Analytical Study of the Dust Acoustic Waves in a Magnetized Dusty Plasma with Many Different Dust Grains

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Abstract

The nonlinear dust waves in a magnetized dusty plasma with many different dust grains are analytically investigated. New analytical solutions for the governing equation of this system have been obtained for the dust acoustic waves in a dusty plasma for the first time. We derive exact mathematical expressions for the general case of the nonlinear dust waves in magnetized dusty plasma which contains different dust grains.

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1 Introduction

Nowadays, there is a growing interest in the study of different types of collective process in dust plasma and other nonlinear physics.[1-5] It has been shown, both theoretically and experimentally, that the presence of extremely massive and highly charged dust grains modifies the existing plasma wave spectra.[2] Motivated by some theoretical and experimental studies,[3-6] some authors have studied the dust acoustic solitary structure in a dust plasma model consisting of negatively charged dust fluid and isothermal or non-isothermal ions, and also non-thermal dust.[7,8] Recently, Ma et al.,[9] Xie et al.[10,11,12] and Chen et al.[13] have considered the effects for the dust charge variation and have shown the existence of dust acoustic solitons. Recent numerical simulation studies [14,15] on linear and nonlinear dust acoustic waves exhibit a significant amount of ion trapping in the wave potential. Clearly, there is a departure from the Boltzmann ion distribution and one encounters vortex-like ion distribution in phase space. Y.V. Kartashov et al.[16] have investigated the
vortex solitons supported by azimuthally modulated lattices and reveal how the global lattice discrete symmetry has fundamental implications on the possible topological charges of solitons. Duan et al.[17-19] have investigated the nonlinear dust acoustic waves in dusty plasmas with many different dust grain, and have shown that the nonlinear dust acoustic wave can be described by the modified Korteweg-de Vries equation, the modified Kadomtsev-Petviashvili equation and the Zakharov-Kuznetsov equation, respectively. Lin. et al.[20-23] have analytically investigated the nonlinear waves in nonlinear physics and the nonlinear Debye screening in plasmas by using the formally variable separation approach[24-26]. These theoretical results have established scientific values and application prospects for the fundamental research in dusty plasmas.

In this paper, We reconsider the dust size distribution for dust acoustic wave in a magnetized dusty plasma with many different dust grains.[17] We have fruitfully investigated the solutions of the nonlinear dust waves in dusty plasma by using the formally variable separation approach. New analytical solutions for the governing equation of this system, avoiding the reductive perturbation technique, have been obtained for nonlinear dust waves in magnetized dusty plasma for the first time. We derive exact mathematical expressions for the nonlinear dust waves in a magnetized dusty plasma with many different dust grains. It is found that the equations of the motion for this system due to the formally variable separation approach causes the nonlinear dust waves to manifold describe and new kinds of the analytical solutions for the nonlinear dust waves are produced. These new analytical solutions, considering the formally variable separation and many different dust grains, generate an intense analytical characteristic along the mathematical approach.

2 Mathematical Formalism

We consider the three-component dusty plasma consisting of massive, negatively charged dusty fluid, Boltzmann distributed electrons, and ions in the presence of an external static magnetic field(along the z direction). The equations of motion for this system are[17]

$$\frac{\partial n_{dj}}{\partial t} + \frac{\partial}{\partial x}(n_{dj} u_{dj}) + \frac{\partial}{\partial y}(n_{dj} v_{dj}) + \frac{\partial}{\partial z}(n_{dj} w_{dj}) = 0,$$

$$\frac{\partial u_{dj}}{\partial t} + u_{dj} \frac{\partial u_{dj}}{\partial x} + v_{dj} \frac{\partial u_{dj}}{\partial y} + w_{dj} \frac{\partial u_{dj}}{\partial z} = \frac{Z_{dj}}{m_{dj}} \frac{\partial \phi}{\partial x} + \omega_{cd} \frac{Z_{dj}}{m_{dj}} v_{dj},$$

$$\frac{\partial v_{dj}}{\partial t} + u_{dj} \frac{\partial v_{dj}}{\partial x} + v_{dj} \frac{\partial v_{dj}}{\partial y} + w_{dj} \frac{\partial v_{dj}}{\partial z} = \frac{Z_{dj}}{m_{dj}} \frac{\partial \phi}{\partial y} - \omega_{cd} \frac{Z_{dj}}{m_{dj}} u_{dj},$$

(2.1) (2.2) (2.3)
formally variable separation equations are

\begin{align}
\frac{\partial w_{d_j}}{\partial t} + u_{d_j} \frac{\partial w_{d_j}}{\partial x} + v_{d_j} \frac{\partial w_{d_j}}{\partial y} + w_{d_j} \frac{\partial w_{d_j}}{\partial z} &= \frac{Z_{d_j}}{m_{d_j}} \frac{\partial \phi}{\partial x},
\end{align}

(2.4)

\begin{align}
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} &= \sum_{j=1}^{N} n_{d_j} Z_{d_j} + n_e - n_i,
\end{align}

(2.5)

where \( n_i = \mu \exp(-s \phi), n_e = \nu \exp(\beta s \phi) (\beta = T_i/T_e, s = 1/(\mu + \nu \beta)) \). \( n_{d_j}, u_{d_j}, v_{d_j} \) and \( w_{d_j} \) are the number density, and the velocities in the \( x-, y- \) and \( z- \) directions, respectively. \( \phi \) is electrical potential.

In this paper, we extend the formally variable separation approach [24–26] to the governing equations for this system. Some new analytical solutions for dust acoustic waves in magnetized dusty plasma are also obtained. The formally variable separation equations are

\begin{align}
\varphi_t &= K_1(\varphi), \quad \varphi_x = K_2(\varphi), \quad \varphi_y = K_3(\varphi), \quad \varphi_z = K_4(\varphi),
\end{align}

(2.6)

where \( \varphi = \varphi(t, x, y, z) \) is a scalar function of \( |t, x, y, z|, K_i(\varphi)(i = 1, 2, 3, 4) \) are functions of \( \varphi \). The sole possible solutions of the compatible nature condition \([K_1, K_2] = 0, [K_1, K_3] = 0, [K_1, K_4] = 0, [K_2, K_3] = 0, [K_2, K_4] = 0, [K_3, K_4] = 0 \) are

\begin{align}
K_1 &= \alpha_1 K(\varphi), \quad K_2 = \alpha_2 K(\varphi), \quad K_3 = \alpha_3 K(\varphi), \quad K_4 = \alpha_4 K(\varphi),
\end{align}

(2.7)

where \( \alpha_1, \alpha_2, \alpha_3, \alpha_4 \) are constants.

We consider the nonlinear dust acoustic waves as some analytical functions, which contact with the formally variable as the physical quantities depending on \( \varphi \),

\begin{align}
\phi(t, x, y, z) &= \Phi(\varphi), \quad n_{d_j}(t, x, y, z) = N_{d_j}(\varphi), \quad u_{d_j}(t, x, y, z) = U_{d_j}(\varphi),
\end{align}

(2.8)

\begin{align}
v_{d_j}(t, x, y, z) &= V_{d_j}(\varphi), \quad w_{d_j}(t, x, y, z) = W_{d_j}(\varphi).
\end{align}

Substituting Eqs. (2.6) – (2.8) to Eqs. (2.1) – (2.5), the functions \( K, \Phi, N_{d_j}, U_{d_j}, V_{d_j}, W_{d_j} \) satisfy the following differential equations:

\begin{align}
\alpha_1 N_{d_j}'(\varphi) + \alpha_2 (N_{d_j}(\varphi) U_{d_j}'(\varphi) + U_{d_j}(\varphi) N_{d_j}'(\varphi)) + \alpha_3 (N_{d_j}(\varphi) V_{d_j}'(\varphi) + V_{d_j}(\varphi) N_{d_j}'(\varphi)) + \alpha_4 (N_{d_j}(\varphi) W_{d_j}'(\varphi) + W_{d_j}(\varphi) N_{d_j}'(\varphi)) &= 0,
\end{align}

(2.9)

\begin{align}
\left[ \alpha_1 + \alpha_2 U_{d_j}(\varphi) + \alpha_3 V_{d_j}(\varphi) + \alpha_4 W_{d_j}(\varphi) \right] U_{d_j}'(\varphi) &= \alpha_2 \frac{Z_{d_j}}{m_{d_j}} \Phi'(\varphi) + \omega_{cd} \frac{Z_{d_j}}{m_{d_j} K(\varphi)} V_{d_j}(\varphi),
\end{align}

(2.10)

\begin{align}
\left[ \alpha_1 + \alpha_2 U_{d_j}(\varphi) + \alpha_3 V_{d_j}(\varphi) + \alpha_4 W_{d_j}(\varphi) \right] V_{d_j}'(\varphi) &= \alpha_3 \frac{Z_{d_j}}{m_{d_j}} \Phi'(\varphi) - \omega_{cd} \frac{Z_{d_j}}{m_{d_j} K(\varphi)} U_{d_j}(\varphi),
\end{align}

(2.11)
where $C$ easily integrated and one finds

$$\int \frac{d\Phi}{\pm 2/\left(\alpha_2^2 + \alpha_3^2 + \alpha_4^2\right) f \left[\sum_{j=1}^{N} N_{d_j}(\Phi) Z_{d_j} + n_e(\Phi) - n_i(\Phi)\right] d\Phi + C'}{2} = \int \frac{1}{K(\varphi)} d\varphi,$$

(2.13)

For such a case, from $d\varphi = \varphi dt + \varphi_x dx + \varphi_y dy + \varphi_z dz$ the function $\Phi$ is easily integrated and one finds

$$\int \frac{d\Phi}{\pm 2/\left(\alpha_2^2 + \alpha_3^2 + \alpha_4^2\right) f \left[\sum_{j=1}^{N} N_{d_j}(\Phi) Z_{d_j} + n_e(\Phi) - n_i(\Phi)\right] d\Phi + C'}{2} = \alpha_1 t + \alpha_2 x + \alpha_3 y + \alpha_4 z + E.$$

Here the above equation bring into playing a decisive action for the analytical investigation of the nonlinear dust acoustic waves in magnetized dusty plasma. From Eqs.(2.9) – (2.12), we obtain the following analytical results:

$$N_{d_j}(\varphi) = \frac{A_{d_j}}{\alpha_1 + \alpha_2 U_{d_j}(\varphi) + \alpha_3 V_{d_j}(\varphi) + \alpha_4 W_{d_j}(\varphi)},$$

(2.15)

$$\Phi'(\varphi)[\alpha_3 dU_{d_j}(\varphi) - \alpha_2 dV_{d_j}(\varphi)] = \frac{\omega_{cd}}{K(\varphi)} \left[V_{d_j}(\varphi) dV_{d_j}(\varphi) + U_{d_j}(\varphi) dU_{d_j}(\varphi)\right],$$

(2.16)

for the nonlinear dust acoustic waves in a magnetized dusty plasma with many different dust grains.

For such a case, from Eq. (2.16), we obtain the following analytical results:

$$-\alpha_2 U_{d_j}(\varphi) = \alpha_3 V_{d_j}(\varphi).$$

(2.17)

By using Eqs.(2.15) – (2.17), $d\Phi = d\Phi$, $dn_{d_j} = dN_{d_j}$, $du_{d_j} = dU_{d_j}$, $dv_{d_j} = dV_{d_j}$, and $dw_{d_j} = dW_{d_j}$, we can determine the mathematical characteristics in Eqs.(2.9) – (2.13), and get some general exact analytical solutions:

$$n_{d_j}(t, x, y, z) = N_{d_j}(\varphi) = \pm \frac{A_{d_j}}{\sqrt{\alpha_1^2 + 2\alpha_4\left(\frac{\alpha_2 Z_{d_j}}{m_{d_j}} \phi + C_{d_j}\right)}},$$

(2.18)

$$u_{d_j}(t, x, y, z) = U_{d_j}(\varphi)$$

$$= \pm \left(\frac{2\alpha_3^2}{\omega_{cd}^2 \left(\alpha_2^2 + \alpha_3^2 + \alpha_4^2\right)}\right)$$

(2.19)

$$\sum_{j=1}^{N} \pm \frac{A_{d_j} m_{d_j}}{\alpha_1^2} \sqrt{\alpha_1^2 + 2\alpha_4\left(\frac{\alpha_2 Z_{d_j}}{m_{d_j}} \phi + C_{d_j}\right)} + \frac{\nu}{\beta s} e^{\beta \phi} + \frac{\mu}{s} e^{-\mu \phi} + C'\right)^{\frac{1}{2}},$$
Analytical study of the dust acoustic waves

$v_{dj}(t, x, y, z) = V_{dj}(\varphi)$

$$= \pm \left\{ \frac{2\alpha_2^2}{\omega_{cd}^2 (\alpha_2^2 + \alpha_3^2 + \alpha_4^2)} \right\} \times$$

$$\left[ \sum_{j=1}^{N} \frac{A_{dj} m_{dj}}{\alpha_4^2} \sqrt{\alpha_1^2 + 2\alpha_4 \left( \frac{\alpha_4 Z_{dj}}{m_{dj}} \phi + C_{dj} \right) + \frac{\nu}{\beta s} e^{\beta s \phi} + \frac{\mu}{s} e^{-\beta s \phi} + C'} \right]^\frac{1}{2},$$

$$w_{dj}(t, x, y, z) = W_{dj}(\varphi) = \frac{1}{\alpha_4} \left\{ -\alpha_1 \pm \sqrt{\alpha_1^2 + 2\alpha_4 \left( \frac{\alpha_4 Z_{dj}}{m_{dj}} \phi + C_{dj} \right)} \right\},$$

$$\int \frac{d\phi}{\pm \left\{ 2/(\alpha_2^2 + \alpha_3^2 + \alpha_4^2) \sum_{j=1}^{N} \frac{A_{dj} m_{dj}}{\alpha_4^2} \sqrt{\alpha_1^2 + 2\alpha_4 \left( \frac{\alpha_4 Z_{dj}}{m_{dj}} \phi + C_{dj} \right) + \frac{\nu}{\beta s} e^{\beta s \phi} + \frac{\mu}{s} e^{-\beta s \phi} + C'} \right\}^\frac{1}{2}}$$

$$= \alpha_1 t + \alpha_2 x + \alpha_3 y + \alpha_4 z + E,$$

where

$$A_{dj} = n_{dj o} (\alpha_1 + \alpha_4 w_{dj o}), \quad C_{dj} = \alpha_1 w_{dj o} + \frac{1}{2} \alpha_4 w_{dj o}^2 - \frac{\alpha_4 Z_{dj}}{m_{dj}} \phi_0,$$

$E$ is defined by

$$E = \left( \int \frac{d\phi}{\pm \left\{ 2/(\alpha_2^2 + \alpha_3^2 + \alpha_4^2) \sum_{j=1}^{N} \frac{A_{dj} m_{dj}}{\alpha_4^2} \sqrt{\alpha_1^2 + 2\alpha_4 \left( \frac{\alpha_4 Z_{dj}}{m_{dj}} \phi + C_{dj} \right) + \frac{\nu}{\beta s} e^{\beta s \phi} + \frac{\mu}{s} e^{-\beta s \phi} + C'} \right\}^\frac{1}{2}} \right)^{\phi_0 \rightarrow \phi_0}$$

$$- \alpha_1 t_0 - \alpha_2 x_0 - \alpha_3 y_0 - \alpha_4 z_0.$$

The above constants can be fully confirmed by using the initial conditions of the system.

The mathematical structures of the nonlinear dust acoustic waves in a magnetized dusty plasma with many different dust grains have been described by the analytical solutions, which concerns the formally variable separation approach for the equations of systems motion. Equations (2.18) – (2.22) are the exact analytical solutions, valid for the nonlinear dust acoustic waves in a magnetized dusty plasma with many different dust grains, in which the number density and the velocities of dust particles fully possess the variational character of the exponential function on the electrical potential. It is found that the seeking solution of the electrical potential is the linchpin of the investigation of the nonlinear dust acoustic waves in dusty plasma. By using Eq. (2.22), we get the exact analytical description for the electrical potential $\phi$ in a magnetized dusty plasma. Then the exact analytical solutions have been obtained with respect to the nonlinear acoustic dust waves in a magnetized dusty plasma with many different dust grains.

The system of Eqs. (2.18) – (2.22) describes the nonlinear dust acoustic waves in a magnetized dusty plasma with many different dust grains for the
general cases more accurately. The results will be an important academic foundation for the numerical simulation studies on the nonlinear dust acoustic waves for magnetized dusty plasma.

3 The symmetry distributions of the nonlinear dust waves

We consider the general exact analytical solution (2.22) for the electrical potential $\phi$ of nonlinear dust waves in a magnetized dusty plasma with many different dust grains

$$F(\phi, t, x, y, z) = -\alpha_1 t - \alpha_2 x - \alpha_3 y - \alpha_4 z - E +$$

$$\int \pm \frac{2}{(\alpha_2^2 + \alpha_3^2 + \alpha_4^2)^{\frac{1}{2}}} \left[ \sum_{j=1}^{N} \left( \frac{A_{\phi_{mj}}}{\alpha_1^2} \sqrt{\alpha_1^2 + 2\alpha_4 (\frac{\alpha_4 Z_{\phi_j}}{m_{dj}} \phi + C_{dj}) + \frac{\nu}{\beta s} e^{\beta s \phi} + \frac{\mu}{s} e^{-s \phi} + C'} \right) \right]^{\frac{1}{2}} = 0. \tag{3.23}$$

From eq. (3.23), we obtain the following analytical results:

$$\phi_i = \pm \left( \frac{2\alpha_i^2}{\alpha_2^2 + \alpha_3^2 + \alpha_4^2} \sum_{j=1}^{N} \frac{A_{\phi_{mj}}}{\alpha_1^2} \sqrt{\alpha_1^2 + 2\alpha_4 (\frac{\alpha_4 Z_{\phi_j}}{m_{dj}} \phi + C_{dj}) + \frac{\nu}{\beta s} e^{\beta s \phi} + \frac{\mu}{s} e^{-s \phi} + C'} \right)^{\frac{1}{2}}, \tag{3.24}$$

$$\left( \phi_i = \phi_t, \phi_x, \phi_y, \phi_z; \alpha_i = \alpha_1, \alpha_2, \alpha_3, \alpha_4 \right)$$

for the nonlinear dust waves in a magnetized dusty plasma with many different dust grains. It is shown that the variational ratio of the electrical potential, for the time and space, have fully possessed the same mathematical properties containing the some different constants for the nonlinear dust acoustic waves in magnetized dusty plasma.

The above results gives the symmetry distributions of the nonlinear dust waves in a magnetized dusty plasma with many different dust grains as the following relations:

$$\phi_i = \pm \frac{\alpha_i^2}{\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2} \left[ (\phi_t)^2 + (\phi_x)^2 + (\phi_y)^2 + (\phi_z)^2 \right] \frac{\partial \phi}{\partial t}, \tag{3.25}$$

$$\left( \phi_i = \phi_t, \phi_x, \phi_y, \phi_z; \alpha_i = \alpha_1, \alpha_2, \alpha_3, \alpha_4; i = t, x, y, z \right)$$

$$\phi_i \frac{\partial F}{\partial x} \frac{\partial F}{\partial y} \frac{\partial F}{\partial z} = \phi_x \frac{\partial F}{\partial y} \frac{\partial F}{\partial z} \frac{\partial F}{\partial t} = \phi_y \frac{\partial F}{\partial z} \frac{\partial F}{\partial t} \frac{\partial F}{\partial x} = \phi_z \frac{\partial F}{\partial t} \frac{\partial F}{\partial x} \frac{\partial F}{\partial y}. \tag{3.26}$$

The system of Eqs.(3.24)~(3.26) describes the mathematical characteristic of the symmetric distributions for the nonlinear dust acoustic waves in a magnetized dusty plasma with many different dust grains for the general cases accurately.
4 Conclusion

We have fruitfully investigated the nonlinear dust waves in a magnetized dusty plasma with many different dust grains. New analytical solutions for the governing equation of this system have been obtained for the nonlinear dust waves in magnetized dusty plasma for the first time. We have accurately solved the motion equation of the nonlinear problem of the system, when the formally variable separation approach are considered. We derive exact mathematical descriptions for the nonlinear dust waves in a magnetized dusty plasma with many different dust grains. It is found that the equations of the motion for this system due to the formally variable separation approach causes the nonlinear dust waves to manifold describe and new kinds of the analytical solutions for the nonlinear dust waves are produced. These new analytical solutions, including the mathematical symmetry and considering many different dust grains and formally variable separation, generate an intense analytical characteristic along the mathematical approach. We regard it as the generalization of the nonlinear dust waves in a magnetized dusty plasma with many different dust grains. This can be important in the investigation of plasma wave spectra for the fundamental characteristic in magnetized dusty plasma by using the above new mathematical results.

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References


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