

Inverse Quantum Mechanics of the Hydrogen Atom:

A General Solution

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Abstract

The possible existence of fractional quantum states in the hydrogen atom has been debated since the advent of quantum theory in 1924. Interest in the topic has intensified recently due to the claimed experimental findings of Randell Mills at Blacklight Power, Inc., Cranbury, New Jersey of 137 inverse principal quantum levels, which he terms the “hydrino” state of hydrogen. This paper will show that the general wave equation predicts exactly that number of reciprocal energy states.

Keywords: hydrinos, wave-to-particle conversion, inverse quantum states

INTRODUCTION

Intensive laboratory research over much of the past decade at the Technical University of Eindhoven¹ and at Blacklight Power, see Ref. [3] for a review of the several publications in refereed journals, on what has come to be known as the “hydrino” state of hydrogen has sent theorists scurrying to explain the experimental spectroscopic observations on the basis of known and trusted physical laws. Jan Naudts of the University of Antwerp, for example, offers the argument that the Klein-Gordon equation of relativistic quantum mechanics allows an inverse quantum state.² Andreas Rathke of the European Space Agency’s Advanced Concepts Team insists that the wave equation cannot produce square integrable fractional quantum states.³ Randell Mills of Blacklight Power employs a theory based on the Bohr concept of a particle in orbit around a much heavier central

mass.⁴ Based on the results of the following analysis, Naudts and Mills appear to be right.

PRESENTATION

Poincaré's four-dimensional potential equation

$$\square \Psi = 0 \quad (1)$$

is normally written in rectangular coordinates x, y, z , and w as

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} + \frac{\partial^2 \Psi}{\partial w^2} = 0 \quad (2)$$

We define the generating function Ψ as

$$\Psi(x, y, z, w) = \Psi(x, y, z, \pm ict) \quad (3)$$

In this case we must have

$$\frac{\partial^2 \Psi}{(\partial w)^2} = \frac{\partial^2 \Psi}{[\partial(\pm ict)]^2} \quad (4)$$

which yields

$$\frac{\partial^2 \Psi}{\partial w^2} = -\frac{1}{c^2} \frac{\partial^2 \Psi}{\partial t^2} \quad (5)$$

Substitution of (5) in (2) yields

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 \Psi}{\partial t^2} \quad (6)$$

We see that the function Ψ is a general wave function of some kind. The three real space variables x, y, z describe a three-dimensional entity, and the variable t describes its behavior in time. The absence of cross derivative terms in the equation requires the existence of an orthogonal geometry.

In order to analyze the motion of translation of mass, we write the wave function

$$\Psi = XYZT \quad (7)$$

where each factor in the product is a function of the given variable only. The use of (7) in (6) provides

$$\frac{1}{X} \frac{\partial^2 X}{\partial x^2} + \frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} = \frac{1}{c^2 T} \frac{\partial^2 T}{\partial t^2} \quad (8)$$

Each term in (8) is a function of one variable only. Since the equation must be identically satisfied for all values of the variables, each term is a constant. Since we must assume the direction of any linear motion to be arbitrary, we assume it to be in the direction of the z axis. Then we have

$$\frac{1}{Z} \frac{d^2 Z}{dz^2} = \pm b^2 \quad (9)$$

where the ordinary derivative notation is used since only one variable appears. In the event that b is not zero, solutions are

$$Z = e^{\pm bz} \quad (10)$$

and

$$Z = e^{\pm ibz} \quad (11)$$

except for the presence of multiplying constants. Equation (10) is that of a function which becomes infinite or zero with increasing z . Since matter does neither, the solution is rejected. Equation (11) is that of a wave extending to infinity on the z axis. In that case, we reject (11) also. The only remaining possibility is given by

$$\frac{d^2 Z}{dz^2} = 0 \quad (12)$$

Then we find

$$Z = C_1 z + C_2 \quad (13)$$

where C_1 and C_2 are arbitrary constants of integration. With the proper choice on these, we have

$$Z = z \quad (14)$$

As a result of the given solution, we conclude that a particle can exist at any point on the z axis. We interpret (14) to represent the instantaneous location of a particle in motion in the direction of z .

The analysis of orbital motion requires a transformation to a more suitable system of coordinates. In the case of an electron in the hydrogen atom, we have a cyclic motion of the nature of a stationary state. This requirement must be imposed on the right member of (8). Then we write

$$\frac{1}{T} \frac{d^2 T}{dt^2} = \pm \alpha^2 \quad (15)$$

Since c , the speed of light, is a constant, there is no need to include it in (15). Then for the description of cyclic states, we write

$$\frac{d^2 T}{dt^2} + \alpha^2 T = 0 \quad (16)$$

Solutions are provided by

$$T = e^{\pm i\alpha t} \quad (17)$$

except for the multiplying constant. The simplest condition is then

$$\alpha = \omega = 2\pi f \quad (18)$$

in terms of a cyclic frequency, where ω represents the angular frequency of the motion. The general wave function may now be written

$$\Psi = \Psi_o e^{\pm i\omega t} \quad (19)$$

The use of (19) in (6) yields

$$\frac{\partial^2 \Psi_o}{\partial x^2} + \frac{\partial^2 \Psi_o}{\partial y^2} + \frac{\partial^2 \Psi_o}{\partial z^2} + \frac{\omega^2}{c^2} \Psi_o = 0 \quad (20)$$

to represent the general wave equation for stationary states. In terms of spherical coordinates r, θ, ϕ , equation (20) is written

$$\frac{\partial^2 \Psi_o}{\partial r^2} + \frac{2}{r} \frac{\partial \Psi_o}{\partial r} + \left\{ \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \Psi_o}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2 \Psi_o}{\partial \phi^2} \right\} + \frac{\omega^2}{c^2} \Psi_o = 0 \quad (21)$$

Calculation can be eased by use of

$$\mu = \cos \theta \quad (22)$$

whose use in (21) produces

$$\frac{\partial^2 \Psi_o}{\partial r^2} + \frac{2}{r} \frac{\partial \Psi_o}{\partial r} + \frac{1}{r^2} \left\{ \frac{\partial}{\partial \mu} \left[(1 - \mu^2) \frac{\partial \Psi_o}{\partial \mu} \right] + \frac{1}{1 - \mu^2} \frac{\partial^2 \Psi_o}{\partial \phi^2} \right\} + \frac{\omega^2}{c^2} \Psi_o = 0 \quad (23)$$

If in equation (23) the transformation

$$\Psi_o = R\Theta\Phi \quad (24)$$

is applied with the understanding that each factor in the product is a function of the corresponding variable only, a separation can be made in the form

$$\frac{1}{R} \frac{\partial^2 R}{\partial r^2} + \frac{2}{rR} \frac{\partial R}{\partial r} + \frac{1}{r^2} \left\{ \frac{1}{\Theta} \frac{\partial}{\partial \mu} \left[(1 - \mu^2) \frac{\partial \Theta}{\partial \mu} \right] + \frac{1}{(1 - \mu^2)\Phi} \frac{\partial^2 \Phi}{\partial \phi^2} \right\} + \frac{\omega^2}{c^2} = 0 \quad (25)$$

The term containing the equatorial angle ϕ can be separated and shown to be a constant

$$\frac{1}{\Phi} \frac{d^2 \Phi}{d\phi^2} = -m^2 \quad (26)$$

resulting in the differential equation

$$\frac{d^2\Phi}{d\phi^2} + m^2\Phi = 0 \quad (27)$$

Solutions of equation (27) are found by

$$\Phi = e^{\pm im\phi} \quad (28)$$

except for a multiplying constant, where m is the orbital magnetic quantum number with values $0, \pm 1, \pm 2, \pm 3, \pm \dots$

The quantity within braces in equation (25) can now be written as

$$\frac{1}{\Theta} \frac{d}{d\mu} \left[(1 - \mu^2) \frac{d\Theta}{d\mu} \right] - \frac{1}{1 - \mu^2} m^2 = -k^2 \quad (29)$$

which, when both sides are multiplied by Θ , yields

$$\frac{d}{d\mu} \left[(1 - \mu^2) \frac{d\Theta}{d\mu} \right] - \frac{m^2\Theta}{1 - \mu^2} + k^2\Theta = 0 \quad (30)$$

Multiplying each term by $1 - \mu^2$,

$$(1 - \mu^2) \frac{d}{d\mu} \left[(1 - \mu^2) \frac{d\Theta}{d\mu} \right] - m^2\Theta + (1 - \mu^2)k^2\Theta = 0 \quad (31)$$

A solution in the form of the Maclaurin series

$$\Theta = a_l \mu^l \quad (32)$$

produces in (31)

$$(1 - \mu^2) \frac{d}{d\mu} [(1 - \mu^2) l a_l \mu^{l-1}] - m^2 a_l \mu^l + (1 - \mu^2) k^2 a_l \mu^l = 0 \quad (33)$$

where

$$\frac{d}{d\mu} [(1 - \mu^2) l a_l \mu^{l-1}] = l(l-1) a_l \mu^{l-2} - l(l+1) a_l \mu^l \quad (34)$$

which, when substituted in (33) produces

$$(1 - \mu^2) [l(l-1) a_l \mu^{l-2} - l(l+1) a_l \mu^l] - m^2 a_l \mu^l + (1 - \mu^2) k^2 a_l \mu^l = 0 \quad (35)$$

We expand (35)

$$l(l-1) a_l \mu^{l-2} - l(l+1) a_l \mu^l - l(l-1) a_l \mu^l + l(l+1) a_l \mu^{l+2} - m^2 a_l \mu^l + k^2 a_l \mu^l - k^2 a_l \mu^{l+2} = 0 \quad (36)$$

We group like terms and divide each term by μ^{l+2} to obtain

$$[l(l+1) - k^2] a_l + [-l(l+1) - l(l-1) - m^2 + k^2] a_l \mu^{-2} + [l(l-1)] a_l \mu^{-4} = 0 \quad (37)$$

Since we want the series to terminate, we eliminate terms with negative exponents on μ since they go to infinity for zero values of θ in $\mu = \cos\theta$. A good approximation of the function, therefore, is provided by use of the first term on the left of the equality, providing

$$[l(l+1) - k^2]a_l = 0 \quad (38)$$

where a_l is not zero. Thus

$$k^2 = l(l+1) \quad (39)$$

Equation (25) can now be written

$$\frac{1}{R} \frac{d^2 R}{dr^2} + \frac{2}{Rr} \frac{dR}{dr} + \frac{1}{r^2} [-k^2] + \frac{\omega^2}{c^2} = 0 \quad (40)$$

or

$$\frac{1}{R} \frac{d^2 R}{dr^2} + \frac{2}{Rr} \frac{dR}{dr} + \frac{1}{r^2} [-l(l+1)] + \frac{\omega^2}{c^2} = 0 \quad (41)$$

which further yields

$$\frac{1}{R} \frac{d^2 R}{dr^2} + \frac{2}{Rr} \frac{dR}{dr} + \left[\frac{\omega^2}{c^2} - \frac{l(l+1)}{r^2} \right] = 0 \quad (42)$$

Multiplying through by R yields

$$\frac{d^2 R}{dr^2} + \frac{2}{r} \frac{dR}{dr} + \left[\frac{\omega^2}{c^2} - \frac{l(l+1)}{r^2} \right] R = 0 \quad (43)$$

and, after extraction of r^2 from the bracket, leaves

$$\frac{d^2 R}{dr^2} + \frac{2}{r} \frac{dR}{dr} + \left[\frac{(r\omega)^2}{c^2} - l(l+1) \right] \frac{R}{r^2} = 0 \quad (44)$$

The electron spin is found from the series representation

$$R = a_s r^s \quad (45)$$

which produces an electron spin of $[-\frac{1}{2}]$. To obtain the other spin, we employ the series

$$R = a_s r^{-s} \quad (46)$$

which produces spin of $[\frac{1}{2}]$

To obtain inverse energy levels, we suppress the spin in equation (44) by use of $R = \rho r^{-1/2}$, which results in

$$\frac{d^2 \rho}{dr^2} + \frac{1}{r} \frac{d\rho}{dr} + \left[\frac{(r\omega)^2}{c^2} - (l + \frac{1}{2})^2 \right] \frac{\rho}{r^2} = 0 \quad (47)$$

We now replace $(r\omega)^2$ with v^2 to obtain

$$\frac{d^2 \rho}{dr^2} + \frac{1}{r} \frac{d\rho}{dr} + \left[\frac{v^2}{c^2} - (l + \frac{1}{2})^2 \right] \frac{\rho}{r^2} = 0 \quad (48)$$

The inverse principal quantum numbers are obtained from use of $\rho = a_1 r^{\frac{1}{n}}$ in equation (48) yielding

$$\frac{1}{n} = \sqrt{(l + \frac{1}{2})^2 - \frac{v^2}{c^2}} \quad (49)$$

where $\frac{1}{n}$ is rational. In the region of the first Bohr orbit⁵ where $n = 1$ and $v = 0.0073c$, equation (49) produces $n = l + \frac{1}{2} = 1$. This marks the beginning of the inverse quantum states. When $v = c$, $n = \frac{1}{137}$. This marks the end of the 137 inverse quantum states. The analysis seems to have support in the metallic hydrides where magnesium hydride, for example, requires an incredible 800K to release the hydrogen. Mills' reports in over thirty refereed papers of experimentally found tight hydrogen bonds is also noteworthy and cannot be dismissed.

We examine a transverse component of the electron matter wave on the y, z plane

$$y = A \sin \omega t \quad (50)$$

where the amplitude A is also the radius of a cylindrical standing wave. Taking the derivative yields

$$v = \frac{dy}{dt} = A\omega \cos \omega t \quad (51)$$

which, for $\cos \omega t = 1$, provides

$$v = A\omega = 2\pi fA \quad (52)$$

where we see that

$$\lambda = 2\pi A \quad (53)$$

In other words, the circumference of the front circular face of the electron cylinder is equal to the electron's wavelength. The radius of the cylinder is

$$A = \frac{\lambda}{2\pi} \quad (54)$$

which provides a volume⁶ of

$$V = \frac{\lambda^3}{4\pi} \quad (55)$$

We observe that as the wavelength reduces, the standing wave becomes more and more particle-like, which makes sense considering what happens in the K-capture process. It is interesting to observe that an induced K-capture process is thought responsible for transforming protons to neutrons.⁷ Thus it appears the hydrogen atom is a site where the electron becomes transformed from wave to particle.

CONCLUSION

The four-dimensional potential equation indicates that fractional quantum states exist. The solution is square integrable, satisfying a fundamental tenet of quantum physics. Mills' claim of 137 different inverse energy levels seems confirmed, as is Naudts' relativistic analysis showing that at least one reciprocal state can exist.

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