Introduction

This article follows up on a previous article proposing that the phenomenon of Inertia may be explained as being caused by curved spacetime during acceleration [Masreliez, 2007a]. In this previous paper an attempt was made to formally preserve Special Relativity (SR) and the Lorentz Transformation (LT).

The present paper takes a different approach by investigating consequences of changing spacetime metrics during acceleration. These changing metrics are given by a *dynamic metrical factor* that depends on relative velocity. Since this metrical factor is a function of position during acceleration it models a dynamically curved spacetime field and explains Inertia as being a gravitational-type phenomenon generated by this field.

The present paper demonstrates that several results familiar from SR may be deduced directly from a certain line-element of General Relativity (GR) that models Inertia.

The new interpretation is observationally indistinguishable from Special Relativity (SR) in most situations. However, the Twin Paradox is unambiguously resolved and a temporal reference is shown to exist. It also provides additional insight into the role of the Lorentz Transformation (LT). Several of the results from SR may by the new theory be viewed as being curved spacetime phenomena.

As examples of previous work on the origin of Inertia we mention research at the Calphysics Institute proposing that Inertia is caused by interaction with the Zero Point Field [Puthoff, 2002], [Rueda and Haisch, 2005]. A different approach to the origin of Inertia has been advanced by James Woodward (http://physics.fullerton.edu/~jimw/general/inertia/index.htm) based on the Sciama's proposal that Inertia is caused by acceleration in relation to a cosmological gravitational background field [Sciama, 1953].

The author is not aware of any previous work along the lines proposed in this article.

2. Background

A recent paper by the author [Masreliez, 2007a] proposes that acceleration locally curves spacetime as experienced by an accelerating particle. It suggests that Inertia may be explained as a gravitational-type phenomenon if the metrical coefficients of a line-element were to change with position during acceleration. A certain dynamical, metrical factor that multiplies all four metrical coefficients in the Minkowskian line-element has the interesting property that it changes with location during acceleration in such a way that all accelerating motion will take place on geodesics of General Relativity (GR). *This would explain Inertia.*

In this scenario the velocity depends on spatial position; the metrics change with location. In other words, an accelerating particle experiences changing spacetime metrics that depend on its location. This could be interpreted as a gravitational-type field, the 'inertial field' acting on an accelerating particle, which generates the inertial force. This previous paper also explores the connection between the metrical factor and SR, showing that there might be a close relationship, which suggests a modified version of SR. This modified version formally retains all features of SR including the Lorentz transformation, and like in SR any inertial frame may be considered to be an inertial reference frame. However, in order to model Inertia by this approach GR is generalized to allow for Dynamic Incremental Scale Transition (DIST) whereby the scale-factor may change incrementally in order to preserve the Minkowskian line-element. This generalization is proposed and justified in [Masreliez, 2007b].

The present paper takes a different point of view by investigating the consequences of changing spacetime metrics during acceleration.

3. Modeling Inertia - the Inertial Line-Element

Consider the Minkowskian line-element, but with an added dynamic metrical factor, which we assume depends on the velocity relative to an inertial frame that is stationary to the observer. This frame will be denoted the Stationary Inertial Frame (SIF).

$$ds^2 = \left[1-(v/c)^2\right]\left[(cdt)^2 - dx^2 - dy^2 - dz^2\right] \qquad (3.1)$$

This will be called the 'Inertial Line-element' (ILE). It will be assumed that the coordinates *(t, x, y, z)* refer to the SIF. Primed coordinates *(t', x', y', z')* will refer to a Moving Inertial Frame denoted MIF. This line-element holds even for a varying velocity. In [Masreliez, 2007a] it is shown that *with the ILE the geodesic equation is an identity; all trajectories are geodesics of GR*.

This implies that an accelerating particle, which is subjected to an inertial force, would be in a situation similar to that of a particle suspended in a gravitational field, which is supported against the gravitational force.
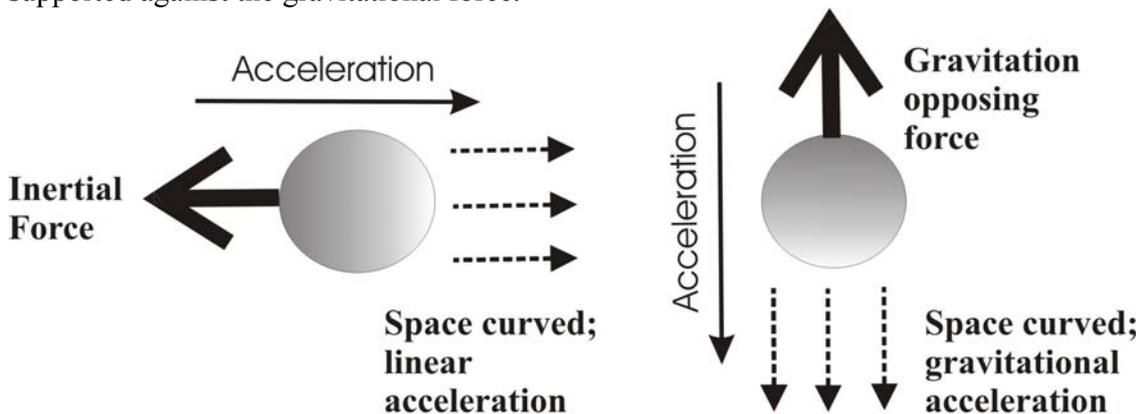

Figure 1: Acceleration and Gravitation both curve space

Furthermore, if the velocity is constant, the two postulates upon which Einstein based his SR theory [Einstein, 1905] will still hold true since:
1. Line-elements with different *constant* metrical factors are physically equivalent because Einstein's GR equations are identical.
2. The speed of light is the same in all these scaled Minkowskian frames.

The fundamental feature of the cosmos, that line-elements differing by a constant factor are physically equivalent, will be denoted *'scale-equivalence'*; the line-elements are said to be *'scale-equivalent'*.

This suggests that the ILE with constant velocities might model inertial frames at different relative velocities.

The 'inertial metrical factor' defined by (1-$(v/c)^2$) is related to the factor γ familiar from SR:

$$\gamma = \frac{1}{\sqrt{1-\left(v/c\right)^2}} \qquad (3.2)$$

This connection to SR is explored in [Masreliez, 2007a]. Here we will take a different approach based on the ILE (3.1).

4. New findings - observations based on the inertial line-element

Let us assume that spacetime is curved relative to an accelerating particle as given by the ILE. Let us further assume that after an accelerating boost when the velocity again is constant the line-element of the MIF is Minkowskian and that the two inertial frames with relative velocity v are related by:

$$ds'^2 = \left(cdt'\right)^2 - dx'^2 - dy'^2 - dz'^2 = \left(1-\left(v/c\right)^2\right)\left(\left(cdt\right)^2 - dx^2 - dy^2 - dz^2\right) \quad (4.1)$$

The primed coordinates refer to the MIF and the unprimed to the SIF.
This allows the following observations:

First observation:
The relationship between temporal increments at fixed locations in each frame with $dx=dx=dz=0$ and $dx'=dy'=dz'=0$ is given by:

$$dt' = \sqrt{1-\left(v/c\right)^2}\,dt \quad (4.2a)$$

Thus, the time-dilation of SR here appears as *gravitational-type phenomenon* caused by different temporal metrics. We are already familiar with this relation in circular motion where the centrifugal acceleration by GR produces a gravitational time-dilation that matches (4.2a).
On the other hand letting $dt=dt'=0$ there is *length contraction*:

$$dl' = \sqrt{1-\left(v/c\right)^2}\,dl$$
$$dl = \sqrt{dx^2 + dy^2 + dz^2} \quad (4.2b)$$

Distance increments are shorter by the factor $1/\gamma$ in the moving frame regardless of direction. This differs from SR where length contraction only occurs in the direction of motion.

Second observation:
Letting $dx=dy=dz=0$ the corresponding velocity in the moving frame is $v'=-v$ and we find from (4.1):

$$\left(\left(cdt'\right)^2 - dx'^2 - dy'^2 - dz'^2\right) = \left(cdt'\right)^2\left(1-\left(v'/c\right)^2\right) = \left(cdt'\right)^2\left(1-\left(v/c\right)^2\right) = \left(cdt\right)^2\left(1-\left(v/c\right)^2\right)$$

This implies:

$$dt' = dt \qquad (4.3)$$

This means that an observer in the MIF, who is moving in the MIF while remaining fixed in the SIF, finds that her clock runs at the same pace as the stationary clock. In other words, by observing a stationary clock in the SIF all moving observers agree that this clock agrees with the corresponding clock in their own moving frame (with their own coordinates) and therefore they agree on elapsed time intervals. This resolves the Twin Paradox in an unambiguous manner; aging always progresses at the same pace regardless of motion. This also means that all observers agree on a cosmological time, which might revive Newton's concept of an absolute cosmological reference frame.

Third observation:

If the observer moves in the SIF so that the velocity matches that of the MIF we have from (4.1) with *dx'=dy'=dz'=0*:

$$\left(cdt' \right)^2 = \left(1 - \left(v/c \right)^2 \right)\left(\left(cdt \right)^2 - dx^2 - dy^2 - dz^2 \right) = \left(1 - \left(v/c \right)^2 \right)^2 \left(cdt \right)^2$$

$$dt' = (1 - \left(v/c \right)^2)dt \qquad (4.4)$$

This implies that a clock in the SIF moving with a velocity *v* that matches that of the MIF does not agree with the clock in the moving frame. In this respect the perspective from the two frames is asymmetrical; it depends on the motion with respect to the stationary frame.

Fourth observation:

Relation (4.1) may also be written:

$$\left(\left(cdt \right)^2 - dx^2 - dy^2 - dz^2 \right) = \frac{1}{1 - \left(v/c \right)^2}\left(\left(cdt' \right)^2 - dx'^2 - dy'^2 - dz'^2 \right) \quad (4.5)$$

For an observer fixed in the SIF with *dx=dy=dz=0* we have:

$$\left(cdt \right)^2 = \frac{1}{1 - \left(v/c \right)^2}\left(\left(cdt' \right)^2 - dx'^2 - dy'^2 - dz'^2 \right) \qquad (4.6)$$

A particle that is fixed in the SIF moves with velocity *–v* in the MIF:

$$dt^2 = \frac{dt'^2}{1 - \left(v/c \right)^2}\left[1 - \left(v/c \right)^2 \right] \qquad (4.7)$$

We already saw that this implies that *dt=dt'*, and therefore we find after dividing both sides by dt^2 (=dt'^{2}) an *identity* that gives two different perspectives on the particle given by observers in the two frames:

$$1 = \frac{1}{1 - \left(v/c \right)^2}\left[1 - \left(v/c \right)^2 \right] \qquad (4.8)$$

Multiplying both sides by *(m$_0$c^2)^2* yields:

$$m_0^2 c^4 = \frac{m_0^2 c^4}{1-(v/c)^2} - p^2 c^2 \qquad (4.9)$$

This expresses conservation of relativistic momentum. The left hand is the relativistic momentum of a stationary particle; the right hand is the same momentum seen from a moving frame. Thus the ILE *implies* conservation of relativistic momentum. The spatial momentum *p* is given by:

$$p = \frac{m_0 v}{\sqrt{1-(v/c)^2}} \qquad (4.10)$$

Rearranging (4.9) we get the relativistic energy:

$$E^2 = \frac{m_0^2 c^4}{1-(v/c)^2} = m_0^2 c^4 + p^2 c^2 \qquad (4.11)$$

We find that the relativistic expressions for momentum and energy are direct consequences of the inertial line-element.

Fifth observation:
Next, consider the relativistic energy given by (4.11):

$$E = \frac{m_0 c^2}{\sqrt{1-(v/c)^2}} = \frac{E_0}{\sqrt{1-(v/c)^2}} \qquad (4.12)$$

Since this relation holds true even for a dynamic velocity that depends on position *it implies the existence of Inertia.* Differentiating with respect to position:

$$\frac{dE}{dl} = \frac{m_0 v \dfrac{dv}{dl}}{\left(1-(v/c)^2\right)^{3/2}} = \frac{m_0 a}{\left(1-(v/c)^2\right)^{3/2}} \qquad (4.13)$$

This implies that a corresponding force exists given by *F=dE/dl,* which is the *inertial force.* We then have the relativistic version of Newton's second law:

$$F = \frac{m_0 a}{\left(1-(v/c)^2\right)^{3/2}} \qquad (4.14)$$

The same conclusion follows from the relation *F=dp/dt* using (4.10).

Note that in SR the expression for relativistic energy (4.12) cannot be derived solely from the Minkowskian line-element or from the Lorentz transformation without additional assumptions, for example conservation of momentum. In classical physics Newton's second law is no more than an assumption, but here we find that the conservation of relativistic momentum, the inertial force, and Newton's second law all follow from a line-element that models Inertia.

We have found a dynamic spacetime geometry that *implies* two fundamental laws of nature. This supports the proposition that the ILE models motion in general, whether accelerating or not.

Sixth observation:

Consider the Doppler shift. The frequency of light from an approaching source will, by classical physics, increase:

$$f_{MIF} = f_0(1 + v/c) \qquad (4.15)$$

However, this equation, which expresses the frequency of a moving source in the MIF, may be adjusted to the SIF by applying the temporal scale correction (4.2a), which relates time intervals in the MIF to those in the SIF. Since a time interval in the moving frame will appear shorter than in the SIF, the resulting frequency shift increases by the factor γ:

$$f_{SIF} = \frac{f_{MIF}}{\sqrt{1 - (v/c)^2}} = f_0 \frac{(1 + v/c)}{\sqrt{1 - (v/c)^2}} = f_0 \sqrt{\frac{1 + v/c}{1 - v/c}} \qquad (4.16)$$

Equation (4.16) is the relativistic Doppler shift. Another way to obtain this result is to note that the energy of a photon is $E_0 = hf_0$, where h is Planck's constant. According to (4.12) we then have:

$$f_{SIF} = \frac{f_{MIF}}{\sqrt{1 - (v/c)^2}} = f_0 \sqrt{\frac{1 + v/c}{1 - v/c}} \qquad (4.17)$$

Note that all these observations may be made without considering any coordinate transformation between the two frames like the Lorentz transformation. The observations listed above mostly agree with SR but with two important differences, namely that the Twin Paradox disappears and that a temporal simultaneity exists.

5. *A few comments on Special Relativity and the Lorentz Transformation*

These observations are based on the proposition that motion curves spacetime as modeled by the inertial line-element. The question arises how this relates to SR.

In his paper on SR [Einstein, 1905] Einstein based this theory on two postulates:

1. The laws of physics hold true in all inertial frames.
2. The speed of light is constant, isotropic and the same in all inertial frames.

Since the inertial line-element (4.1) relates scale-equivalent spacetimes these two postulates still hold true.

It is commonly believed that the two postulates above *imply* the Lorentz transformation, i.e. that they necessarily lead to SR. However, as we shall see this is not true.

Einstein made an additional assumption, which could be taken as a third postulate. He assumed that all inertial frames are equivalent in all respects. By this postulate inertial frames ought to have identical spacetime geometries and metrics, and Einstein therefore assumed that inertial frames are related by the LT, which replicates the Minkowskian line-element. Therefore, by SR and the LT all inertial frames are Minkowskian.

However, it will be shown that this is not a necessary requirement for achieving equivalence and symmetry between inertial frames. There is another transformation similar to the LT, but which, unlike the LT, is consistent with the inertial line-element.

6. The Voigt Transform

In 1887 Woldemar Voigt published a paper proposing a coordinate transformation between two frames with one of them moving relative to the other along the x-axis:

$$t' = t - (v/c)^2 x$$
$$x' = x - vt$$
$$y' = y\sqrt{1 - (v/c)^2} = y/\gamma \qquad (6.1)$$
$$z' = z\sqrt{1 - (v/c)^2} = z/\gamma$$

In [Masreliez, 2007a] I independently re-derived this transformation, being unaware of the Voigt Transformation (VT), calling it the Scaled Lorentz Transformation (SLT). As in the LT the velocity *v* is assumed to be *constant*. It becomes identical to the Lorentz transformation if the factor *γ* is applied to all four relations:

$$t' = \gamma\left(t - (v/c^2)x\right)$$
$$x' = \gamma\left(x - vt\right) \qquad (6.2)$$
$$y' = y$$
$$z' = z$$

Since these linear relations also hold for increments in the coordinates, the line-element corresponding to the VT is consistent with the ILE.

$$(cdt')^2 - dx'^2 - dy'^2 - dz'^2 = \left(1 - (v/c)^2\right)\left((cdt)^2 - dx^2 - dy^2 - dz^2\right)$$

The inverse of the VT is:

$$t = \gamma^2(t' + (v/c^2)x')$$
$$x = \gamma^2(x' + vt') \qquad (6.3)$$
$$y = \gamma y'$$
$$z = \gamma z'$$

Thus, the VT is asymmetrical in that the forward transformation differs from its inverse. We find that the transformation that formally implements the ILE for constant velocities actually preceded the Lorentz transformation! Apparently Hendrik Lorentz did not know about this aspect of Voigt's work; he is on record as saying that he could have taken these transformations into his theory of electrodynamics rather than developing his own, if only he had known of them [Woldermar Voigt, Wikipedia]. In the paper by Ernst and Hsu [Ernst and Hsu, 2001] the following letter may be found:

H. A. Lorentz to W. Voigt Leiden, July 30, 1908

Dear friend,

I would like to thank you very much for sending me your paper on Doppler's principle

together with your enclosed remarks. I really regret that your paper has escaped my notice.
I can only explain it by the fact that many lectures kept me back from reading everything, while I was already glad to be able to work a little bit.
Of course I will not miss the first opportunity to mention that the concerned transformation and the introduction of a local time has been your idea.

Sincerely,

Your H. A. Lorentz

The VT differs from the LT by a constant scale-factor γ, which in the LT has the effect of restoring the Minkowskian line-element, thereby hiding the scale adjustment that models Inertia. The two transformations are physically equivalent and work equally well in Maxwell's equations as well as in physics in general, since they are scale-equivalent. It is ironic to note that if Lorentz had adopted the VT, Einstein might perhaps have chosen the VT instead of the LT in his SR-paper, and the origin of Inertia might have been found a long time ago.
Additional comments on the Voigt transform may be found in [Gluckman, 1968] and on the Internet at [Ernst, 2005].

7. Two different approaches to modeling 'motion'

In classical as well as in modern physics 'motion' is usually modeled in terms of coordinate locations that change with time, and transformations like the LT or the VT relate these locations in different coordinate frames. These transformations relate positions in space and time so that positions in the SIF correspond to positions in the MIF in one-to-one correspondence. For example, $x=t=0$ implies $x'=t'=0$. Furthermore, it is implicitly assumed that these two frames have identical coordinate metrics, so that coordinate increments in one frame may be directly compared to those in the other frame. However, Inertia is in this paper modeled as a curved spacetime phenomenon using dynamic spacetime metrics given by the inertial line-element. With this new interpretation the VT and LT transformations (which only apply if the velocity is constant) should be viewed as continuous variable transformations in GR relating covariant coordinate representations. We saw that this interpretation leads to the inertial line element and allows time-dilation to be modeled as a curved spacetime phenomenon.
However, had we instead interpreted the VT transformation as a generalization of the classical Galilean transformation, time-dilation would not be evident because the VT does not allow us to relate time intervals at fixed locations in the SIF to time intervals at fixed locations in the MIF.
Similarly SR interprets LT as relating corresponding locations in two inertial coordinate systems, and as we know, this leads to several important and valid results. However, if

the LT is seen as a coordinate transformation in GR, both inertial frames will have Minkowskian line-elements and their clock rates at fixed locations will be the same, which means that the time-dilation disappears.

Therefore, the geometric interpretation investigated in this paper favors the VT.

Woldemar Voigt's objective with his transformation was to make the wave equation invariant, while Lorentz desired to preserve Maxwell's equations, and Einstein based the LT on his two postulates of SR (which also hold for the VT). However, from the geometrical point of view by which motion effects the scale of spacetime the reason for the success of the VT and LT could be that they both preserve the Minkowskian character of spacetime and, because the corresponding GR equations are identical, they preserve all physics as well. Therefore the non-intuitive term $-xv/c^2$ in the temporal coordinate transformation might not be primarily associated with motion, but rather with coordinate transformations in GR expressing the relationship between line-elements of inertial frames, which preserve all physics.

In this sense only Voigt's transformation is consistent with a dynamic metric that models Inertia.

8. Resolving the Twin Paradox

The Clock Paradox, popularized as the Twin Paradox by Paul Langevin [Langevin, 1911], has been discussed at length ever since the introduction of SR in 1905 and several proposals on how to resolve it have been put forward over the years. Nowadays many believe that the Twin Paradox has been resolved one way or the other, although it appears that a universally accepted resolution still is missing. In SR problems arise when comparing predictions made by observers in different inertial frames. This is particularly bothersome in the context of the Twin Paradox where each twin concludes that the other twin ages slower.

There is strong experimental evidence that the pace of atomic clocks change with motion, and we have seen that clocks moving in relation to a stationary reference frame run at a slower pace as indicated by relation (4.2a). Based on this it is tempting to conclude that the twin who accelerates ages slower and therefore returns younger.

However, according to relation (4.3) and the Voigt transformation, a clock in the MIF that moves so that it remains fixed in the SIF always agrees with a fixed clock in the SIF. Since this is the case both on the outward leg and on the inward leg of the travel, the twins always age at the same pace!

The inertial line-element allows comparison of coordinate increments in the SIF and the MIF. This comparison is only made possible if the coordinates are in the same GR manifold with metrics related by relation (4.1). However, if another inertial frame were to be selected as being the SIF a different manifold in GR would be selected as reference that will not be related to the previous manifold by any continuous coordinate transformation. However, since elapsed time intervals in moving frames always agree, no matter which frame is considered stationary; no contradiction arises, like it does in SR.

In the context of SR the LT implies that all inertial frames have Minkowskian line-elements, which could hide a metrical scale change that models Inertia. Therefore, the LT transformation could be seen as the result of continuous metric scale adjustment according to (4.1) combined with discrete scale transition. If this is the case, the interpretation of the LT will depend on which frame is considered a SIF. The metrical scale of the coordinates of the MIF obtained by the LT will be different from the metrical scale of the SIF coordinates. However, the LT does not recognize this hidden adjustment of the metrics, and might compensate for this difference by applying the factor γ. As a consequence, the twins disagree on their ages, although they might actually be in agreement taking into account their different temporal measures.

With the geometric interpretation of motion the primed and unprimed coordinates of the LT cannot be directly compared.

This resolution to the infamous Twin Paradox eliminates the rather strange proposition that a person may extend her life expectancy simply by traveling. Thus, the proposed model resolves the Twin Paradox unambiguously; aging always proceeds at the same pace regardless of motion and reference frame.

It is commonly believed that the Hafele-Keating experiment [Hafele-Keating, 1972] confirmed time-dilation in accordance with SR by a time difference observed in aircrafts that circumnavigated the world in opposite directions with four atomic caesium-beam clocks onboard. However, this experiment has been severely challenged by A. G. Kelly [Kelly, 1996], who had access to the raw data. He offers several points of criticism one being that if one of the four clocks in the experiment were to be removed from the data the three remaining clocks do not indicate any time dilation. Here is a quote from the Kelly paper:

> '*The USNO standard station had some years previously adopted a practice of replacing at intervals whichever clock was giving the worst performance. On a similar basis, the results of Clock 120 should have been disregarded. That erratic clock had contributed all of the alteration in time on the Eastward test and 83% on the Westward test, as given in the 1971 report. Discounting this one totally unreliable clock, the results would have been within 5ns and 28ns of zero on the Eastward and Westward tests respectively. This is a result that could not be interpreted as proving any difference whatever between the two directions of flight.*'

Also, Louis Essen [Essen, 1988], the inventor of the atomic clock, mentioned the (in his opinion) inadequate accuracy of the experiment. However, neither of these two references criticizing the Hafele-Keating experiment may be found in peer-reviewed sources.

Hopefully the Hafele-Keating experiment will be repeated using for example the space shuttle.

9. Symmetry between inertial frames – simultaneity

In the discussion above I have considered relative motion between a stationary frame and a moving frame. But, by the relativity principle of SR we should be able to pick any one of these two frames as being stationary. This is true; the inertial scale-factor equally well applies to *1-[(v-v₀)/c]²* where v_0 is an arbitrary constant velocity, which implies that any inertial frame may be considered 'stationary' even when moving relative to some arbitrary reference frame. Therefore the velocity *v* is the relative velocity, and in the particular arbitrary frame that is chosen to be the SIF we have *v=0*. Since the development presented above applies with any inertial frame acting as the SIF, it implies that each inertial frame 'sees' other moving frames from the same relative 'perspective' as given by the inertial line-element (3.1).

According to this scenario a frame that appears to be Minkowskian in a certain inertial reference frame will in another inertial frame, which is moving in relation to the reference frame, no longer be Minkowskian but 'scaled Minkowskian'. However, this perspective is reversed when exchanging the two frame designations so that the moving frame becomes the stationary reference.

At first the proposition that spacetime curvature is relative might seem strange, but we are already familiar with this situation from GR where a local Minkowskian frame always exists, and where spacetime at other locations may appear curved in relation to this local frame. Therefore, the concept of inertial spacetime curvature is *relative*; each inertial frame is offering a similar perspective of moving frames as given by the inertial line-element.

This kind of symmetry may be illustrated by a simile, letting the 4D Minkowskian spacetimes of inertial frames be represented by flat 2D surfaces. Consider two different positions on a spherical surface, each with a local tangent plane. Although the relative perspectives from these two planes are the same, they are rotated relative to each other. This is illustrated in Figure 2 where the relative orientation is reflected by the angle Phi.
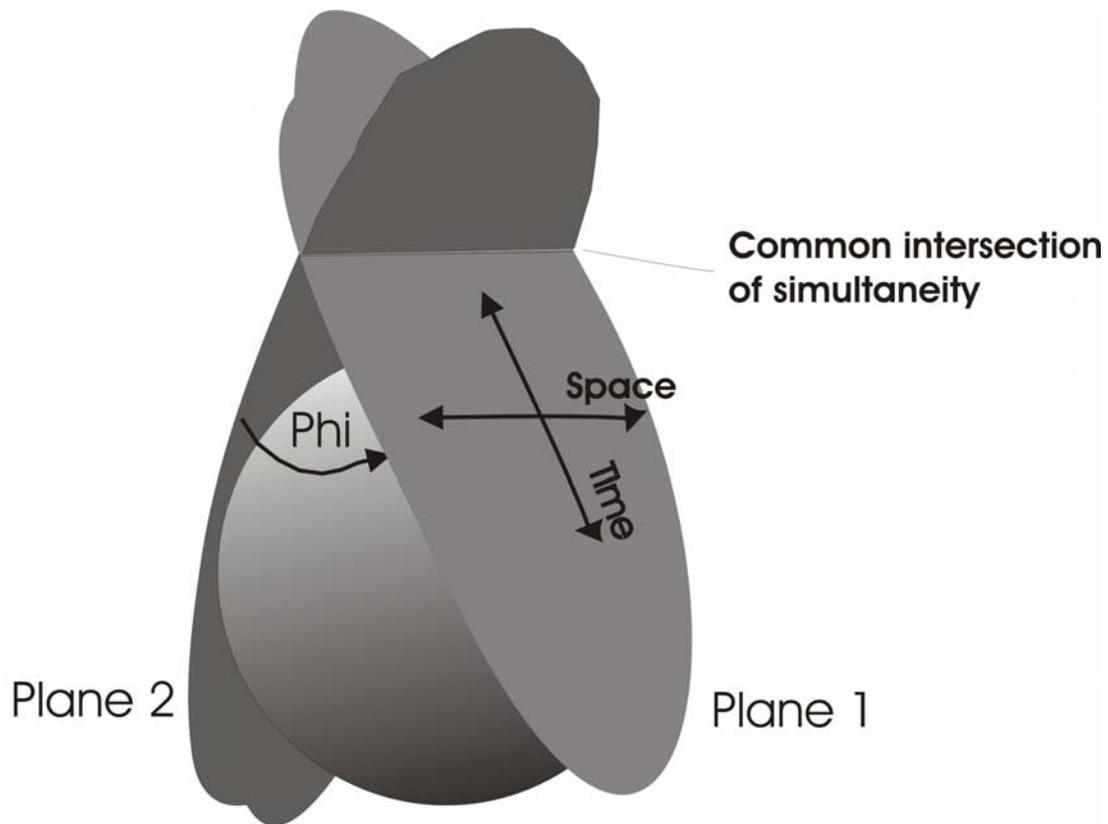
Figure 2: Simile illustrating inertial frame symmetry

Note that this simile collapses the four spacetime dimension plus the scale 'dimension' into three dimensions by merging three spatial dimensions into one.

The two planar surfaces of Figure 2 represent 4D spacetimes. Obviously it is impossible to transform coordinates in one of these surfaces into the other coordinates without taking into account the separating curvature, which corresponds to the dynamic scale. This is also true with Minkowskian line-elements of inertial frames, which cannot be geometrically related by a transformation in 4D spacetime like the Lorentz transformation.

This symmetry between inertial frames might also preserve temporal symmetry since the projection of a temporal increment is the same on each surface. For example, let the angle Phi between the two tangent planes be related to the relative velocity by v/c=sin(Phi). Then the projection of a time interval of one plane onto the other plane is proportional to $\cos(Phi)=(1-(v/c)^2)^{1/2}$ emulating time-dilation.

It also implies that there is a *common temporal reference*, which in Figure 2 could be represented by the intersection line which is common to both planar surfaces. In 4D spacetime this line would correspond to a 3D space of simultaneity.

For example, if the 'temporal' coordinate directions were perpendicular to the intersection line in both planes, this line would represent simultaneity in both planes (3D spaces). As was demonstrated above, all observers agree on elapsed time intervals, regardless of which inertial frame is selected as being 'stationary'. This is discussed further in Appendix I.

The geometrical simile of Figure 2 merely illustrates the concept of relative scale-symmetry in three dimensions rather than in five and has no particular physical significance other than to illustrate a dynamic spacetime scale; for example, in Figure 2 the projections of spatial increments are not contracted. However, the figure shows how transitioning from one inertial frame to another reverses the relative scaling so that the perspective from both frames remain the same.

10. Summary and concluding comments

This paper proposes that motion in general might involve dynamic spacetime metrics. With this proposition General Relativity extends its traditional role of modeling static Gravitation. A first step in modeling dynamic spacetime was taken in cosmology by Friedman [Friedman, 1922], who was followed by many others including Einstein and de Sitter [Einstein and de Sitter, 1932]. In these articles the cosmological expansion is usually modeled by dynamic spatial metrics that depend on time as in the Big Bang theory. More recently this approach has been generalized by considering expansion of all four metrics in the Scale Expanding Cosmos (SEC) theory [Masreliez, 1999, 2006, 2007b]. In the present paper the role of General Relativity is further generalized by modeling Inertia via dynamic spacetime metrics.

A previous paper [Masreliez, 2007a] proposed an alternative to Special Relativity that is observationally indistinguishable from Special Relativity but also models Inertia. A dynamical scale-factor for the Minkowskian line-element has the interesting property that all accelerating trajectories are geodesics of General Relativity. Inertia is thus explained as being a curved spacetime phenomenon similar to Gravitation.

This previous paper discusses this possibility from the point of view of an observer in an accelerating system, showing how Special Relativity and the Lorentz Transformation could be preserved by considering changing spacetime metrics combined with dynamic incremental scale transition, with the implication that General Relativity must be generalized to allow for discrete scale adjustments, an idea further explored in [Masreliez, 2007b]. This previous approach models Inertia while preserving the Minkowskian line-element, and thus formally retains Special Relativity.

The current paper takes a different point of view by demonstrating that several results familiar from Special Relativity may be derived directly from the properties of the inertial metrics without the use of any coordinate transformation. In fact, in the new theory no continuous coordinate transformation exists between Minkowskian inertial frames. However, if the velocities are constant, Einstein's two Special Relativity

postulates will still hold true and a coordinate transformation will exist. This is the transformation originally proposed by Woldemar Voigt in 1887, which actually preceded the Lorentz transformation. Apparently Lorentz was ignorant about Voigt's transformation and admitted that he might have adopted it and used it instead of his own had he known about it. And, in this case Einstein might have selected the Voigt transformation instead of the Lorentz transformation in his paper on Special Relativity in 1905, which might have resolved the question of Inertia a long time ago.

It is interesting and significant that the inertial line-element proposed in this article implies conservation of (relativistic) momentum, and that this in turn implies the existence of the inertial force and Newton's second law. Thus, Newton's second law of motion, which is postulated but never derived in classical physics, is implicit with a line-element that models Inertia. In other words, a line-element for which all motion become General Relativity geodesics implies Newton's second law. This supports the proposition that dynamic spacetime geometry plays a decisive role in modeling motion.

In Special Relativity there is no indication as to what might cause Inertia, because if there is complete equivalence between inertial frames, these frames ought to have identical spacetime geometries. Therefore, there seems to be no sufficient reason why motion between them should be resisted by a gravitational-type inertial force that depends on the spacetime geometry.

The solution to the puzzle proposed here could not have been found without General Relativity (which had not yet been developed in 1905), and recognizing that four-dimensional dynamical scale-equivalence is a phenomenon of major cosmological importance. It allows the existence of different, scaled, spacetime manifolds with identical General Relativity equations. These scaled manifolds are physically equivalent in that the laws of physics are valid but they differ conceptually since they refer to different spacetime manifolds; there is no continuous coordinate transformation between them if the scale-factor is dynamic.

The implication is that the coordinates in the two frames related by the Lorentz transformation have different metrics. The transformed metrics differ by the factor γ in the Lorentz Transformation, which restores the Minkowskian line-element. On the other hand, the opposite would be the case if the roles of the frames were exchanged. Therefore coordinate increments in the two frames as given by the Lorentz Transformation cannot be directly compared because their metrics are different. This would explain the contradiction encountered with Special Relativity regarding the Twin Paradox.

The new theory unambiguously resolves the Twin Paradox since travelling twins always age at the same pace.

Furthermore, by symmetry inertial frames may be considered physically equivalent in the sense that all laws of physics hold true, but an observer in a particular frame will find that the spacetime geometries of other inertial frames differ in a relative sense. This difference, which is reciprocal, explains Inertia. Since all moving observers agree on elapsed time intervals an absolute temporal reference exists, reviving the pre-relativistic concept of an absolute cosmological time.

Summarizing, the paper proposes a new model that offers the following advantages over Special Relativity:

- It explains Inertia.
- It reconciles Special Relativity with General Relativity.
- Preservation of momentum may be derived from the inertial line-element.
- Newton's second law of motion also directly follows from this line-element.
- The Twin Paradox is resolved.
- It admits simultaneity and a temporal reference.
- The role of the Lorentz Transformation is clarified.

In practice the proposed model is indistinguishable from Special Relativity, except in the context of the Twin Paradox. Also, it allows simultaneity regardless of motion.

The new approach proposed in this paper, according to which motion influences the spacetime metrics, implies that the Lorentz Transformation falls short, since it does not model Inertia.

Perhaps the most important aspect of the new proposition advanced in this article is that motion in general not only implies that the 4D coordinates change but also a changing 4D scale during acceleration.

In retrospect SR and the Lorentz transformation represent the best possible model for inertial motion available at the time when it was proposed by Einstein, eleven years before he published his paper on general relativity.

Because the new theory may be of general interest even to the layman an attempt is made in Appendix II to informally outline its main features.

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Appendix I: Simultaneity in scaled frames

The following discussion provides support for the contention that clocks in inertial frames run at the same pace.

Consider a light-clock in a SIF frame according to figure A below, where a light beam is bouncing between two mirrors spaced the distance L. The period for the light is t=2L/c. Another light-clock with mirrors spaced L'=sL is located in a MIF. Let's assume that in this moving frame all distances and time intervals are scaled by the factor s =$[1-(v/c)^2]^{1/2}$ relative to the SIF.

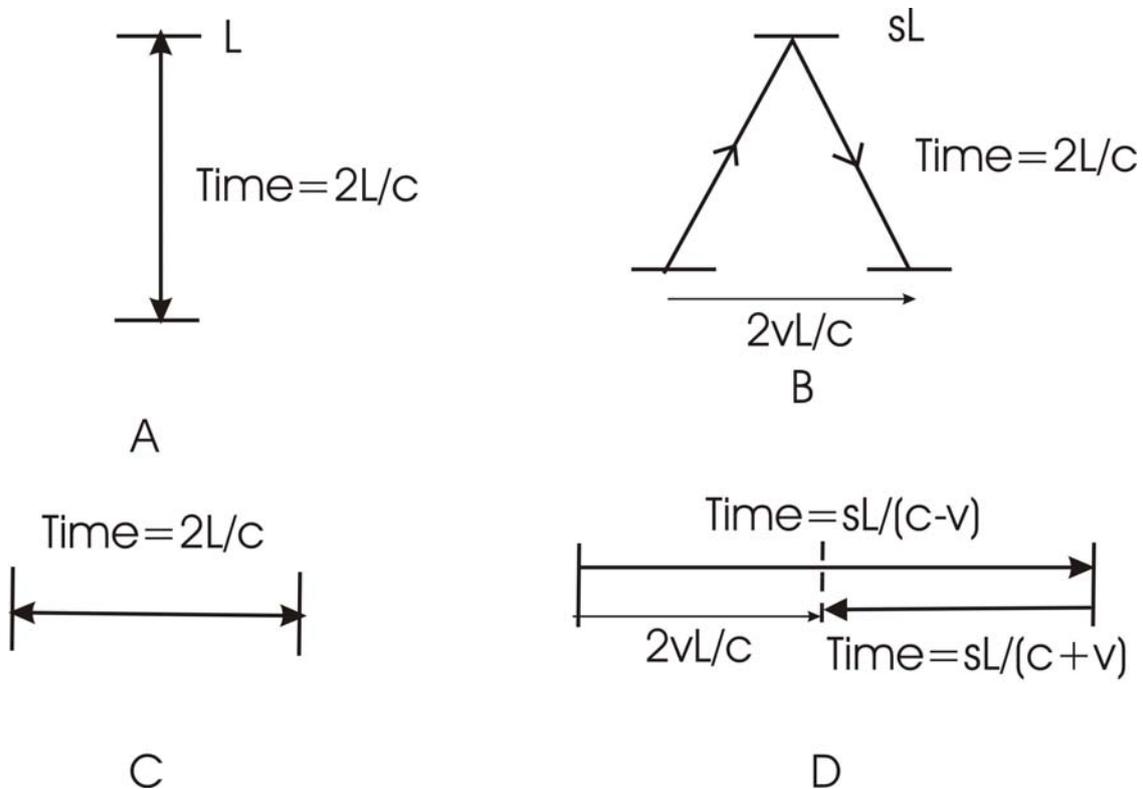Figure B shows this moving clock as seen by a SIF observer. The clock period is still t=2L/c because the path length is unchanged.

Now consider a clock in the MIF that moves at the same speed in the MIF frame but in opposite direction so that it is stationary in the SIF. Since this clock in the MIF is identical to the one in the SIF *it should run, and does run, at the same pace* as seen from either the SIF or the moving observer in the MIF, who is stationary in the SIF.

Therefore, clocks moving in the MIF so that they are stationary in the SIF run at the same pace regardless of velocity.

Now consider a clock oriented in the direction of motion as in figure C. The time interval in the fixed frame is of course unchanged t=2L/c. The time interval in the MIF as seen from the SIF and shown in figure D is t=[sL /(c+v)+ sL /(c-v)]=2L/cs. However, since the time for a *stationary* clock in the MIF runs slower we have t'=st and therefore t'=2L/c again showing that a moving clock runs at the same pace. The same conclusions may be drawn with the roles of the SIF and the MIF interchanged.

This argument supports the proposition that all clocks in inertial frames always measure out the same cumulative time intervals, which resolves the Twin Paradox and makes possible an absolute temporal reference.

Appendix II: An informal presentation of the new theory of motion

Since the subject of this paper could be of general interest to experts and laymen alike due to the newness of the ideas and their philosophical implications, it invites a more general discussion. The ideas presented in this paper run counter to the standard approach to modeling motion and are perhaps difficult to assimilate. Therefore, I will attempt to present the theory in plain language.

The most important aspect of the new idea proposed in this paper is the recognition that the properties of space and time experienced by a particle might change during acceleration as expressed by the 'metrics' of spacetime in general relativity. If the metrics of space and time change for a particle during acceleration we might wonder why. The reason could be that an accelerating particle compensates for a changing light-speed by adjusting its metrics. For example, if during acceleration photons should begin to go slower than light in a certain direction, a particle might compensate for this by 'shrinking' the length of a meter (or foot) in the same direction so that it seems like the speed of light has not changed. Alternatively, it might make the duration of a second longer, which also would cause the measured speed of light to be the same. By adjusting the metrics of both space and time (spacetime) the particle may preserve a constant speed of light in all directions. We might say that spacetime of a particle 'morphs' to accommodate acceleration. After acceleration has ceased, when the velocity again is constant, a co-accelerating observer will find that the particle's spacetime appears exactly as it did before the acceleration boost.

The morphing spacetime remain hidden from inertial observers, but during acceleration the changing spacetime metrics make their presence felt by the inertial force. Therefore, like the gravitational force the inertial force is caused by changing spacetime metrics.

Each frame moving with constant velocity (inertial frame) locally looks the same (with Minkowskian line-elements) after the metrics have adjusted to a new velocity because measuring rods ands clocks have also adjusted to the new metrics, but it appears that the metrics of other frames moving at different velocities have changed in relation. This is a symmetric situation; all inertial observers will find that their local metrics differ from frames in relative motion, and this relative perspective is the same for all inertial observers. In short, there is symmetry between inertial frames, but their metrics differ in a relative sense. This difference explains Inertia. This relative kind of symmetry is illustrated in Figure 3.

Perspective from A

Perspective from B

Figure 3 Each inertial frame has the same relative perspective

The relative scale perspective of figure 3 is understandable; if the scale were to increase in one inertial frame the *relative* scale of other frames should decrease. This is a natural consequence of spacetime curvature caused by a variable spacetime scale.

This is to be compared to Special Relativity according to which the metrics in the moving frame are believed to be the same as in the local frame. When Einstein developed SR, he was correct in assuming that all inertial frames have identical Minkowskian line-elements, and he might also have thought that all inertial frames must have identical spacetime *metrics in a relative sense*. But, this is not a necessary requirement for symmetry between inertial frames because relative geometries do not have to be the same; it is sufficient that all inertial frames 'see' other frames from the same relative 'perspective'. Relative geometries of moving frames, *as expressed in the local frames coordinates,* could differ from the local geometry.

Therefore Einstein might have believed that the coordinates of inertial frames should be related by a continuous transformation like the LT that preserves the geometry of spacetime. But, in the new theory proposed here, the LT cannot be interpreted as a coordinate transformation in General Relativity; no continuous transformation exists relating the Minkowskian spacetime coordinates of inertial frames. As a consequence increments like a meter or a second in the two frames *as given by the Lorentz Transformation* cannot be directly compared.

Comparing this to the new theory, we find that *relative* metrics of inertial frames, which are given by the inertial line-element, differ and that this difference explains Inertia. However, the changing metrics are not recognized by Special Relativity, which explains why the proposed explanation to Inertia has not been found earlier.

We might also wonder why Nature should preserve the speed of light in all directions. One possibility could be that particles are resonating states in the metrics of spacetime that preserve the conditions necessary for their existence, in particular an isotropic speed of light, regardless of their motion. (In this context we should remember that Voigt derived his transformation by demanding invariance of the wave function.)

Thus, the proposed theory of motion demystifies Special Relativity's assumption that the speed of light is constant. We might say that the velocity if light only *appears* to remain constant because of the morphing metrics. The new theory explains the success of Special Relativity while resolving contradictions like the Twin Paradox that has been the subject of considerable confusion over the years. And, it allows a temporal reference in the universe, which is needed in modeling the cosmological expansion as well as to allow non-local influences in quantum theory.

What's new and perhaps at first difficult to accept with the new theory of motion proposed in this paper is that motion might not only take place in space and time but also in *scale*. The scale enters as a new 'dimension' that changes spacetime in a relative sense, keeping it Minkowskian locally (where v=0). One familiar example of this relative type of symmetry is the 'perspective' whereby distant objects seem smaller as shown in figure 3. The perspective is the same when seen from the other direction. The same is true with the scale.

This means that if we model motion we must take into account the changing 4D scale. This might seem confusing since spacetime locally remains Minkowskian in all inertial frames, which suggests the Lorentz Transformation. But, if we ignore the scale parameter we will run into contradictions like the Twin Paradox. Taking into account the changing scale, no transformation *in GR* exists that properly will transform the coordinates of one Minkowskian inertial frame into those of another inertial Minkowskian frame.