A New Gauge Mediation Theory

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Abstract

We propose a class of models with gauge mediation of supersymmetry breaking, inspired by simple brane constructions, where $R$-symmetry is very weakly broken. The gauge sector has an extended $N = 2$ supersymmetry and the two electroweak Higgses form an $N = 2$ hypermultiplet, while quarks and leptons remain in $N = 1$ chiral multiplets. Supersymmetry is broken via the $D$-term expectation value of a secluded $U(1)$ and it is transmitted to the Standard Model via gauge interactions of messengers in $N = 2$ hypermultiplets: gauginos thus receive Dirac masses. The model has several distinct experimental signatures with respect to ordinary models of gauge or gravity mediation realizations of the Minimal Supersymmetric Standard Model (MSSM). First, it predicts extra states as a third chargino that can be observed at collider experiments. Second, the absence of a $D$-flat direction in the Higgs sector implies a lightest Higgs behaving exactly as the Standard Model one and thus a reduction of the ‘little’ fine-tuning in the low $\tan \beta$ region.

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This breaking of supersymmetry can be easily implemented in string theory models.

1 Introduction

The supersymmetric flavor problem is one of the guidelines for constructing realistic models at the electroweak scale with deep implications at LHC. Supersymmetry is generically broken in a hidden sector and transmitted to the observable sector either by gravity or by gauge interactions giving rise to the so-called gravity [1] or gauge-mediated [2] models. In gravity-mediated scenarios the soft-breaking masses are generated at the Planck scale and there is no symmetry reason why they should be flavor-invariant or why they should not mediate large contributions to flavor-changing neutral current processes. In fact this idea motivated the introduction of gauge-mediated theories where the generated soft terms are flavor-blind and therefore they feel flavor breaking only through Yukawa interactions.

In gauge mediated (GMSB) theories supersymmetry is broken in a secluded sector such that the Goldstino field belongs to a chiral superfield \( X \) which acquires a vacuum expectation value (VEV) along its auxiliary \( F \)-component as \( \langle X \rangle = M + \theta^2 F \), where it is usually assumed that \( F \ll M^2 \) so that supersymmetry breaking can be treated as a small perturbation. The theory also contains a vector-like messenger sector \( (\Phi, \Phi^c) \) coupled to \( X \) by a superpotential coupling \( \int d^2 \theta \Phi^c X \Phi \) and with Standard Model (SM) quantum numbers providing one-loop Majorana masses to gauginos \( \sim \alpha/4\pi F/M \) and two-loop positive squared masses to all sfermions as \( \sim (\alpha/4\pi)^2 F^2/M^2 \), where \( \alpha = g^2/4\pi \) and \( g \) is the corresponding gauge coupling. The corresponding effective operators giving rise to these masses are \( (1/M) \int d^2 \theta X W^\alpha W_\alpha \) and \( (1/M^2) \int d^4 \theta X^\dagger X Q^\dagger Q \), where \( W^\alpha \) are the chiral field strengths of the SM vector superfields and \( Q \) the SM chiral superfields. Fixing all masses in the TeV range one obtains a relation between \( F \) and \( M \). In particular if gravity mediated contributions are required not to reintroduce the flavor problem, the flavor changing Planck suppressed contributions should remain smaller than the flavor conserving gauge-mediated ones, i.e. \( M \lesssim (\alpha/4\pi) M_{P\ell} \sim 10^{16} \text{ GeV} \), implying \( \sqrt{F} \lesssim 10^{11} \text{ GeV} \). In gauge mediation the \( R \)-symmetry is broken by
the same mechanism that breaks supersymmetry which constrains the corresponding ultraviolet (UV) completion of the theory. The main problem of any fundamental theory, as e.g. string theory, is then to provide the required mechanism of supersymmetry and $R$-symmetry breaking.

In the simplest compactifications of a class of string theories, leading to intersecting branes at angles, the gauge group sector is often in multiplets of extended supersymmetry while matter states come in $N = 1$ multiplets [3]. A simple way of breaking supersymmetry is by deforming the intersection angles from their special values corresponding to the supersymmetric configuration. A small deformation $\epsilon$ of these angles breaks supersymmetry via a $D$-term vacuum expectation value, associated in the $T$-dual picture to a magnetized $U(1)$ in the internal compactification space, $\epsilon = D\ell_s^2$ with $\ell_s$ the string length. In this case, the secluded sector is given by an (extended) vector multiplet containing an $N = 1$ vector superfield $V^0$. The Goldstino is then identified with the gaugino of $V^0$ which acquires a VEV: $\langle V^0 \rangle = \frac{1}{2} \theta^2 \bar{\theta}^2 D$ or $\langle W_\alpha^0 \rangle = \theta_\alpha D$. Of course, this supersymmetry breaking preserves the $R$-symmetry and can only give Dirac masses to the gauginos of the extended gauge sector.

A prototype model is based on type II string compactifications on a factorizable six-torus $T^6 = \otimes_{i=1}^3 T_i^2$ with appropriate orbifold and orientifold planes and two sets of brane stacks: the observable set $\mathcal{O}$ and the hidden set $\mathcal{H}$ [4, 5]. The SM gauge sector corresponds to open strings that propagate with both ends on the same stack of branes that belong to $\mathcal{O}$: it has therefore an extended $N = 2$ or $N = 4$ supersymmetry. Similarly, the secluded gauge sector corresponds to strings with both ends on the hidden stack of branes $\mathcal{H}$. The SM quarks and leptons come from open strings stretched between different stacks of branes in $\mathcal{O}$ that intersect at fixed points of the three torii $T_i^2$ and have therefore $N = 1$ supersymmetry. The Higgs sector however corresponds to strings stretched between different stacks of branes in $\mathcal{O}$ that intersect at fixed points of two torii and that are parallel along the third one: it has therefore $N = 2$ supersymmetry. Finally the messenger sector contains strings stretched between stacks of branes in $\mathcal{O}$ and the hidden branes $\mathcal{H}$, that intersect at fixed points of two torii and are parallel along a third torus. It has therefore also $N = 2$ supersymmetry. Moreover, the two stacks of branes along the third torus are separated by a distance $1/M$, which introduces a supersymmetric
mass $M$ to the hypermultiplet messengers. The latter are also charged under the supersymmetry breaking $U(1)$ and they are given corresponding supersymmetry breaking squared-masses $\pm D$.

In this paper (inspired by the previous brane constructions) we propose a new gauge mediated theory (NGMSB), alternative to the usual GMSB, where $R$-symmetry remains unbroken by the mechanism of supersymmetry breaking. The observable gauge sector is a set of $N = 2$ gauge multiplets corresponding to the SM gauge group. In general an $N = 2$ gauge multiplet contains an $N = 1$ vector multiplet $V = (A_\mu, \lambda_1, D)$ and a chiral multiplet $\chi = (\Sigma, \lambda_2, F_\chi)$. It can be described by the $N = 2$ chiral vector superfield [6]

$$A = \chi + \tilde{\theta} W + \tilde{\theta}^2 D D \chi,$$

(1.1)

where $\tilde{\theta}$ is the second $N = 2$ Grassmannian coordinate and $D$ the $N = 1$ super-covariant derivative. The secluded sector is identified with the $N = 2$ chiral vector superfield $A^0$ (whose $N = 1$ vector superfield is $V^0$) where supersymmetry is broken by a hidden $D$-term as

$$\langle A^0 \rangle = \tilde{\theta} \langle W^0 \rangle = \tilde{\theta} \theta D$$

(1.2)

For the messenger sector, we choose a (set of) $N = 2$ hypermultiplet(s) denoted as $(\Phi^c, \Phi)$, with field contents $\Phi = (\phi, \psi, F_\Phi)$ and $\Phi^c = (\phi^c, \psi^c, F_{\Phi^c})$, charged under the secluded $U(1)$ (with charges $\pm 1$) and with a supersymmetric mass $M$.

The content of the paper is as follows. In section 2 we present the structure of our model. In particular the transmission of supersymmetry breaking from the hidden to the observable sector is calculated by loop diagrams involving messenger fields. Its main outputs are Dirac masses for gauginos of the $N = 2$ gauge sector ($R$-symmetry is preserved by the $D$-breaking) as well as soft masses for $N = 1$ sfermions. In section 3 we study the generation of the soft-breaking terms for the $N = 2$ Higgs sector, as well as the electroweak symmetry breaking. The main departure of the latter from the MSSM is

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3As we will see later, gravitational interactions produce $R$-symmetry breaking at a sub-leading level in the observable sector.

4We are assuming here that some dynamical mechanism in the hidden sector generates the $D$-breaking of supersymmetry given by the VEV in Eq. (1.2). A particular realization is given by the angle deformation of intersecting branes described above, leading to an $N = 2$ Fayet-Iliopoulos term.
the absence of $D$-flat directions along which the Higgs potential becomes unstable. As a consequence, after electroweak symmetry breaking, the SM-like Higgs has a $\tan \beta$-independent mass and couplings to SM fermions while the rest of the Higgs sector has $\tan \beta$-independent masses (at the tree-level) and $\tan \beta$-dependent couplings to ordinary matter. In section 4 we present a few comments about the gravitino mass and the generation of an $F$-breaking by gravitational interactions, possible dark matter candidates in our scenario, its experimental signatures at the next high energy colliders (LHC and ILC) and the possibility of unification at the GUT scale. In section 5 our conclusions are drawn. Finally in appendix A we give a description of the supersymmetry breaking potential, based on a Fayet-Iliopoulos (FI) term, for the scalar messengers and the scalar field in the chiral multiplet in the secluded gauge sector. We show that the supersymmetry $D$-breaking minimum is a local (metastable) one. However we also show that it is absolutely stable on cosmological times.

2 THE MODEL STRUCTURE

The interaction Lagrangian of the messengers with the different gauge sectors is written as

$$\int d^4 \theta \left\{ \Phi^\dagger e^V \Phi + \Phi^c e^{-V} \Phi^{c\dagger} \right\} + \int d^2 \theta \Phi^c \left[ M - \sqrt{2} \chi \right] \Phi + h.c. \right\} \tag{2.1}$$

where $V = \sum_A g_A T^A V^A$ contains all gauge fields (in the hidden and observable sectors) and similarly for $\chi = \sum_A g_A T^A \chi^A \tag{5}$.

From (2.1) and after replacing the $V^0$ VEV from Eq. (1.2), we find that the Dirac spinor $\Psi = (\psi, \bar{\psi}^e)^T$ acquires a Dirac mass $M$ while the scalar components have a squared mass matrix given by

$$(\phi^1, \phi^c)M^2 \left( \begin{array}{c} \phi^1 \\ \phi^{c\dagger} \end{array} \right), \quad M^2 = \left( \begin{array}{cc} M^2 + D & 0 \\ 0 & M^2 - D \end{array} \right) \tag{2.2}$$

Notice that in the absence of the supersymmetric mass $M$, the origin in the $(\phi, \phi^c)$ field space is a saddle point. However since we are assuming that $M^2 > D$ the origin becomes a minimum along the $(\phi, \phi^c)$ directions. Assuming

\footnote{We are normalizing the generators such that $Tr T^A T^B = 1/2 \delta^{AB}$ in the fundamental representation.}
that supersymmetry is broken by a FI mechanism the $\Sigma^0$-scalar in the extended gauge sector is a flat direction of the potential. However in the presence of supersymmetry breaking it will acquire a radiative soft mass, see Eq. (2.13), and the origin $\langle \Sigma^0 \rangle = \langle \phi \rangle = \langle \phi^c \rangle = 0$ becomes a local minimum. Of course there exists a global minimum where supersymmetry is restored and gauge symmetry is broken, at $\langle \Sigma^0 \rangle = M/\sqrt{2}$ and $|\langle \phi^c \rangle|^2 = D$. The barrier height separating these minima is very small as compared to the distance between them if $M^2 \gg D$. In that case the tunneling probability per unit space-time volume from the local minimum to the global one is

$$P \sim e^{-\kappa M^4/D^2}$$

(2.3)

where $\kappa > 1$ is a dimensionless constant. For $M^2 \gg D$ the probability (2.3) is so small that the false vacuum is essentially stable on cosmological times. More details can be found in appendix A.

Using now the Lagrangian (2.1), the coupling of the SM gauginos with the messenger fields is written as

$$-\sqrt{2} (\phi^c \bar{\lambda}_1 \Psi - \phi^\dagger \bar{\lambda}_2 \Psi) + h.c.$$ (2.4)

where we have defined the four-component symplectic-Majorana spinors $\lambda_i = (\lambda_i, \epsilon_{ij} \bar{\lambda}_j)^T$. A Dirac mass $m^D_a$ for the Dirac gaugino $\lambda_a$ is radiatively generated from the diagram of Fig. 1 which gives a finite value

$$m^D_a = k_a \frac{\alpha_a}{4\pi} \mathcal{N} \frac{D}{M} \left[ 1 + \mathcal{O} \left( \frac{D^2}{M^4} \right) \right], \quad a = 1, 2, 3$$ (2.5)

Notice that if we define the Dirac spinor $\lambda = (\lambda_1, \bar{\lambda}_2)^T$, it satisfies the identity $\bar{\lambda} \lambda = \frac{1}{2} (\lambda_1 \lambda_1 - \lambda_2 \lambda_2)$. 

\[ \text{Figure 1: Feynman diagrams contributing to the Dirac mass of the gaugino } \lambda^a \]
with \( k_1 = 5/3, k_3 = k_2 = 1, \alpha_a = g_a^2/4\pi, \) and \( N \) the number of messengers. This value for the Dirac gaugino mass can be equivalently understood from the effective operator [7, 4]

\[
\sim \frac{1}{M} \int d^2\theta W^0 \text{Tr}(W\chi) + h.c. \quad (2.6)
\]

This operator is actually consistent with \( N = 2 \) supersymmetry since it is generated by a manifest \( N = 2 \) supersymmetric Lagrangian (2.1). The operator (2.6) can be rewritten in an explicit \( N = 2 \) supersymmetric way:

\[
\sim \frac{1}{M} \int d^2\theta d^2\tilde{\theta} A^0 \text{Tr}(A)^2 + h.c. \quad (2.7)
\]

where \( A^0 \) is the secluded \( U(1) \) \( N = 2 \) vector superfield.

In Ref. [5] such an operator was computed in string theory for the same physical setup of brane configurations discussed in section 1. The result was found to be topological, in the sense that it is independent of the massive string oscillator modes. It receives contributions only from the field theory Kaluza-Klein (KK) part of the torus along which the messengers brane intersection of the observable stack \( \mathcal{O} \) with the hidden stack \( \mathcal{H} \) is extended. The separation \( \ell \) of the two stacks along a direction within this torus determines the messengers mass \( M = \ell \) in string units. The gaugino mass (in the limit \( D\ell_s^2 \ll 1 \)) can then be written as an integral over the real modulus parameter \( t \) of the worldsheet annulus having as boundaries the two brane stacks \(^7\):

\[
m_{1/2}^D \simeq \frac{\alpha}{2} ND \int_0^{+\infty} \sum_{n=-\infty}^{+\infty} (nR + \ell)e^{-2\pi t(nR+\ell)^2}, \quad (2.8)
\]

where for simplicity we have chosen the brane separation to be along one of the two dimensions of an orthogonal torus of radius \( R \). The integration can be performed explicitly with the result:

\[
m_{1/2}^D \simeq \frac{\alpha}{4\pi} ND \left\{ \frac{1}{\ell} - \frac{2}{R^2} \sum_{n \geq 1} \frac{\ell^2/R^2}{n^2 - \ell^2/R^2} \right\}. \quad (2.9)
\]

\(^7\)Note a factor of \( t \) misprint in Eq. (3.22) of Ref. [5]. Here, we also restored the numerical prefactor.
The first term in the right-hand side reproduces precisely the field theory expression (2.5), while the second term represents the (sub-leading) contribution of the messengers KK excitations.

The superpotential term in Eq. (2.1) gives rise to the $F$-term potential

$$\text{Tr}\left\{2\phi^\dagger\Sigma^\dagger\Sigma\phi + 2\phi^c\Sigma^\dagger\Sigma\phi^c - \sqrt{2}M\left[\phi^\dagger(\Sigma + \Sigma^\dagger)\phi + \phi^c(\Sigma + \Sigma^\dagger)\phi^c\right]\right\}.$$  

By using now the one-loop diagrams of Fig. 2 we obtain for the real and imaginary parts of the field $\Sigma^a$, a squared mass splitting $m_{\Sigma^a}^2 = m_{\Sigma^a}^2 \simeq 8$. This behaviour can be understood from the effective operator

$$\sim \frac{1}{M^2} \int d^2\theta (W^0)^2 \text{Tr}(\chi)^2.$$  

This problem was discussed at length in Ref. [8] where several solutions were proposed.

One possibility can be based on the fact that, as we will see in section 4, cancellation of the cosmological constant when supergravity interactions are turned on will generically induce an $F$-breaking of the order of $D$ giving rise to a spurion superfield $X = \theta^2 F$ in the hidden brane $\mathcal{H}$. Another possibility is that global supersymmetry be spontaneously broken in the hidden sector by an O’Raifeartaigh model generating the $F$ component of some chiral superfield

$^8$We thank Yuri Shirman for pointing out a mistake in an earlier version when computing the $\Sigma^a$ squared mass.
A new gauge mediation theory

Interactions between $X$ in the hidden sector and $\chi$ can give rise through the operator

$$\sim \frac{1}{M^2} \int d^4 \theta X^\dagger X \chi^\dagger \chi$$

(2.12)

to masses

$$m^2_{\Sigma a} \sim D^2/M^2.$$  

(2.13)

If the $X$-superfields are located at an intersection of hidden stack of branes separated from the observable sector three-brane along some extra dimension, an operator similar to (2.12) does not appear for fields in the observable sector and the corresponding transmission of supersymmetry breaking is forbidden by locality or suppressed by powers of the Planck scale $\sim F/M_{Pl}$. The actual value of the $\Sigma$ mass is to a large extent model dependent but we will implicitly assume that Eq. (2.13) is at most suppressed by one-loop factors just to cancel the negative contribution from diagrams in Fig. 2.

The sector of quarks and leptons is made of $N = 1$ chiral multiplets, that we generically denote as $Q = (Q, q_L)$. Its interactions with the gauge sector are given by the Lagrangian

$$\int d^4 \theta Q^\dagger e^V Q$$

(2.14)

In principle, since the messenger sector has SM quantum numbers, there are quartic interactions (from integration of the SM $D$-terms) as

$$\frac{1}{2} g_a^2 \left( \phi_i^\dagger \phi_j - \phi_i^c \phi_j^c \right) Q_k^l Q_l^i \sum_A (T^A_{\phi})_{ij} (T^A_Q)_{kl}$$

(2.15)

where $T^A_{\phi}$ are the generators of the gauge group in the representation of $\phi$. From Eq. (2.15) and by performing a one-loop contraction of the messenger fields $\phi$ and $\phi^c$, one could in principle provide the SM sfermions $Q$ with a phenomenologically unacceptable large squared mass $\sim D$. For non-abelian group factors this contribution vanishes since it is proportional to $\text{Tr}(T^A_{\phi}) = 0$. However, this cancellation does not automatically take place for the case of $U(1)$ factors, as the hypercharge. Moreover, the sign of the different sfermion
squared masses depends on their hypercharge and tachyonic masses can thus be generated. A solution to this problem appears if \( \phi \) is in a complete representation of \( SU(5) \) in which case it is guaranteed that \( \text{Tr}(Y_\phi) = 0 \) 9. This mechanism is similar to that proposed in Ref. [9] in conventional gauge mediation. In fact, by making a \( \pi/4 \) rotation on the fields \((\Phi, \Phi^\dagger)\), the mass matrix (2.2) rotates to

\[
\begin{pmatrix}
  M^2 & D \\
  D & M^2
\end{pmatrix}
\]  (2.16)

which coincides with the supersymmetry breaking mass matrix in usual gauge mediation via an \( F \)-term breaking when the messenger sector is invariant under a “messenger parity” by just making the identification \( F = F^\dagger = D \).

In view of the previous identification and given that the adjoint superfields \( \Sigma \) do not have direct interactions with \( N = 1 \) matter, the two-loop diagrams that contribute to the sfermions masses are those in usual gauge mediation models (see the diagrams shown for instance in Refs. [10, 11]) with the result

\[
m^2_Q \propto (\alpha/4\pi)^2 D^4/M^6
\]

where a strong cancellation with respect to the usual gauge mediation of the \( F \)-supersymmetry breaking due to the structure of the messenger mass matrix is found 10. This contribution is however subleading since one-loop diagrams involving Dirac gauginos in the effective theory where messengers have been integrated out (three-loop diagrams in the underlying theory) provide finite masses given by [12, 7]

\[
m^2_Q = \sum_{j=1}^{3} k_j C_j(Q) \frac{\alpha_j}{\pi} (m_j^D)^2 \log \left[ \frac{m^2_\Sigma}{(m_j^D)^2} \right]
\]  (2.17)

where \( C_j(N) = (N^2 - 1)/2N \) for the fundamental representation of \( SU(N) \).

There could be an extra contribution to the squared scalar masses, coming from the extra \( F \)-term used to make the \( \Sigma \)'s to have a positive mass, if this \( F \)-term generates a non-zero supertrace. This contribution is negative and log-enhanced as \( \log \Lambda/M \) where \( \Lambda \) is the ultraviolet cutoff [11], and it can produce

9Notice that this solution uses the fact that within the states of the representation \( \phi \) the mass matrix is proportional to the unity and hence does not involve the vector-like character of the hypermultiplet.

10We thank Yuri Shirman for a discussion on this point.
tachyonic masses for scalar matter fields. If the extra breaking of supersymmetry comes from a renormalizable interaction (as it could appear in spontaneous breaking of global supersymmetry in an O’Raifeartaigh-type model) then the supertrace has to be canceled by some heavy messengers with mass $M_H < \Lambda$ that make the replacement $\log \Lambda/M \rightarrow \log M_H/M$ in the supertrace formula. In addition there could also be threshold effects on the squared masses coming from the heavy messengers that should go as $\delta M^4_H/M^2_H$ where $\delta M^2_H$ is the squared mass splitting of the heavy supermultiplets. If the mass splitting inside the heavy messengers is (as we will assume) of the order of the scale of supersymmetry breaking in the observable sector all those effects are negligible. However if the extra breaking comes from non-renormalizable interactions (as e.g. supergravity) then nothing prevent the theory from getting a non-null supertrace and we expect on general grounds a large negative logarithmically enhanced contribution to the squared masses. In that case the only possibility is to have an ultraviolet cutoff (string scale) sufficiently close to the scale $M$. Low string scale scenarios exist in the literature and there is nothing wrong with them except that unification should eventually hold sufficiently close to the string scale to avoid entering the non-perturbative regime.

Finally associated with the superpotential term $\int d^2 \theta h_t H^c Q U$ there exists the relevant supersymmetry breaking parameter $A_t$ that appears in the $(R$-symmetry breaking) Lagrangian $-A_t h_t H^c Q U$. This term generates a mixing between the left and right-handed stops and plays a crucial role in the radiative corrections contributing to the lightest Higgs mass. In our model we have $A_t = 0$ at the scale $M$ and furthermore, since the $D$ supersymmetry breaking does not break the $R$-symmetry, this null value is not modified by the renormalization group equations to any order in perturbation theory, unlike the case of usual gauge mediation where the gaugino Majorana masses contribute to the one-loop renormalization of $A_t$. However since, as we shall see, $R$-symmetry is broken by tiny effects required for electroweak symmetry breaking (and in particular also by gravitational interactions) there will be a small trilinear parameter $A_t$ but irrelevant for phenomenological purposes.

\footnote{This mechanism was discussed in length in Ref. [11].}
3 ELECTROWEAK BREAKING

Concerning the Higgs sector, it belongs to an $N = 2$ hypermultiplet $\mathbb{H} = (\mathcal{H}^c, \mathcal{H})$ and its interactions with the gauge sector are given by the Lagrangian

$$\int d^4\theta \left\{ \mathcal{H}^\dagger e^V \mathcal{H} + \mathcal{H}^c e^{-V} \mathcal{H}^c\dagger \right\} - \left\{ \sqrt{2} \int d^2\theta \mathcal{H}^c \chi \mathcal{H} + \text{h.c.} \right\}$$ \hspace{1cm} (3.1)

The Higgs scalars acquire three-loop masses from the Feynman diagrams leading to doublet slepton masses (2.17) and the two-loop diagrams of Fig. 3 that come from the $N = 2$ superpotential gauge interactions in Eq. (3.1).

![Feynman diagrams](image)

Figure 3: Feynman diagrams from $\mathbb{N} = 2$ superpotential contributing to the $H$ squared mass. The same diagrams contribute also to the $H^c$ squared mass (make the change $H \leftrightarrow H^c$ in all graphs).

The contribution from the superpotential interactions is easily written as

$$m_H^2 = 2 \sum_{j=1}^{2} k_j C_j(H) \left( \frac{\alpha_j}{4\pi} \right)^2 \frac{D^2}{M^2} \left[ 1 + \mathcal{O} \left( \frac{D^2}{M^4} \right) \right]$$ \hspace{1cm} (3.2)

that can be understood as coming from the effective operator

$$\sim \frac{1}{M^2} \int d^2\theta W^2 \mathcal{P}_C [\mathcal{H}^\dagger \mathcal{H} + \mathcal{H}^c \mathcal{H}^c\dagger].$$ \hspace{1cm} (3.3)

where $\mathcal{P}_C$ is the non-local operator \cite{13, 14}

$$\mathcal{P}_C = \Box^{-1} \bar{D} \bar{D} \bar{D} \bar{D} \bar{D}$$ \hspace{1cm} (3.4)
that, acting on a real superfield, produces a chiral one such that its lowest component contains the lowest component of the real superfield. In our case the lowest component of $P_C \text{Tr} H^\dagger H$ contains $\text{Tr} H^\dagger H$.

Note that the Higgs potential is corrected with some additional contributions coming from Eq. (2.6) [7]. The resulting Lagrangian for the $\Sigma^A$ scalars, including the radiative mass terms in Eq. (2.13) can then be written as

$$\begin{align*}
-m_2^D D^a (\Sigma^a + \Sigma^a) - m_1^D Y (\Sigma_Y + \Sigma_Y^\dagger) - m_2^2 |\Sigma^a|^2 - m_1^2 |\Sigma_Y|^2
\end{align*}$$

(3.5)

where $\Sigma^a$ and $D^a$ are the adjoint scalar and auxiliary field of the $SU(2)$ vector superfield, $\Sigma_Y$ and $D_Y$ are the neutral scalar and auxiliary field of the $U(1)$-hypercharge vector superfield, and $m_2^D$ and $m_1^D$ represent the corresponding gaugino Dirac masses. We can now integrate out the adjoint fields $\Sigma^a$ and $\Sigma_Y$. In the absence of the mass terms (2.13) this integration would yield $D^a = D_Y = 0$ which would be a phenomenological drawback for this kind of models. However, in the presence of the mass terms (2.13) the integration yields:

$$\begin{align*}
\Sigma^a &= -\frac{m_2^D D^a}{m_2^2}, \\
\Sigma_Y &= -\frac{m_1^D D_Y}{m_1^2}
\end{align*}$$

(3.6)

Replacing now (3.6) in the Lagrangian generates an extra term (quartic in the Higgs fields)

$$\begin{align*}
\left(\frac{m_2^D}{m_2^2}\right)^2 \bar{D}^2 + \left(\frac{m_1^D}{m_1^2}\right)^2 D_Y^2
\end{align*}$$

(3.7)

which is a small correction to the tree-level Lagrangian $-\frac{1}{2}(\bar{D}^2 + D_Y^2)$ and can thus be neglected in the calculation of the Higgs scalar potential. On the other hand, from Eq. (3.6), the VEV of the neutral component of the $SU(2)$ triplet $\Sigma^3$ gives rise to a small violation of the $SU(2)$ custodial symmetry when the neutral Higgses acquire a VEV. It would contribute to the $\rho$-parameter as

$$\begin{align*}
\rho_0 - 1 = 4\frac{(\Sigma^3)^2}{v^2} = g^2 \left(\frac{m_2 v}{m_2^2}\right)^2 \cos^2 2\beta
\end{align*}$$

(3.8)
where \( v = 174 \text{ GeV} \) and as usual \( \tan\beta \) is the ratio of the two Higgs VEVs, 
\[ \tan\beta = \frac{v_2}{v_1} \]. Here we also used \( \langle D^3 \rangle = 1/2 \ g v^2 \cos 2\beta \), following from the usual minimization of the Higgs potential. Present experimental bounds \([15]\),
\[ \rho_0 - 1 < 6 \times 10^{-4} \] are then easily satisfied. Similarly, there is an induced VEV for the singlet \( \Sigma_Y \) that contributes to the \( \mu \) parameter but without any phenomenological impact.

Apart from the Higgs soft masses, there are two other key ingredients of the Higgs sector in order to successfully break \( SU(2) \times U(1) \): namely the \( \mu \) and \( m_3^2 \) terms. The former represents a supersymmetric mass for Higgses and Higgsinos that can be written as \( \mu \int d^2\theta H^c H + h.c. \). Notice that this superpotential mass term is perfectly consistent with the \( N = 2 \) supersymmetry of the Higgs sector. However, its origin and correct size is one of the main problems of supersymmetric model building \(^{12}\). Moreover the \( m_3^2 \) term is a soft mass for the scalar components of \( H \) and \( H^c \), as \( m_3^2 H^c H + h.c. \), which is required to explicitly break the Peccei-Quinn symmetry. The simplest possibility to generate a \( \mu \)-term would be the existence in the \( N = 1 \) observable sector of an extra singlet chiral superfield \( S \), coupled to the Higgs sector in the superpotential \( \int d^2\theta S H^c H \) and that acquires a VEV \( \langle S \rangle \sim \text{TeV} \) when supersymmetry is broken, together with an \( F \)-component VEV generated (by some O’Rafeartaigh mechanism or even by gravitational interactions as we will see in the next section) at the scale \( \sqrt{F_S} \sim \text{TeV} \). This breaking would be subleading with respect to the \( D \)-term breaking and thus will not alter any of the general conclusions previously obtained. Finally notice that, as we will comment in the next section, \( D \)-breaking implies, in the presence of gravitational interactions and through the cancellation of the cosmological constant, an \( F \)-breaking that could be used to solve the \( \mu \)-problem by some of the existing solutions in the literature either using gravitational \([16]\) or gauge \([17, 19]\) interactions.

The soft masses in the Higgs sector of Eq. (3.2) are ‘lowest order’ masses \(^{13}\), that correspond to given boundary conditions at the scale \( M \), while the renor-

\(^{12}\)Actually, as mentioned above, in our model such a term is generated through the \( N = 2 \) superpotential in Eq. (3.1), upon the VEV of the scalar component \( \Sigma_Y \) of the superfield \( \chi_Y \) in Eq. (3.6). However its magnitude is too low for phenomenological purposes and an additional source of \( \mu \) is needed.

\(^{13}\)Both masses are equal because gauge interactions respect the \( N = 2 \) structure of the Higgs hypermultiplet.
malization group equations (RGE) evolution should be considered from $M$ to the weak scale $m_Z$. In particular, the Higgs coupled to the top quark will get a large negative squared mass proportional to the squared top-Yukawa coupling, as in the MSSM\textsuperscript{14}. We can now write the Higgs potential in the usual notation where $H_{1,2}$ are the lowest components of the chiral superfields $\mathcal{H}$ and $\mathcal{H}^c$ respectively. For the neutral components of the Higgs doublets the potential is\textsuperscript{15}
\begin{align}
V &= m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - m_3^2 (H_1 H_2 + \text{h.c.}) \\
&+ \frac{1}{8} (g^2 + g'^2) (|H_1|^2 - |H_2|^2)^2 + \frac{1}{2} (g^2 + g'^2) (1 + \delta_t) |H_1 H_2|^2 
\end{align}
where $m_i^2 = m_{H_i}^2 + \mu^2$ are the mass parameters at low energy including the soft- and the $\mu$-terms. In the last line, the first quartic term is the usual $D$-term of the MSSM, whereas the second is a genuine $N = 2$ effect similar to the one in Eq. (3.1) where we have also included a radiative $N = 1$ term coming from the running of the ($N = 1$) top-Yukawa coupling between the messenger $M$ and the supersymmetry breaking scale. In summary whereas the tree level potential exhibits the $N = 2$ structure, radiative corrections from the $N = 1$ sector provide a breaking of the $N = 2$ structure. One can evaluate this effect by solving the RGE for the superpotential coupling that generates this term in the leading logarithmic approximation\textsuperscript{16}. While radiative corrections that respect the $N = 2$ structure of the theory can be reabsorbed in a redefinition of the $N = 2$ quartic coupling, the leading contribution to the $N = 2$ breaking $\delta_t$ goes as follows:
\begin{align}
\delta_t \simeq - \frac{3h_t^2}{8\pi^2} \log \frac{M}{m_Q} 
\end{align}
Having an extra quartic term in the potential has interesting consequences for its minimization. Indeed, the origin in the Higgs field space can either be

\textsuperscript{14}Since the Yukawa couplings only respect $N = 1$ supersymmetry we expect the renormalization of the two Higgs masses to be different.

\textsuperscript{15}We are assuming here that the $\Sigma$ soft masses are generated (at most) at one-loop order, as required by the cancellation of the instability generated by the diagrams of Fig. 2, while $H_{1,2}$ soft masses are generated at two-loop. In this way in the (effective) potential (3.9) $\Sigma$-scalars are integrated out non-supersymmetrically.

\textsuperscript{16}RGE for the case of zero hypercharge singlets and $SU(2)$ triplets coupled to the Higgs can be found in [18].
a maximum or a saddle-point, whereas in the MSSM it can only be a saddle-
point due to the existence of a flat direction of the $D$-term ($|H_1| = |H_2|$). The
conditions to have a vacuum that breaks electroweak symmetry at the correct
value are:

$$
\frac{m_Z^2}{2} = -\mu^2 + \frac{1}{\tan^2 \beta - 1} \left( m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta \right) \quad (3.11)
$$

$$
m_A^2 = m_{H_1}^2 + m_{H_2}^2 + 2\mu^2 + m_Z^2(1 + \delta_t) \quad (3.12)
$$

where $m_A^2 = 2m_3^2/\sin 2\beta$ is the squared mass of the pseudo-scalar. Notice the
important fact that the MSSM stability condition $2|m_3^2| < m_{H_1}^2 + m_{H_2}^2 + 2\mu^2$
is not required in the case of an $N = 2$ Higgs sector. Actually the MSSM
minimization condition $m_A^2 = m_{H_1}^2 + m_{H_2}^2 + 2\mu^2$ is replaced by Eq. (3.12) so
that for the same input mass parameters the value of $m_A$ is larger than that
of the MSSM. Another feature of the potential is that the “little” electroweak
fine-tuning of the MSSM is reduced for values of $\tan \beta$ close to one, as we will
see below.

We can now calculate the spectrum of the Higgs sector. As in the MSSM,
it is controlled by $m_A$. However it has no dependence on $\tan \beta$, except for a
small piece proportional to $\delta_t$:

$$
m_h^2 = m_Z^2(1 + \delta_t \sin^2 2\beta)
$$

$$
m_H^2 = m_A^2 - m_Z^2 \delta_t \sin^2 2\beta
$$

$$
m_{H^\pm}^2 = m_A^2 + m_W^2(2 + \delta_t) \quad (3.13)
$$

Moreover, if we neglect $\delta_t$ the rotation matrix $R_\alpha$ from $H_1, H_2$ to $h, H$ is trivial
leading to $\alpha = \beta - \pi/2$ which means that the coupling $g_{Zhh}$ is the SM one, while
$g_{ZHH} = 0$; therefore $h$ behaves always like a SM Higgs and $H$ plays no role
in electroweak symmetry breaking. This leads to a different phenomenology
from that of the MSSM as we will describe in the next section. Of course the
size of the effect of $\delta_t$ depends on the scale of the messenger masses and it
can be easily kept under control if we impose an upper bound on the absolute
value of radiative corrections by mean of a corresponding upper bound on
$M$. For instance if we want to have it below 30% we must impose for $M$ an
upper bound $\sim 10^7$ GeV, for TeV squark masses. For the light Higgs mass
A new gauge mediation theory 661

this represents a 16% effect for the case of small tan $\beta$ (no effect for the case of large tan $\beta$ of course) which would mean a tree level mass around 76 GeV. Still this result is very different from the MSSM one for which obviously the tree level mass for large $m_A$ and tan $\beta=1$ is zero! In our case a tree-level Higgs mass of around 76 GeV for tan $\beta = 1$ equals that in the MSSM for tan $\beta \simeq 3.4$, which is not excluded by experimental data since one still has to include the one-loop radiative corrections from the squark mass scale to the electroweak scale and the threshold effects at the squark mass. In our case we will have more room if either M is smaller than $10^7$ GeV and/or tan $\beta$ is larger than one.

The spectrum from Eq. (3.13) is modified by radiative corrections in the non-supersymmetric theory below $m_Q$. At leading order, the mass matrix for the neutral states can be written as:

$$M = \begin{pmatrix}
    m_Z^2 \cos^2 \beta + m_A^2 \sin^2 \beta & \left(m_A^2 - m_Z^2 (1 + 2\delta_t)\right) \cos \beta \sin \beta \\
    \left(m_A^2 - m_Z^2 (1 + 2\delta_t)\right) \cos \beta \sin \beta & m_Z^2 \sin^2 \beta + m_A^2 \cos^2 \beta + \epsilon_t
\end{pmatrix}$$

(3.14)

where $\epsilon_t$ is the leading one loop correction which can be written as:

$$\epsilon_t = \frac{3m_t^4}{4\pi^2 v^2} \left(\log \frac{m_{t_1}^2 + m_{t_2}^2}{m_t^2} + \frac{X_t^2}{2M_S} \left(1 - \frac{X_t^2}{6M_S^2}\right)\right)$$

(3.15)

where $m_{t_{1,2}}^2$ are the two stop masses, $X_t = A_t - \mu / \tan \beta \simeq -\mu / \tan \beta$ is the stop mixing mass parameter, and $M_S \simeq \sqrt{(m_{t_1}^2 + m_{t_2}^2)}/2$ represents the common soft supersymmetry breaking scale. It should be noted that for large $m_A$ and any value of tan $\beta$ this corresponds to the limit of the MSSM at large tan $\beta$ so the Higgs mass in this model is always maximal in the decoupling limit for the given value of $X_t$. In particular for values of tan $\beta \sim O(1)$ we get a mixing $X_t^2 \sim \mu^2$. On the other hand, for values of $m_A$ close to $m_Z$ the value for $m_h$ should be below the present bound on the Higgs mass although such small values should be normally excluded by the electroweak symmetry breaking condition (3.12).

Finally it is easy to see that the “little” fine-tuning in this model is greatly reduced with respect to that of the MSSM in the low tan $\beta$ region. The origin of the “little” fine-tuning in the MSSM is that the Higgs mass only increases logarithmically with the stop mass $m_Q$ while it appears quadratically in the
determination of $m_Z^2$. In fact since the tree-level mass of the Higgs in the MSSM (in the large $m_A$ limit) goes to zero in the limit $\tan \beta \to 1$, in order to cope with the LEP limit on the Higgs mass, one should go to the region of very large values of $m_Q$ and thus to a very severe “little” fine-tuning. This fine-tuning is softened in the region of large $\tan \beta$. Since in our model the tree-level Higgs mass does not depend on $\tan \beta$ (it coincides with the MSSM one in the $\tan \beta \to \infty$ limit) the “little” fine-tuning for any value of $\tan \beta$ coincides with that of the MSSM in the large $\tan \beta$ limit.

4 SOME PHENOMENOLOGICAL ASPECTS

In this section we discuss different phenomenological aspects of the model presented above.

THE GRAVITINO MASS AND GRAVITATIONAL EFFECTS

At the supergravity level, enforcing the cancellation of the cosmological constant leads to the presence of an extra source of supersymmetry breaking through an $F$-term for some chiral field. The goldstino is then a combination of the fermionic partners of this field and the fermionic partner of the $U(1)$ gauge boson with non-vanishing $D$-term. Through the super-Higgs mechanism it is absorbed by the gravitino which gets a mass of order

$$m_{3/2} \simeq \frac{D}{M_{Pl}}$$

(4.1)

where $M_{Pl} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass. We will assume here that $F$ is of the same order as $D$:

$$F \simeq m_{3/2} M_{Pl} \simeq D$$

(4.2)

In fact, the relative sizes of $D$ and $F$ are very model dependent. While normally $F \gtrsim D$, models with $F \ll D$ can also be engineered [21].

The additional $F$-breaking source can be arranged to reside in the hidden sector. It will be communicated only through gravitational interactions to the observable sector. The associated effects qualitatively differ from those from $D$-terms by the fact that they could break $R$-symmetry, generate Majorana masses and are not compelled to be flavor independent. This implies
in particular that they must be subleading with respect to the gauge mediating mechanism presented in this paper in order to not re-introduce a flavor problem. As the typical size of soft-mass terms induced by gravitational interactions is \( \sim F/M_{Pl} \), if Eq. (4.2) holds the condition for gravity mediated contributions to be subleading is that \( M \lesssim (\alpha/4\pi) M_{Pl} \sim 10^{16} \text{ GeV} \) which in turn implies the bound \( m_{3/2} \lesssim 1 \text{ TeV} \). However if \( F \ll D \) then it may be possible to suppress gravitational interactions even with \( m_{3/2} \gtrsim 1 \text{ TeV} \).

Note also that anomaly-mediated supersymmetry breaking (AMSB) \cite{22} is subleading with respect to the soft-breaking induced by the gauge mediation mechanism, provided that \( M \ll M_{Pl} \). For instance, the gaugino Majorana masses \( M_a \) induced by anomaly mediation are

\[
M_a \sim \frac{\alpha_a}{4\pi} m_{3/2} \sim \left[ \frac{M}{M_{Pl}} \right] m_a^D \tag{4.3}
\]

and similarly for the squark and Higgs masses.

Fixing \( m_a^D \sim 1 \text{ TeV} \) in Eq. (2.5) one can write \( D \) as a function of \( M \) as

\[
D \sim M \times 10^5 \text{ GeV} \tag{4.4}
\]

and plugging it into (4.1) one obtains

\[
m_{3/2} \sim \left[ \frac{M}{10^9 \text{ GeV}} \right] \text{ keV} \tag{4.5}
\]

Preferred ranges for values of the scales \( M \) and \( \sqrt{D} \) can then be obtained if one imposes certain cosmological constraints on the gravitino mass \cite{23}. For instance as a warm dark matter component a light gravitino mass should lie below \( \sim 16 \text{ eV} \) \cite{24} in order not to have unwanted cosmological consequences, which translates into \( M \lesssim 10^7 \text{ GeV} \). Furthermore as a cold dark matter component the mass of the gravitino should be above few keV, which translates into \( M \gtrsim 10^9 \text{ GeV} \), and the reheating temperature has to be such that the density of gravitinos does not overclose the universe \cite{24}.

**Collider phenomenology**

One of the main differences between the usual gauge mediation and the models we have studied is the spectrum of supersymmetric particles since in our
case the scalars get masses at three-loops, in a very similar way to theories with *gaugino mediation* [25]. Although the precise spectrum depends on the particular details on every model.

On the other hand there are three different experimental signatures of these models which are quite general. The first one, and common to the usual gauge mediation scenarios, arises when the gravitino is very light (the LSP) and therefore the NLSP could have a decay suppressed by the SUSY breaking scale. In this situation and if the NLSP is charged (a very common case) it could yield a track in the detector before decaying.

The second feature in our models is the $N = 2$ structure of the Higgs sector. As we noticed in section 3, the lightest Higgs $h$ behaves as a SM Higgs with no $\tan \beta$ dependence on its couplings to fermions; therefore its production and decay rates are those typical of the SM rather than the MSSM. On the other hand, the couplings to matter of the heaviest Higgs $H$, the pseudo-scalar $A$ and the charged Higgses $H^\pm$, as well as all self-couplings within the Higgs sector do depend on $\tan \beta$: in principle one could then distinguish these models from others (such as the MSSM or the non-supersymmetric two-Higgs doublet models) by measuring these couplings, although this may be the realm for ILC rather than LHC.

Finally the last signature comes from the $N = 2$ structure of the gauge sector and consequently from the fact that gauginos have their main mass contribution forming a Dirac particle. This translates into having different decay channels of gluinos, which are difficult to measure but, more importantly, into the existence of *three* charginos and *six* neutralinos, the latter being paired into three pseudo-Dirac neutral fermions. Discovery of three charginos should constitute the smoking gun of this scenario.

**Unification**

Let us finish this phenomenological section with some comments on gauge coupling unification. The model as it is, with $N = 2$ gauge sector and a single Higgs hypermultiplet, together with the usual $N = 1$ chiral matter, has the following beta-function coefficients: $\beta_1 = \frac{33}{5}$, $\beta_2 = 3$, $\beta_3 = 0$. Evolution of the three gauge couplings with the previous beta-functions does not lead to unification. However, this can be achieved by adding appropriate extra states [26, 7].
For instance, if one adds to the above spectrum one hypermultiplet with the quantum numbers of the Higgs plus two hypermultiplets with the quantum numbers of a right-handed lepton (unifons) the new beta-function coefficients read:

\[ \beta_1 = \frac{48}{5}, \quad \beta_2 = 4, \quad \beta_3 = 0 \]  

(4.6)

and one recovers the one-loop unification at the MSSM GUT scale \( M_{\text{GUT}} \sim 2 \times 10^{16} \text{ GeV} \). It should be noted that the unification scale is not affected by the messengers, as long as their number is not so large to spoil pertubativity before \( M_{\text{GUT}} \), since they form complete \( SU(5) \) representations. Also since we are supposing that the string scale is large we should not worry about string states contributing to the running of gauge couplings. Moreover, the unifons come in vector-like representations and can be given the desired (supersymmetric) mass.

5 Conclusions

In this paper we have proposed a new model of gauge mediation where supersymmetry is broken by a \( D \)-term expectation value in a secluded local \( U(1) \) sector, thus conserving \( R \)-symmetry in the global limit. The NGMSB model, an alternative to the usual GMSB where supersymmetry is broken by the \( F \)-term of a secluded chiral sector, is easily realized in intersecting brane models of type I string theory [5]. Its main feature is the presence of an extended \( N = 2 \) supersymmetry in the gauge, as well as in the Higgs and messengers sectors. As a result, the transmission of supersymmetry breaking by the hypermultiplet messengers generates Dirac masses for gauginos at the one-loop level. The observable matter sector is contained in chiral \( N = 1 \) multiplets (localized in brane intersections) that get radiative masses at the one-loop order in the effective theory where messengers have been integrated out. The NGMSB model shares some features with the usual GMSB models:

- It solves the supersymmetric flavor problem.

- It provides a common supersymmetry breaking scale in the observable sector, the masses of all supersymmetric particles being proportional to powers of gauge couplings.
• In sensible models, the gravitino is very light and thus a viable candidate to describe the Dark Matter of the Universe.

However, NGMSB departures from usual GMSB in a number of features:

• Gauginos are the heaviest supersymmetric particles while squarks and sleptons are one-loop suppressed with respect to them.

• The gaugino sector contains twice as many degrees of freedom as that of the MSSM assembled into (quasi)-Dirac fermions with Dirac masses. Finding three (instead of two) charginos, and six (instead of four) neutralinos should then be the smoking gun of this class of models at LHC.

• The two Higgs superfields of the MSSM are contained in one hypermultiplet. Thus, the Higgs phenomenology for low tan β at LHC is completely different from that of the MSSM. This alleviates the fine-tuning problem of low tan β and opens up the corresponding window in the supersymmetric parameter space.

There is a number of issues whose detailed analysis was outside the scope of the present paper but that should be worth of further study. On the phenomenological side, one should translate LEP data into bounds on the Higgs masses and work out more in detail the LHC phenomenology of both the scalar Higgs and the chargino and neutralino sectors. Finally at the string theory level, the construction of realistic intersecting brane models realizing the ideas contained in this paper.

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Exclusion plots should look very different from those in the MSSM.
A new gauge mediation theory

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A Tunneling probability density

In this appendix we will first analyze the potential structure of the \( N = 2 \) supersymmetric \( U(1) \) secluded sector \( A^0 \) with gauge coupling \( g_0 \) in the presence of the messenger hypermultiplet \((\Phi, \Phi^c)\) with charges normalized to \( \pm 1 \). We will assume that \( N = 2 \) supersymmetry is broken by the FI parameter \( \xi \). The interaction of the messenger hypermultiplet \((\Phi, \Phi^c)\) with the secluded \( U(1) \) is described by the Lagrangian

\[
\int d^4\theta \left\{ \Phi^\dagger e^{g_0 V^0} \Phi + \Phi^c e^{-g_0 V^0} \Phi^c + \xi V^0 \right\} + \int d^2\theta \Phi^c \left[ M - \sqrt{2} g_0 \chi^0 \right] \Phi + h.c. \]

(A.1)

that gives rise to the scalar potential

\[
V = \frac{D^2}{2g^2} + \left\{ |M - \sqrt{2} \Sigma|^2 + D \right\} |\phi|^2 \\
+ \left\{ |M - \sqrt{2} \Sigma|^2 - D \right\} |\phi^c|^2 + \frac{g^2}{2} (|\phi|^2 + |\phi^c|^2)^2
\]

(A.2)

where we have used in (A.2), to simplify the notation, \( D = g_0^2 \xi \), \( \Sigma = g_0 \Sigma^0 \) and \( g = g_0 \).

The potential (A.2) has two minima:

- A local non-supersymmetric minimum at the origin, \( \phi = \phi^c = 0 \), which is a flat direction along the \( \Sigma \)-field. This minimum has a vacuum energy \( \langle V \rangle = D^2/2g^2 \). The flat direction is lifted by radiative corrections and extra contributions that provide a mass to \( \Sigma \) as in Eq. (2.13)

\[
V_{\text{rad}}(\phi, \phi^c, \Sigma) |_{\phi=\phi^c=0} = m^2 |\Sigma|^2, \quad m^2 \simeq \frac{1}{16\pi^2} \frac{D^2}{M^2}
\]

(A.3)

Using Eq. (A.3) the local supersymmetry breaking minimum is then at the origin

\[
\langle \phi \rangle = \langle \phi^c \rangle = \langle \Sigma \rangle = 0
\]

(A.4)
A global supersymmetric minimum at
\[ \langle \phi \rangle = 0, \quad |\langle \phi^c \rangle|^2 = D/g^2, \quad \langle \text{Re}(\Sigma) \rangle = M/\sqrt{2}, \langle \text{Im}(\Sigma) \rangle = 0 \] (A.5)

with zero vacuum energy \(^{18}\).

For any value of \(\Sigma\) at zero mass, the potential has a minimum at the origin along the \(\phi\)-direction and for values of \(\Sigma\) such that \(|M - \sqrt{2} \Sigma|^2 > D\) the potential also has the minimum (A.5) along the \(\phi^c\) direction. However when \(|M - \sqrt{2} \Sigma|^2 = D\) there is an almost flat direction along \(\phi^c\) at the minimum that becomes a maximum for \(|M - \sqrt{2} \Sigma|^2 < D\). Since the flat direction \(\Sigma\) is lifted only by radiative corrections [see Eq.(A.3)] the path followed by the instanton which controls the tunneling from the local (A.4) to the global (A.5) minimum goes along the \(\text{Re}(\Sigma)\)-direction, from the origin up to values of \(\text{Re}(\Sigma)\), \(\text{Re}(\Sigma) = M/\sqrt{2} + \mathcal{O}(D^{1/2})\) where the potential becomes unstable along the \(\phi^c\) direction. Since the instanton has to jump over the path of least slope along \(\text{Re}(\Sigma)\), up to values of \(\text{Re}(\Sigma)\) where the potential becomes unstable, the tunneling problem can be very well approximated by a one-dimensional problem where the instanton field that satisfies the euclidean equation of motion [27]
\[ \frac{d^2 \varphi}{dr^2} + \frac{3 d\varphi}{r dr} = V'(\varphi) \] (A.6)
is \(\varphi \equiv \text{Re}(\Sigma)/\sqrt{2}\). The variable \(r = \sqrt{t^2 + \vec{x}^2}\) in (A.6) makes the \(O(4)\) symmetry of the solution manifest with the boundary conditions \(\varphi \to 0\) at \(r \to \infty\) and \(d\varphi/dr = 0\) at \(r = 0\).

In order to make an estimate of the probability of bubble formation by tunneling from the local (A.4) to the global (A.5) we will follow Ref. [28]. First of all notice that the depth of the global minimum \(\sim D^2/2g^2\) is much larger

\(^{18}\)The general features of our analysis should remain true after introducing the interaction of the messengers with the observable gauge sector. In particular, when the gauge sector \(A^4\) is introduced, this minimum is uplifted and the \(|\langle \phi^c \rangle|^2\) VEV is shifted by the observable \(D\)-term \(g_4^2/2 (\phi^c T^A \phi^c)^1 \), while the \(\Sigma\) direction should have components along the \(\Sigma^A\)-fields of the observable sector. However radiative corrections (2.13) provide masses proportional to \(\text{tr} T^A T^B + \text{tr} T^A \phi^c T^B\). Given that \(T_{\phi,\phi^c}^{U(1)} \propto 1\) and \(\text{tr} T^Y\phi^c = 0\) there is no mixing between the secluded \(U(1)\) and \(U(1)\) and all flat directions are lifted. Since we intend to do only a semi-quantitative analysis of the tunneling probability from the local (A.4) to the global (A.5) minimum we do not include in this appendix gauge interactions from the observable sector.
than the heigh of the barrier $\sim D^2/32\pi^2$ so that the bubble solution is outside the domain of validity of the thin wall approximation. Second, to compute the tunneling probability one can disregard the details of the behaviour of the potential $V(\varphi)$ at $\varphi \gg \varphi_1$ where $V(\varphi_1) \simeq V(0)$. In particular considering a potential $V_{\text{app}}(\varphi)$ that approximates $V(\varphi)$ for $\varphi \lesssim \varphi_1$ is usually a sufficiently good approximation [28]. In our case $\varphi_1 \simeq M/2$ and the simplest such potential is

$$V_{\text{app}}(\varphi) = \frac{1}{2} m^2 \varphi^2 - \frac{\delta}{3} \varphi^3$$

(A.7)

where the mass term is given by Eq. (A.3) and $\delta$ is chosen such that $V(\varphi_1) \simeq 0$, i.e.

$$\delta \simeq \frac{3m^2}{2\varphi_1} \simeq \frac{3}{16\pi^2} \frac{D^2}{M^3}$$

(A.8)

The euclidean action $S_4$ corresponding to the solution $\varphi$ of Eq. (A.6) is given by

$$S_4 \sim 2\left(\frac{10M}{3m}\right)^2 \sim \left(\frac{\kappa_4 M^2}{D}\right)^2$$

(A.9)

where $\kappa_4 \simeq 59$.

At finite temperature one should replace $V(\varphi)$ by $V(\varphi, T)$ where finite temperature effects have been considered. In our case the dominant thermal effects in Eq. (A.7) can be encoded in the thermal mass

$$m^2(T) \simeq \frac{1}{16\pi^2} \left(\frac{D^2}{M^2} + aT^2\right)$$

(A.10)

where $a$ is some order one coefficient. At finite temperature the $O(4)$ symmetric solution of (A.6) should be replaced by the $O(3)$ symmetric one that satisfies the equation [28]

$$\frac{d^2 \varphi}{dr^2} + 2\frac{d\varphi}{r \, dr} = V'(\varphi, T)$$

(A.11)

where now $r^2 = \bar{x}^2$. For the potential (A.7) it is found [28]

$$\frac{S_3}{T} \sim \frac{5}{3} \frac{M^3 m^3(T)}{m^4 T} \gtrsim \left(\frac{\kappa_3 M^2}{D}\right)^2$$

(A.12)
where $\kappa_3 \simeq 45$ and the last inequality holds for any temperature. The tunneling probability per unit time per unit volume is then

$$\mathcal{P} \sim e^{-\mathcal{B}}$$

(A.13)

where $\mathcal{B} = \min [S_4, S_3(T)/T]$ at any temperature. This justifies our Eq. (2.3).

Of course we do expect expressions (A.9) and (A.12), based on the approximated potential (A.7) to be accurate only within factors of order a few. In fact if we approximate the potential by a different one with a negative quartic term, $V_{\text{app}} = 1/2 m^2 \varphi^2 - 1/4 \lambda \varphi^4$ as in Ref. [28], and fix $\lambda$ such that $V_{\text{app}}(\varphi_1) \simeq 0$ the corresponding expressions (A.9) and (A.12) have coefficients $\kappa_4 \simeq 23$ and $\kappa_3 \simeq 19$ respectively, thus related to those in (A.9) and (A.12) by order two factors. Given that in all cases the euclidean actions are dominated by the large $M^4/D^2$-factors, these results are very consistent to each other.

References


[3] For a review see e. g.: A. M. Uranga, Class. Quant. Grav. 22 (2005) S41; and references therein.


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