Electro-Optical Properties of Quantum Dots with an Asymmetric Confinement

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Abstract

We show how to compute the optical response of a Quantum Dot with asymmetric parabolic confinement exposed to a constant electric field applied in the growth direction. The method uses the microscopic calculation of QD eigenfunctions and the macroscopic real density matrix approach to compute the electroabsorption. Results are computed for In$_{0.64}$Al$_{0.36}$As/Al$_{0.3}$Ga$_{0.7}$As Quantum Dots. We obtain asymmetrical Stark shift of the electroabsorption maximum and the dependence on QDs dimensions is also displayed. New effects in absorption line shapes are observed. Fair agreement with experiments is obtained.

Keywords: Quantum Dots, asymmetric parabolic confinement, excitons, electro-optical properties, asymmetric Stark shift

1 Introduction

Recently, there has been much interest in the study of quantum dots (QD’s), because they behave like artificial atoms with unique optoelectronic properties. The QD’s, composed of a low-band-gap material (e.g. In$_{0.64}$Al$_{0.36}$As) and embedded in a wide-band-gap substrate (e.g., Al$_{0.3}$Ga$_{0.7}$As) create a confinement potential for electrons and holes. Mostly the dots are of cylindrical shape, with the symmetry axis $z$, and the confinement for electron and holes in the $z$ direction is of the steplike potential, corresponding to a rectangular potential well. As it was shown by Raymond et al. [10], the confinement in the growth direction can be asymmetric, and the steplike potential is perturbed within the dot by a parabolic-like potential. When a constant electric field is applied in the $z$ direction to a dot with such asymmetric potential, asymmetric Stark effect is observed. Asymmetric means in that case that it is non invariant by the change of the field direction. The standard Stark shift is a downwards directed parabola, with the maximum at $F = 0$. The Stark shift observed by
Raymond et al. [10] has a maximum at a field $F \neq 0$, and different slopes for positive and negative fields. This phenomenon is attributed to the asymmetric shape of the dots considered. In the present paper we show how an asymmetric Stark shift can be related to the asymmetric shape of the dot, and how to compute the Stark shift for asymmetric dots. Since the main contribution to the asymmetric Stark effect is due to the asymmetric confinement in the $z$ direction, we will consider the remaining in-plane confinement as simple as possible. Therefore we choose the in-plane confinement as infinite hard wall potentials for electrons and holes located in the $x-y$ plane at the radius $R$, which corresponds to the geometry of a Quantum Disk. For such a dot we compute the linear optical response, using the method of the real density matrix approach (see, for example, [13], [2], [5], and [6]).

The paper is organized as follows. In the next section we present the theoretical model and explain our method of solution. It is based on a simultaneous solution of the so-called constitutive equations and Maxwell’s equations. The advantage of such approach is that microscopical properties, including the asymmetric confinement and the applied electric field, and macroscopical quantities related to the electromagnetic field of the propagating wave, are treated on the same footing. The constitutive equations for the model under considerations are given also in Section 2. The eigenfunctions and eigenvalues for the carrier motion in the $z$-direction are determined in Section 3. Having in mind experiments of Raymond et al. [10], we derive the parameters of an asymmetric potential in Section 4 (an alternative way to establish the confinement parameters is given in Appendix A).

Having solved the constitutive equations and using parameters derived in Section 4 we have computed the optical properties of an asymmetric QD, choosing the susceptibility and the electroabsorption coefficient, taking as an example In$_{0.64}$Al$_{0.36}$As/Al$_{0.3}$Ga$_{0.7}$As Quantum Dots from experiments of Raymond et al. [10]. We obtained an asymmetric Stark shift with a fairly good agreement with experimental results (Section 5). We close the paper with final remarks in Section 6.

## 2 Constitutive equations

Our goal is to compute the optical response of a QD illuminated by an electromagnetic wave

$$\mathbf{E} = \mathcal{E} e^{-i\omega t}. \quad (1)$$

In the long wave approximation the amplitude $\mathcal{E}$ will be considered as a constant vector inside the dot. The confinement potentials for electrons and holes are

$$V_e(\rho_e) = V_{ez} + V_\parallel(\rho_e),$$
The potentials \( V_e(z), V_h(z) \) are asymmetric, and can be taken in the form

\[
V(z) = \frac{V}{L^2} \left( \frac{L}{2} - z \right)^2 ,
\]

where we adopted the scheme of ref. [10]. In all the above expressions the \( z = 0 \) point is located in the centre of the QD.

In the real density matrix approach the optical properties are obtained by solving a set of equations, the constitutive equations for the electron-hole amplitudes, and the Maxwell’s equations for the electromagnetic field. By the above assumptions, the constitutive equations have the form [6]

\[
(\mathcal{H}_{\text{QD}} - \hbar \omega - i\Gamma) \mathcal{Y} = \mathbf{M} \mathbf{E} ,
\]

with the Hamiltonian

\[
\mathcal{H}_{\text{QD}} = E_g + \frac{\hat{p}_e^2}{2m_e} + \frac{\hat{p}_h^2}{2m_h} - \frac{\hbar^2}{2m_{e\parallel}} \nabla_e (2D)^2 - \frac{\hbar^2}{2m_{h\parallel}} \nabla_h (2D)^2 \\
+ eF(z_e - z_h) + V_e(\rho_e) + V_h(\rho_h) \\
- \frac{\epsilon^2}{4\pi\epsilon_0\epsilon_b \sqrt{(\rho_e - \rho_h)^2 + (z_e - z_h)^2}} ,
\]

\( F \) being the constant electric field strength, \( \hat{p}_e \) the \( z \)-component of the momentum operator, \( m_{e\parallel}, m_{h\parallel} \) the in-plane and \( z \)-hole effective masses, the electron effective mass \( m_e \) is assumed to be isotropic, \( \epsilon_b \) is the static dielectric constant of the QD material, \( \mathbf{M} \) the interband–dipole density, and we denoted

\[
\nabla(2D)^2 = \frac{1}{\rho^2} \left( \frac{\partial}{\partial \rho} \rho^2 \frac{\partial}{\partial \rho} + \frac{\partial^2}{\partial \phi^2} \right) .
\]

We adopted the convention of ref. [10] that the electric field is taken positive when directed from the top of the QD to the substrate. We seek for amplitudes \( \mathcal{Y} \) in the form

\[
\mathcal{Y}(z_e, z_h, \rho_e, \rho_h) = \sum_{j\ell} \psi_{e,j}(z_e) \psi_{h,\ell}(z_h) Y_{j\ell}(\rho_e, \rho_h) ,
\]
where the functions \( \psi_{ej}, \psi_{hl} \) are the eigenfunctions of the \( z \) confinement when the field \( \mathbf{F} \parallel z \) is applied

\[
\left[ \frac{\hbar^2}{2m} \frac{d^2}{dz^2} + \frac{V}{L^2} \left( \frac{L}{2} - z \right)^2 + eF \left( \frac{L}{2} - z \right) \right] \psi_W = E\psi_W, \quad -\frac{L}{2} \leq z \leq \frac{L}{2},
\]
\[
\left[ \frac{\hbar^2}{2m} \frac{d^2}{dz^2} + V \right] \psi_B = E\psi_B, \quad z_e \leq -\frac{L}{2}, \quad \text{and} \quad z \geq \frac{L}{2},
\]

with the sign \(-\) for electrons, \(+\) for holes, and \( W \) denotes the Well and \( B \) the barrier. The above equations have to be solved with appropriate boundary conditions (here given for \( z = L/2 \))

\[
\psi_W (z = L/2) = \psi_B (z = L/2),
\]
\[
\left. \frac{1}{m_W} \frac{d\psi_W}{dz} \right|_{z=L/2} = \left. \frac{1}{m_B} \frac{d\psi_B}{dz} \right|_{z=L/2},
\]

\( m_W \) is the effective mass in the well and \( m_B \) in the barrier. The dipole densities are taken in the form \([6, 12]\)

\[
M (\rho_e, \rho_h, z_e, z_h) = \delta (z_e - z_h) \sum_\nu M_{0\nu} e^{i\nu(\phi_h - \phi_e)} D_\nu (\rho_e, \rho_h),
\]

\( D_\nu (\rho_e, \rho_h) \) being the in-plane transition dipole densities, and \( M_{0\nu} \) the integrated dipole strengths. By this assumption, and in the long wave approximation, we arrive at the constitutive equations for the coherent amplitudes of the following type

\[
\left[ E_g + E_{ej} (F) + E_{h\ell} (F) - \hbar \omega - i\Gamma - \frac{\hbar^2}{2m_e} \nabla_e (2\omega)^2 - \frac{\hbar^2}{2m_h} \nabla_h (2\omega)^2 \right.
\]
\[
+ \left. V_{ef} (|\rho_e - \rho_h|) \right] Y_{j\ell} (\rho_e, \rho_h) = \mathcal{E} \Psi_{j\ell} \sum_\nu M_{0\nu} e^{i\nu(\phi_e - \phi_h)} D_\nu (\rho_e, \rho_h),
\]

where

\[
\Psi_{j\ell} = \int_{-\infty}^{\infty} \psi_{ej} (z) \psi_{h\ell} (z) dz,
\]

\( V_{ef} (|\rho_e - \rho_h|) \) being the effective electron-hole interaction inside the dot, and \( \mathcal{E} \) being a value of the electric field strength in the dot. By the boundary conditions (2) the radial part vanishes at the boundary

\[
Y_{j\ell} (\rho_e, \rho_h) = 0 \quad \text{when} \quad |\rho_e| = R, \quad \text{or} \quad |\rho_h| = R.
\]

The solution of the constitutive equation (14) is a quite complicated problem. As it was shown elsewhere \([6, 12]\), it can be done for a certain model e–h potential, which we take in the form

\[
V_{ef} (|\rho_e - \rho_h|) = -\frac{e^2}{4\pi \epsilon_0 \epsilon_b |\rho_e|},
\]
The justification of such a choice for QDs of cylindrical shape was given in Ref. [12]. With this e-h potential the constitutive equations (14) can be solved, obtaining the coherent amplitudes $Y_{j\ell}$ in terms of electron- and hole eigenfunctions satisfying the conditions (16). The coherent amplitudes so obtained determine the polarization within the QD

$$P(R) = 2 \sum_{\lambda,\mu} \int d^3r M^{\lambda\mu*} Y^{\lambda\mu}(r, R),$$

where $R$ is the e-h centre of mass coordinate, $r = r_e - r_h$ the relative coordinate, respectively, $\lambda, \mu$ indicate the respective valence- and conduction bands, and $M^{\lambda\mu}$ are dipole matrix elements for the (allowed) interband transition $\lambda \rightarrow \mu$. The above equations, together with Maxwell’s equations, contain all the ingredients for the calculation of all optical properties of a QD.

Having the polarization, and in the long wave approximation, we compute the effective QD susceptibility tensor elements from

$$\chi = \frac{1}{V} \int_{QD} d^3R \frac{P(\omega, R)}{\epsilon_0\mathcal{E}},$$

where $\mathcal{E}$ is the electric field amplitude inside the dot, and $V$ the dot volume. In the above expression we assumed that the input wave is linearly polarized with an in-plane component (say $y$) $\mathcal{E}$. So we can compute an effective dielectric function,

$$\epsilon_{\text{eff}} = \epsilon_b + \chi,$$

and the QD absorption coefficient

$$\eta = 2\omega/c \text{ Im} \sqrt{\epsilon_{\text{eff}}}.$$

Taking into account the electronic transitions from the Heavy Hole (H) and Light Hole (L) subbands to the nondegenerate conduction subband, we obtain that the susceptibility tensor has a nonvanishing component $yy$, consisting of two contributions

$$\chi = \chi_{yyH} + \chi_{yyL}.$$

They will depend on the applied electric field strength and on the QD dimensions. To compute the above susceptibility we first determine the eigenfunctions of the motion in the $z$-direction. This will be done in the next section.

### 3 The eigenfunctions for the $z$-motion

The eigenfunctions of the motion in the $z$ direction can be obtained from the equations (10), in terms of parabolic cylinder functions. We intend to solve this
problem in a forthcoming paper. In what follows we propose a simplification, which gives a solution in much simpler way, and, in particular, an analytical expression for the energetic Stark shift. The scheme is as follows. The effective overall confinement potential, considered in the ref. [10] (see Fig. 1), will be replaced by an asymmetric parabolic potential (here given for the electron)

\[ V_e(z) = \frac{1}{2} m_e \omega_{ez}^2 z_e^2 + \frac{v_e}{L^2} \left( \frac{L}{2} - z_e \right)^2 , \]  

(23)

where the first term corresponds to the symmetric rectangular well potential, and the second is a perturbation producing an asymmetric profile of the overall confinement potential. The parameters \( v_e, \omega_{ez} \) will be computed later on. The same shape of potential holds for the holes. When a constant electric field is applied in the \( z \) direction, with the above potential (23), the eigenfunctions and eigenvalues result from the Schrödinger equation

\[ \left[ -\frac{\hbar^2}{2m_e} \frac{d^2}{dz_e^2} + \frac{1}{2} m_e \omega_{ez}^2 z_e^2 + \frac{v_e}{L^2} \left( \frac{L}{2} - z_e \right)^2 - eF \left( \frac{L}{2} - z_e \right) \right] \psi_e = E_{ez} \psi_e . \]  

(24)

The constant factor \( eFL/2 \) will be omitted in the following. Making use of the definitions

\[ \zeta_e = \frac{z_e}{a_e^*} , \]

\[ \frac{2m_e}{\hbar^2} = \frac{1}{R_e a_e^{*2}} , \]

\[ \tilde{a}_{ez} = a_{ez} a_e^* = a_e^* \sqrt{\frac{m_e \omega_{ez}}{\hbar}} = \sqrt{\frac{\hbar \omega_{ez}}{2 R_e}} , \]  

(25)
\[ \varepsilon_{ez} = \frac{E_{ez}}{R_e^*}, \]
\[ f_e = \frac{eF a_e^*}{R_e^*} = \frac{F}{F_{le}}, \]

\( F_{le} \) being the ionization field,

\[ F_{le} = \frac{R_e^*}{e a_e^*}, \quad (26) \]

we obtain from (24) the equation

\[ \left\{ \frac{d^2}{dx^2} + \left[ \tilde{\alpha}_{ez}^4 + \left( \frac{v_e a_e^*}{R_e^* L^2} \right)^2 \right] \right\} \zeta_e + \left( f_e - \frac{v_e a_e^*}{R_e^* L} \right) \zeta_e + \frac{v_e}{4R_e^*} \psi_e = \varepsilon_{ez} \psi. \quad (27) \]

Introducing the quantities

\[ \beta_e^2 = \tilde{\alpha}_{ez}^4 + \frac{v_e a_e^*}{R_e^* L^2}, \]
\[ \varphi_e = f_e - \frac{v_e a_e^*}{R_e^* L} = f_e - \delta_e, \]
\[ \tilde{\varepsilon}_{ez} = \varepsilon_{ez} - \frac{\varphi_e}{4\beta_e^2}, \]
\[ \zeta_e = x + a, \quad a = -\frac{\varphi_e}{2\beta_e^2}, \quad (28) \]

we transform Eq. (27) into the form

\[ \left[ \frac{d^2}{dx^2} - \beta_e^2 x^2 + \left( \tilde{\varepsilon}_{ez} + \frac{\varphi_e^2}{4\beta_e^2} \right) \right] \psi = 0. \quad (29) \]

The above equation is the one–dimensional quantum oscillator equation, with the solutions

\[ \psi_{ejz}(z_e) = C_{ej} H_{j-1} \left[ \beta_e \left( \frac{z_e}{a_e^*} + \frac{\varphi_e}{2\beta_e^2} \right) \right] \exp \left[ -\frac{\beta_e^2}{2} \left( \frac{z_e}{a_e^*} + \frac{\varphi_e}{2\beta_e^2} \right)^2 \right], \quad (30) \]

\( j = 1, 2, \ldots, \), \( H_j(x) \) being the Hermite polynomials, and \( C_{ej} \) the corresponding normalization constants. The energy eigenvalues for the electron are then

\[ E_{ejz} = \left( (2j - 1)\beta_e^2 - \frac{(f_e - \delta_e)^2}{4\beta_e^2} \right) R_e^* + \frac{v_e}{4}. \quad (31) \]

Similar calculations give for the hole

\[ \psi_{hz}(z_h) = C_{hz} H_{\ell-1} \left[ \beta_h \left( \frac{z_h}{a_{hz}^*} - \frac{\varphi_h}{2\beta_h^2} \right) \right] \exp \left[ -\frac{\beta_h^2}{2} \left( \frac{z_h}{a_{hz}^*} - \frac{\varphi_h}{2\beta_h^2} \right)^2 \right], \quad (32) \]
where

\[
\beta_h^2 = \alpha_{hz}^4 + \frac{v_h a_{hz}^2}{R_{hz}^2},
\]

\[
\varphi_h = f_h + \frac{v_h a_{hz}^2}{R_{hz}^2} = f_h + \delta_h,
\]

\[
\frac{2m_{hz}}{\hbar^2} = \frac{1}{R_{hz}^2 a_{hz}^2},
\]

with \(\alpha_{hz}\) defined in Eq. (25). The hole energies are given by

\[
E_{h\ell z} = \left[ (2\ell - 1)\beta_h^2 - \frac{(f_h + \delta_h)^2}{4\beta_h^2} \right] R_{hz}^2 + \frac{v_h}{4}.
\]

In particular, the normalized eigenstates of the lowest energies have the form

\[
\psi_{e1z}(z_e) = \frac{1}{\pi^{1/4}} \sqrt{\frac{\beta_e}{a_e^2}} \exp \left[ -\frac{\beta_e^2}{2} \left( \frac{z_e}{a_e^2} + \frac{\varphi_e}{2\beta_e^2} \right)^2 \right],
\]

\[
\psi_{h1z}(z_h) = \frac{1}{\pi^{1/4}} \sqrt{\frac{\beta_h}{a_{hz}^2}} \exp \left[ -\frac{\beta_h^2}{2} \left( \frac{z_h}{a_{hz}^2} - \frac{\varphi_h}{2\beta_h^2} \right)^2 \right].
\]

Some first consequences can be read off from the Eq. (31) and (34). Consider, for example, the Stark shift for electrons and holes for the ground state \(\ell = 1\)

\[
\Delta_e(F) = E_{e1z}(F) - E_{e1z}(F = 0),
\]

and

\[
\Delta_h(F) = E_{h1z}(F) - E_{h1z}(F = 0).
\]

The electronic Stark shift

\[
\Delta_e(F) = -\frac{(f_e - \delta_e)^2}{4\beta_e^2},
\]

is a down-directed parabola, having the top for the reduced electric field \(f_e = \delta_e > 0\). The shift is asymmetric with respect to the field \(F = 0\), and the energy values \(|\Delta_e(\pm |F|)|\) are greater than \(|\Delta_e(|F|)|\). For the holes the respective parabola has the top at a negative field value \(f_h = -\delta_h\), and the energetic shift is also asymmetric with respect to \(F = 0\). These observations are in agreement with the results of Raymond et al. [10]. This also means that our model reproduces qualitatively the asymmetric Stark in asymmetric QDs.

In the above equations (31), (34) and (30), (32) the energies and the space variable for electrons and holes are expressed in different units. Since we want to compute the overall Stark shift, we have to express them in the same units,
which are the effective Rydbergs \( R^*_H \) and effective Bohr radii \( a^*_H \), given in Table 1 for the case considered, and related to the above used units by the relations

\[
R^*_e = \left( \frac{m_e}{\mu\|H} \right) R^*_H, \quad R^*_h = \left( \frac{m_{hH}}{\mu\|H} \right) R^*_H, \tag{40}
\]

and

\[
a^*_e = \left( \frac{\mu\|H}{m_e} \right) a^*_H, \quad a^*_h = \left( \frac{\mu\|H}{m_{hH}} \right) a^*_H, \tag{41}
\]

with analogous expressions for the light-hole exciton. Using the relations (40,41) we obtain from (28) and (33)

\[
\beta^2_e = \left( \frac{\mu\|H}{m_e} \right)^2 \left[ \left( \frac{\hbar \omega_{ez}}{2 R^*_H} \right)^2 + \left( \frac{\mu\|H}{m_e} \right) \frac{v_e a^2_H}{R^*_H L^2} \right],
\]

\[
\varphi_e = \left( \frac{\mu\|H}{m_e} \right)^2 \left( \frac{F}{F_{1H}} - \frac{v_e a^*_H}{R^*_H L} \right), \tag{42}
\]

\[
\epsilon_{ez} = \left( \frac{\mu\|H}{m_e} \right) \frac{E_{ez}}{R^*_H},
\]

\[
\bar{\epsilon}_{ez} = \left( \frac{\mu\|H}{m_e} \right) \left( \frac{E_{ez}}{R^*_H} - \frac{v_e}{4 R^*_H} \right),
\]

\[
\beta^2_h = \left( \frac{\mu\|H}{m_{hH}} \right)^2 \left[ \left( \frac{\hbar \omega_{hHz}}{2 R^*_H} \right)^2 + \left( \frac{\mu\|H}{m_{hH}} \right) \frac{v_h a^2_H}{R^*_H L^2} \right],
\]

\[
\varphi_h = \left( \frac{\mu\|H}{m_{hH}} \right)^2 \left( \frac{F}{F_{1H}} + \frac{v_h a^*_H}{R^*_H L} \right), \tag{43}
\]

\[
\epsilon_{hHz} = \left( \frac{\mu\|H}{m_{hH}} \right) \frac{E_{hHz}}{R^*_H},
\]

\[
\bar{\epsilon}_{hHz} = \left( \frac{\mu\|H}{m_{hH}} \right) \left( \frac{E_{hHz}}{R^*_H} - \frac{v_h}{4 R^*_H} \right),
\]

with the ionization field \( F_{1H} \)

\[
F_{1H} = \frac{R^*_H}{e a^*_H}, \tag{44}
\]

Now the electron and hole energies can be put into the form

\[
E_{ezH}(F) = \left\{ (2j - 1) \left[ \left( \frac{\hbar \omega_{ez}}{2 R^*_H} \right)^2 + \left( \frac{\mu\|H}{m_e} \right) \frac{v_e a^2_H}{R^*_H L^2} \right] \right\}^{1/2}
\]

\[
- \left( \frac{\mu\|H}{m_e} \right) \left( \frac{F}{F_{1H}} - \frac{v_e a^*_H}{R^*_H L} \right)^2
\]

\[
\frac{1}{4} \left[ \left( \frac{\hbar \omega_{ez}}{2 R^*_H} \right)^2 + \left( \frac{\mu\|H}{m_e} \right) \frac{v_e a^2_H}{R^*_H L^2} \right] R^*_H + \frac{v_e}{4}, \tag{45}
\]
\[ E_{h\ell zH}(F) = \left\{ (2\ell - 1) \left[ \left( \frac{\hbar \omega_{hzH}}{2R_H^*} \right)^2 + \left( \frac{\mu_{||H}}{m_{hzH}} \right) \frac{\nu_h a_H^2}{R_H^* L^2} \right] \right\}^{1/2} \]

\[ - \left( \frac{\mu_{||H}}{m_{hzH}} \right) \left( \frac{F}{F_{1H}} + \frac{\nu_h a_H^2}{R_H^* L} \right)^2 \right] \right\} R_H^* + \frac{\nu_h}{4}. \]  

(46)

The Heavy Hole Stark shift is given by

\[ \Delta E(F) = [E_{e\ell zH}(F) + E_{h\ell zH}(F)] - [E_{e\ell zH}(F = 0) + E_{h\ell zH}(F = 0)], \]

which can be given explicitly by substituting the expressions (45) and (46)

\[ \frac{\Delta_H E(F)}{R_H^*} = - \left( \frac{\mu_{||H}}{m_{ce}} \right) \left( \frac{F}{F_{1H}} - \frac{\nu_c a_c^2}{R_H^* L} \right)^2 \]  

\[ - \left( \frac{\mu_{||H}}{m_{hzH}} \right) \left( \frac{F}{F_{1H}} + \frac{\nu_h a_H^2}{R_H^* L} \right)^2 \right] \right\} + \left( \frac{\mu_{||H}}{m_{ce}} \right) \left( \frac{\nu_c a_c^2}{R_H^* L} \right)^2 \]  

\[ + \frac{\left( \frac{\mu_{||H}}{m_{hzH}} \right) \left( \frac{\nu_h a_H^2}{R_H^* L} \right)^2}{4 \left( \frac{\hbar \omega_{hzH}}{2R_H^*} \right)^2 + \left( \frac{\mu_{||H}}{m_{hzH}} \right) \left( \frac{\nu_h a_H^2}{R_H^* L} \right)^2}. \]  

(48)

Quite analogous expression holds for the Light Hole Stark shift \( \Delta_L E(F) \), and the total Stark shift reads

\[ \Delta E(F) = \Delta_H E(F) + \Delta_L E(F). \]

Note that the above expression contains terms linear and quadratic in \( F \), as was also obtained by Sabathil et al. [11] and Majewski et al. [9]. The maximum of the expression \( \Delta_H E(F) \) is obtained for the field strength \( F_{\text{max}} \), which results from the equation

\[ \left( \frac{\mu_{||H}}{m_{ce}} \right) \left( \frac{F_{\text{max}}}{F_{1H}} - \frac{\nu_c a_c^2}{R_H^* L} \right)^2 + \left( \frac{\mu_{||H}}{m_{hzH}} \right) \left( \frac{F_{\text{max}}}{F_{1H}} + \frac{\nu_h a_H^2}{R_H^* L} \right)^2 = 0, \]  

(50)

equivalent to

\[ \left\{ \begin{array}{c}
\frac{1}{m_{ce} \left( \frac{\hbar \omega_{eH}}{2R_H^*} \right)^2 + \left( \frac{\mu_{||H}}{m_{ce}} \right) \left( \frac{\nu_c a_c^2}{R_H^* L} \right)^2} + \\
\frac{1}{m_{hzH} \left( \frac{\hbar \omega_{hzH}}{2R_H^*} \right)^2 + \left( \frac{\mu_{||H}}{m_{hzH}} \right) \left( \frac{\nu_h a_H^2}{R_H^* L} \right)^2} \end{array} \right\} \frac{F_{\text{max}}}{F_{1H}} \]
Thus we obtained a complete set of solutions of the Schrödinger equation with an asymmetric parabolic potential. These solutions are to be used in the expressions for the effective QD susceptibility.

4 The $z$ confinement parameters

The described method can be used when we define the parameters $\hbar \omega_{ez}$, $\hbar \omega_{hz}$, $v_e$, $v_h$. We will choose them to compare our theoretical results with the experimental findings of Raymond et al. [10] on asymmetric Stark shift in In$_{0.64}$Al$_{0.36}$As/Al$_{0.3}$Ga$_{0.7}$As QDs. The remaining parameters are collected in Table 1.

The $z$ confinement parameters are obtained as follows.

1. We consider a symmetric quantum well with a rectangular confinement potential $V$. The potentials for electrons and holes are chosen as

$$ V_e = 0.67 \cdot V, \quad V_h = 0.33 \cdot V, $$

$$ V = V_e + V_h = E_g(\text{barrier}) - E_g(\text{dot}). $$

(52)

We compute the eigenfunctions and eigenenergies for In$_{0.64}$Al$_{0.36}$As/Al$_{0.3}$Ga$_{0.7}$As QW. For the well thickness of 3.1 nm we obtain one electron state $e_1$ and at least two hole states $h_1, 2$, with the corresponding energies $E_{e1}, E_{h1,2}$. Having these energies, we compute the parameters $\omega_{ez}, \omega_{hz}$ from

$$ E_{e1} = \frac{1}{2} \hbar \omega_{ez}, \quad E_{h1} = \frac{1}{2} \hbar \omega_{hz}. $$

(53)

2. The parameters $v_e, v_h$ can be found either by fitting the experimental spectra, or by a certain procedure. By fitting the experiments, we observe that the electron mass is much lower than the hole mass and thus the main contribution to the Stark shift comes from electrons. Therefore we assume that $v_e \gg v_h$, for example $v_h = 0.2 v_e$, and obtain the parameter $v_e$ from the maximum of the observed Stark shift. In the case of ref. [10] we have $F_{\text{max}} \approx 40$ kV/cm, which yields

$$ \frac{F_{\text{max}}}{F_{1H}} = 27. $$

(54)

Putting

$$ v_h = p v_e, \quad p = 0.2, $$

(55)
into the Eq. (50), which we rewrite in the form

\[
\left( \frac{\mu_{\parallel} H}{m_e} \right) (\lambda - x) + \left( \frac{\mu_{\parallel} H}{m_h z H} \right) (\lambda + px) = 0, \tag{56}
\]

with

\[
\lambda = \frac{F_{\text{max}}}{F_{1H}}, \quad x = \frac{v_e a_H^*}{R_H^* L}, \tag{57}
\]

where we use the values \( \hbar \omega_{ez}, \hbar \omega_{hz} \) from Eq. (53), we obtain the value of \( v_e \). For the given QW we obtained \( x = 155 \), and

\[
 v_e = 73 \text{ meV}, \quad v_h = 14.6 \text{ meV}. \tag{58}
\]

Another way of determining potential parameters \( v_e, v_h \) is described in Appendix A. It gives smaller values for the parameters \( v_e, v_h \) (but \( F_{\text{max}} \) is given not very exactly due to the smooth shape of the curve), but the relation \( v_e \gg v_h \) remains fulfilled.

Having established all parameters, we compute the field dependent energies from equations (45),(46), and the Stark shift from Eq. (48).

Our method gives not only the Stark shift, but also the optical functions from the susceptibility. They will exhibit a resonance behaviour at the energies

\[
E_{Tjmn} = E_g + E_{ejz}(F) + E_{hz}(F) + E_{mn}, \tag{59}
\]

where \( E_{mn} \) is the total in-plane confinement energy: in our model it contains the Coulomb interaction between the electron and the hole.

### 5 Results of specific calculations

We have carried out detailed calculations for In\(_{0.64}\)Al\(_{0.36}\)As/Al\(_{0.3}\)Ga\(_{0.7}\)As Quantum Dots of the heights 3.1, 3.5 and 3.9 nm and in-plane radii 8.5, 9.5 and 10.5 nm, respectively, which are the dimensions of QDs experimentally studied by Raymond et al. [10]. The electric field values change from \(-60 \) kV/cm to \(+60 \) kV/cm. First we have established the values of the energies (symmetric and asymmetric) and of the parameters \( v_e, v_h \), by the method of Appendix A. Their values are given in Table 2.

The resulting Stark shift obtained from the equations (48) for three sets of values \( R, L_z \) is displayed in Fig. 2. We observe an asymmetric Stark shift with different slopes for negative and positive applied field strengths. The position of maximum shifts downwards when the dot height increases. The energy shift values corresponding to maxima are given in Table 3. We observe that these values depend on the QD height: they decrease with the increasing height. In
Figure 2: The Stark energy shifts for three chosen In$_{0.64}$Al$_{0.36}$As/Al$_{0.3}$Ga$_{0.7}$As QD sizes as function of electric field strength applied along the $z$ axis, $L_z = 3.1$ nm, $R = 8.5$ nm (solid line), $L_z = 3.5$ nm, $R = 9.5$ nm (dashed line), and $L_z = 3.9$ nm, $R = 10.5$ nm (dotted line). Experimental values from Ref. [10] are indicated as symbols.
Table 1: Parameter values InAs, In$_{0.64}$Al$_{0.36}$As, Al$_{0.3}$Ga$_{0.7}$As, AlAs, and GaAs. Parameter values for InAs from ref. [3], for AlAs from ref. [8], for GaAs QW from ref. [1], for In$_{0.64}$Al$_{0.36}$As by linear interpolation. Energies in meV, masses in electron free mass $m_0$, ionization fields in kV/cm from Eq. (44), and $\gamma_1, \gamma_2$ are Luttinger parameters.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>InAs</th>
<th>In$<em>{0.64}$Al$</em>{0.36}$As</th>
<th>AlAs</th>
<th>Al$<em>{0.3}$Ga$</em>{0.7}$As</th>
<th>GaAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_g(300$ K)</td>
<td>360</td>
<td>1357</td>
<td>3130</td>
<td>2002</td>
<td>1430</td>
</tr>
<tr>
<td>$m_e$</td>
<td>0.023</td>
<td>0.059</td>
<td>0.124</td>
<td>0.084</td>
<td>0.0665</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>19.67</td>
<td>13.75</td>
<td>3.218</td>
<td>6.85</td>
<td></td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>8.37</td>
<td>5.58</td>
<td>0.628</td>
<td>2.10</td>
<td></td>
</tr>
<tr>
<td>$m_{h</td>
<td></td>
<td>H}$</td>
<td>0.036</td>
<td>0.117</td>
<td>0.26</td>
</tr>
<tr>
<td>$m_{h</td>
<td></td>
<td>L}$</td>
<td>0.088</td>
<td>0.195</td>
<td>0.386</td>
</tr>
<tr>
<td>$\mu_{</td>
<td></td>
<td>H}$</td>
<td>0.014</td>
<td>0.039</td>
<td>0.114</td>
</tr>
<tr>
<td>$\mu_{</td>
<td></td>
<td>L}$</td>
<td>0.018</td>
<td>0.045</td>
<td>0.05</td>
</tr>
<tr>
<td>$m_{hzH}$</td>
<td>0.34</td>
<td>0.40</td>
<td>0.51</td>
<td>0.39</td>
<td>0.34</td>
</tr>
<tr>
<td>$m_{hzL}$</td>
<td>0.027</td>
<td>0.096</td>
<td>0.22</td>
<td>0.13</td>
<td>0.094</td>
</tr>
<tr>
<td>$R^*_H$</td>
<td>0.83</td>
<td>2.82</td>
<td>13.32</td>
<td>3.64</td>
<td></td>
</tr>
<tr>
<td>$R^*_L$</td>
<td>1.07</td>
<td>3.25</td>
<td>19.35</td>
<td>4.34</td>
<td></td>
</tr>
<tr>
<td>$R^*_e$</td>
<td>34.65</td>
<td>4.27</td>
<td>5.76</td>
<td>9.76</td>
<td></td>
</tr>
<tr>
<td>$a^*_H$</td>
<td>57.2</td>
<td>18.6</td>
<td>7.03</td>
<td>15.7</td>
<td></td>
</tr>
<tr>
<td>$a^*_L$</td>
<td>44.5</td>
<td>16.1</td>
<td>4.84</td>
<td>13.2</td>
<td></td>
</tr>
<tr>
<td>$a^*_e$</td>
<td>34.65</td>
<td>12.29</td>
<td>9.76</td>
<td>19.9</td>
<td></td>
</tr>
<tr>
<td>$F_{IH}$</td>
<td>0.145</td>
<td>1.52</td>
<td>2.32</td>
<td>3.29</td>
<td></td>
</tr>
<tr>
<td>$F_{IL}$</td>
<td>0.24</td>
<td>2.02</td>
<td>3.48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_{Ie}$</td>
<td>159.9</td>
<td>159.9</td>
<td>159.9</td>
<td>159.9</td>
<td></td>
</tr>
<tr>
<td>$\epsilon_b$</td>
<td>15.15</td>
<td>13.71</td>
<td>11.16</td>
<td>12.12</td>
<td>12.53</td>
</tr>
</tbody>
</table>

the same Figure some experimental points from Ref. [10] are also indicated. We observe a fairly good agreement.

In Figures 3 and 4 we present the computed Stark energy shift values for electrons and holes separately. It can be seen that the Stark effect originating from electrons is much greater than the Stark shift caused by holes, and that the resulting total shift displayed in Figure 2 is mainly due to electrons.

Similar results as in Ref. [10] have been obtained by Fry et al. [7] for the InAs/GaAs QDs of pyramidal shape. They also observed an asymmetric Stark shift, when a constant electric field was applied in the growth direction. Their results can be reproduced by the above described method, assuming a similar asymmetric parabolic confinement potential, and by making use of the
Table 2: Values of $\theta$, $\theta_{01}$, $\theta_{02}$, asymmetric QW energy eigenvalues for the lowest states $E_{e1z}$, $E_{h1z}$ and parameters $\nu_e$ and $\nu_h$. For comparison we also present the energy eigenvalues of symmetric QW for electrons and heavy (H) and light (L) holes ($\frac{1}{2}h\omega_{ez}$, $\frac{1}{2}h\omega_{hzH}$ and $\frac{1}{2}h\omega_{hzL}$, respectively). All energies and parameters $\nu_e$ and $\nu_h$ in meV, and QD height $L_z$ in nm.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$L_z$</th>
<th>$\theta$</th>
<th>$\theta_{01}$</th>
<th>$\theta_{02}$</th>
<th>$E_{hz}$</th>
<th>$\frac{1}{2}h\omega$</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>electron</td>
<td>3.1</td>
<td>0.828</td>
<td>1.456</td>
<td>1.289</td>
<td>184.190</td>
<td>172.016</td>
<td>28.523</td>
</tr>
<tr>
<td></td>
<td>3.5</td>
<td>0.877</td>
<td>1.654</td>
<td>1.432</td>
<td>162.048</td>
<td>152.038</td>
<td>24.564</td>
</tr>
<tr>
<td></td>
<td>3.9</td>
<td>0.920</td>
<td>1.843</td>
<td>1.596</td>
<td>143.546</td>
<td>135.233</td>
<td>21.169</td>
</tr>
<tr>
<td>heavy hole</td>
<td>3.1</td>
<td>1.112</td>
<td>2.677</td>
<td>2.316</td>
<td>48.932</td>
<td>46.844</td>
<td>5.994</td>
</tr>
<tr>
<td></td>
<td>3.5</td>
<td>1.152</td>
<td>3.023</td>
<td>2.618</td>
<td>41.229</td>
<td>39.630</td>
<td>4.650</td>
</tr>
<tr>
<td></td>
<td>3.9</td>
<td>1.186</td>
<td>3.368</td>
<td>2.917</td>
<td>35.184</td>
<td>33.933</td>
<td>3.716</td>
</tr>
<tr>
<td>light hole</td>
<td>3.1</td>
<td>0.790</td>
<td>1.314</td>
<td>1.138</td>
<td>102.548</td>
<td>95.371</td>
<td>16.114</td>
</tr>
<tr>
<td></td>
<td>3.5</td>
<td>0.840</td>
<td>1.484</td>
<td>1.285</td>
<td>90.933</td>
<td>84.972</td>
<td>14.106</td>
</tr>
<tr>
<td></td>
<td>3.9</td>
<td>0.884</td>
<td>1.653</td>
<td>1.432</td>
<td>81.085</td>
<td>76.087</td>
<td>12.317</td>
</tr>
</tbody>
</table>

Table 3: Values of parameters $p$, the Stark shift (meV) and the field $F_{\text{max}}$ (kV/cm) for which the Stark shift attains maximum.

<table>
<thead>
<tr>
<th>$L_z$</th>
<th>$p$</th>
<th>$F_{\text{max}}$</th>
<th>$\Delta E(F)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>0.210</td>
<td>52.956</td>
<td>0.399</td>
</tr>
<tr>
<td>3.5</td>
<td>0.189</td>
<td>40.451</td>
<td>0.300</td>
</tr>
<tr>
<td>3.9</td>
<td>0.176</td>
<td>31.309</td>
<td>0.228</td>
</tr>
</tbody>
</table>
Figure 3: The Stark energy shifts computed for the electrons for three chosen dot sizes as function of electric field strength applied along the $z$ axis, the dot sizes as in Fig. 2.

Appropriate parameters given in Table 1 and for the geometric disk parameters given in ref. [7]. The electric field changes from -300 kV/cm to +300 kV/cm. Note that for comparison of our theoretical results with the observations of Fry et al. [7] we have to change the sign of the electric field, since in ref. [7] the field is taken positive when directed from the substrate to the top of the dot. In the experiments of Fry et al. [7] two types of polarization of the incoming wave were used, the TE and the $z$ polarization.

As concerning the optical properties, we compute the electroabsorption by the formula (21), restricting the considerations to the lowest in-plane hole confined state $m = 0$, $n = 1$ (given a QD radius $R$). We use the equation (22), where

$$\chi_{yyH} = \epsilon_b \left( \frac{m_e}{\mu || H} \right) \Psi_{11H} I_H \left| \langle \psi_{e01} | \psi_{h01H} \rangle \right|^2 \frac{\Delta_{LTH}}{E_{1101H} - \hbar \omega - i \Gamma_H},$$

(60)

$$E_{1101H} = E_g + E_{e1z}(F) + E_{h1zH}(F) + \varepsilon_{e01} R^*_e + \frac{\mu || H}{m_{h|| H}} \left( \frac{x_{0,1} a_H^*}{R} \right)^2 R^*_H,$$

(61)

with quite analogous equations for the ligh hole contribution,

$$\psi_{h01H}(\rho a_e^*) = N_{01} J_0 \left( \frac{\rho a_e^*}{R} x_{0,1} \right).$$
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Figure 4: The Stark energy shifts computed for the heavy holes and for three chosen dot sizes as function of electric field strength applied along the $z$ axis, the dot sizes as in Fig. 2.

$$N_{01} = \frac{\sqrt{2}}{R} |J_1(x_{0,1})|^{-1}.$$  

(62)

For the electron in–plane motion we take the lowest eigenfunction [4]

$$\psi_{e01} = N_{e01} \left[ 1 - 2\rho + \frac{2\tilde{R} - 1}{R^2} \rho^2 \right],$$

(63)

$N_{e01}$ being the normalization factor from

$$\int_0^R \rho d\rho \psi_{e01}^2 = 1.$$  

(64)

The electron energy $\varepsilon_{e01}$ is given by [4]

$$\varepsilon_{e01} = \frac{4 \left( 1 - \tilde{R}^2 \right)}{R^2}.$$  

(65)

The integrals $I$ and $\Psi_{11}$ are given by (92) and (93), respectively. All the above relations hold for heavy hole- and light hole excitons.

We have performed the calculations for In$_{0.64}$Al$_{0.36}$As/Al$_{0.3}$Ga$_{0.7}$As QDs with the in–plane radii 8.5 nm, 9.5 nm and 10.5 nm, and heights 3.1 nm, 3.5
Figure 5: The absorption coefficient computed by the formulae (21), (60)-(65) for $F = 0$ and a In$_{0.64}$Al$_{0.36}$As/Al$_{0.3}$ Ga$_{0.7}$As QD of 3.1 nm height and 8.5 nm radius (solid line), 3.5 nm height and 9.5 nm radius (dashed line), and 3.9 nm height and 10.5 nm radius (dotted line). Heavy hole- exciton and light hole exciton transitions are indicated.
nm, and 3.9 nm, respectively. The absorption coefficient for the three QDs and without electric field is displayed in Fig. 5

Next we take the applied electric field in the calculations. The result for the electroabsorption is displayed in Fig. 6. We observe anisotropy effects in shapes and positions of the absorption curves. The height of the electroab-

<table>
<thead>
<tr>
<th>Energy [meV]</th>
<th>Absorption coefficient [cm(^{-1})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1677.5</td>
<td>2.5 x 10^5</td>
</tr>
<tr>
<td>1678</td>
<td>3 x 10^5</td>
</tr>
<tr>
<td>1678.5</td>
<td>3.5 x 10^5</td>
</tr>
<tr>
<td>1679</td>
<td>4 x 10^5</td>
</tr>
<tr>
<td>1679.5</td>
<td>4.5 x 10^5</td>
</tr>
</tbody>
</table>

Figure 6: The electroabsorption coefficient computed by the formulae (21), (60)-(65) for a disk of 3.1 nm height and 8.5 nm radius.

sorption curve is proportional to the corresponding oscillator strength, which is displayed in Fig. 7. Here we also observe anisotropy effects. The oscillator strength increases for positive fields up to a field strength of about 100 kV/cm, then decreases. This is a new effect due to the anisotropy since in symmetrical cases the oscillator strength almost always decreases with increasing electric field, see, for example, Ref. [6], Figs. 20 and 27. For the inverse bias it decreases from the value for \( F = 0 \) with increasing negative fields.

6 Conclusions

We have developed a simple mathematical procedure to calculate the electrooptical functions of asymmetric QDs of cylindrical shape. The method takes into account the Coulomb interaction between electrons and holes and the coherence of the electron-hole pair with the radiation field. We have made some simplifications such as the assumption of infinite barriers for the carriers
in-plane motion and large hole masses. We obtained analytical expressions for the Stark shift containing linear and quadratic terms in the field strength. We have applied our approach to In\_0.64Al\_0.36As/Al\_0.3Ga\_0.7As QDots of various sizes and exposed to a constant electric field applied in the growth direction, and have computed the Stark shift and the electroabsorption coefficient. We obtained asymmetric shapes for both quantities. The Stark shift depends also on the QD dimension in the growth direction. A good agreement of our theory with experiment is obtained.

**ACKNOWLEDGEMENTS.** One of us (G.C.) acknowledges the support from the U.V.O- R.O.S.T.E section of UNESCO under contract no. 875 668.0 (P-6), and Scuola Normale Superiore, Pisa, Italy, for invitation and hospitality. We would like to acknowledge fruitful discussions with G. F. Bassani and G. La Rocca.

**A Estimation of the asymmetric parabolic potential parameters**

The parameters $v_e, v_h$ can be established as follows. First, we consider a rectangular symmetric QW of the depth $V$, and compute the eigenfunctions of the

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Figure 7: Oscillator strength of the lowest excitonic transition as a function of the applied electric field, for a disk of 3.1 nm height and 8.5 nm radius.
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\[ -\frac{\hbar^2}{2} \frac{1}{dz} \frac{d}{dz} + V(z) \] \( \psi(z) = E\psi(z), \] (66)

where

\[ V(z) = \begin{cases} 0 & \text{for } -L/2 \leq z \leq L/2, \\ V & \text{for } z > L/2, \text{ or } z < -L/2, \end{cases} \] (67)

\[ m(z) = \begin{cases} m_W & \text{for } -L/2 \leq z \leq L/2, \\ m_B & \text{for } z > L/2, \text{ or } z < -L/2. \end{cases} \] (68)

Having obtained the ground state eigenfunction \( \psi_1 \), we compute the mean value of the perturbation \( \Delta V(z) \)

\[ \Delta V = \begin{bmatrix} \psi_1 \psi_1 \end{bmatrix} \begin{bmatrix} \frac{V}{L^2} (L/2 - z)^2 \end{bmatrix} \psi_1, \] (69)

which is about \( V/3 \). The value \( \Delta V \) so obtained is added to the potential \( V \), to obtain an asymmetric QW with the potential \( V(z) \)

\[ V(z) = \begin{cases} V_1 = V + \Delta V & \text{for } z \leq -L/2, \\ 0 & \text{for } -L/2 \leq z \leq L/2, \\ V_2 = V & \text{for } z \geq L/2, \end{cases} \] (70)

\[ m(z) = \begin{cases} m_W & \text{for } -L/2 \leq z \leq L/2, \\ m_B & \text{for } z > L/2, \text{ or } z < -L/2. \end{cases} \] (71)

We take, for simplicity, the same effective mass for both barriers. The solution of the Schrödinger equation (66) with the potential (70) is obtained by standard methods, which give the eigenvalues by the equation

\[ \frac{F_1 + F_2}{1 - F_1 F_2} - \tan 2\theta = 0, \] (72)

where

\[ F_1 = \sqrt{m_W \left( \frac{\theta_0^2}{\theta^2} - 1 \right)}, \] (73)

\[ F_2 = \sqrt{m_W \left( \frac{\theta_0^2}{\theta^2} - 1 \right)}, \]

\[ \theta_{01,2} = \frac{L}{2} \sqrt{\frac{2m_W}{\hbar^2} V_{1,2}} = \frac{L}{2a^*} \sqrt{\left( \frac{m_{hz}}{\mu_\parallel} \right)} \frac{V_{1,2}}{R^*}, \] (74)

\[ \theta = \frac{1}{2} \sqrt{\frac{E L}{R^* a^*}}, \]
and the respective parameter values for electrons and holes are to be used. Having computed the lowest energy values for electrons and holes, $E_{e1}$ and $E_{h1}$, respectively, we put them on the left side of the formulae (45) and (46)

$$E_{e1z} = E_{e1z}(F = 0), \quad E_{h1z} = E_{h1z}(F = 0). \quad (75)$$

Since the values $\hbar \omega_{ez}/2, \hbar \omega_{hz}/2$ are known (they correspond to the eigenvalues for the corresponding symmetric QW), we obtain equations for the parameters $v_e, v_h$.

Consider, for example, a In$_{0.64}$Al$_{0.36}$As (Well)/ Al$_{0.3}$Ga$_{0.7}$As (Barrier) QW of thickness 3.1 nm. Taking $\Delta V = V/3$, we obtain for the electron

$$V_1 = 576.2 \text{ meV}, \quad V_2 = 432.15 \text{ meV}$$

$$m_{B1} = 0.084 \, m_0, \quad m_W = 0.059 \, m_0, \quad m_{B2} = 0.084 \, m_0. \quad (76)$$

The parameters $\theta_{01,2}$ take the values

$$\theta_{01} = \frac{L}{2a^*_e} \sqrt{\frac{V_1}{R^*_e}} = 1.465, \quad \theta_{02} = \frac{L}{2a^*_e} \sqrt{\frac{V_2}{R^*_e}} = 1.269. \quad (77)$$

From Eq. (72) we obtain $\theta = 0.829$ and

$$E_{e1z} = 184.5 \text{ meV}. \quad (78)$$

Similar procedure can be applied for the heavy holes yielding

$$V_1 = 283.8 \text{ meV}, \quad V_2 = 212.85 \text{ meV},$$

$$m_{B1} = 0.4 \, m_0, \quad m_W = 0.4 \, m_0, \quad m_{B2} = 0.4 \, m_0. \quad (79)$$

Here we obtain

$$\theta_{01} = 2.667, \quad \theta_{02} = 2.319, \quad \theta = 1.105, \quad (80)$$

and

$$E_{h1z} = 48.344 \text{ meV}. \quad (81)$$

Substituting the above values to the equations (45) and (46) and making use of the symmetric QW eigenvalues

$$\frac{1}{2} \hbar \omega_{ez} = 171.8 \text{ meV}, \quad \frac{1}{2} \hbar \omega_{hzH} = 46.60 \text{ meV}, \quad (82)$$

we obtain the parameters $v_e, v_h$ (see Table 2)

$$v_e \approx 30 \text{ meV}, \quad v_h \approx 4.2 \text{ meV}, \quad (83)$$
with the relation
\[ p = \frac{v_h}{v_e} \approx 0.14, \] (84)
which confirms our previous assumption \( v_e \gg v_h \). Using the above parameters we compute the Stark shift which now attains its maximum value for the field
\[ \frac{F}{F_{1H}} \approx 14, \] (85)
in agreement with the data obtained from fitting the results of Ref. [10]. Results for three different QD heights are collected in Table 2.

### B Calculation of Integrals \( \Psi'_{11} \) and \( \Psi_{11} \)

We have to compute the integral
\[ \Psi'_{11} = \frac{1}{L_z} \int_{L_z/2}^{L_z/2} \psi_{e1a}(z) \psi_{h1z}(z) \, dz, \] (86)
where (see relations (35,36))

\[ \psi_{e1z}(z) = \frac{1}{\pi^{1/4}} \frac{\beta_e}{a_e^*} \exp \left[ -\frac{\beta_e^2}{2} \left( z \frac{\varphi_e}{a_e^*} + \frac{\varphi_e}{2\beta_e^2} \right)^2 \right], \] (87)

\[ \psi_{h1z}(z) = \frac{1}{\pi^{1/4}} \frac{\beta_h}{a_{hz}^*} \exp \left[ -\frac{\beta_h^2}{2} \left( z \frac{\varphi_h}{a_{hz}^*} - \frac{\varphi_h}{2\beta_h^2} \right)^2 \right]. \] (88)

By relations (41)

\[ \frac{\beta_e^2}{2} \left( \frac{z}{a_e^*} + \frac{\varphi_e}{2\beta_e^2} \right)^2 + \frac{\beta_h^2}{2} \left( \frac{z}{a_{hz}^*} - \frac{\varphi_h}{2\beta_h^2} \right)^2 \]
\[ = \frac{1}{2} \left( A \xi^2 + B \xi + C \right) = \frac{A}{2} \left[ \left( \xi + \frac{B}{2A} \right)^2 - \frac{B^2}{4A^2} \right], \] (89)

where

\[ \xi = \frac{z}{a^*}, \quad A = \frac{1}{\mu^2} \left( \beta_e^2 m_e^2 + \beta_h^2 m_{hz}^2 \right), \]

\[ B = \frac{m_e \varphi_e - m_{hz} \varphi_h}{\mu}, \]

\[ C = \frac{\varphi_e^2}{4\beta_e^2} + \frac{\varphi_h^2}{4\beta_h^2}. \] (90)
Thus (86) becomes

\[
\Psi_{11}' = \frac{1}{L_z} \frac{1}{\sqrt{\pi}} \sqrt{\frac{\beta_e \beta_h}{a_e^* a_{h_z}^*}} a^* \exp \left( -\frac{C}{2} + \frac{B^2}{8A} \right) \times \int_{-L_z/2a^*}^{L_z/2a^*} \exp \left[ -\frac{A}{2} \left( \xi + \frac{B}{2A} \right)^2 \right] \, d\xi = \frac{1}{a^*} I, \quad (91)
\]

where

\[
I = \frac{2}{\sqrt{\pi}} \left( \frac{a^*}{L_z} \right)^2 \sqrt{\frac{m_e m_{h_z} \beta_e \beta_h}{\mu^2}} \exp \left( -\frac{C}{2} + \frac{B^2}{8A} \right) \times \int_{-1}^{1} \exp \left[ -\frac{A}{8} \left( \frac{L_z}{a^*} + \frac{B}{A} \right)^2 \right] \, dt. \quad (92)
\]

Analogously

\[
\Psi_{11} = \int_{-\infty}^{\infty} \psi_{e1z}(z) \psi_{h1z}(z) \, dz = \frac{1}{\sqrt{\pi}} \sqrt{\frac{\beta_e \beta_h}{a_e^* a_{h_z}^*}} a^* \exp \left( -\frac{C}{2} + \frac{B^2}{8A} \right) \times \int_{-\infty}^{\infty} \exp \left[ -\frac{A}{2} \left( \xi + \frac{B}{2A} \right)^2 \right] \, d\xi \quad (93)
\]

\[
= \sqrt{\frac{2}{A}} \sqrt{\frac{m_e m_{h_z} \beta_e \beta_h}{\mu^2}} \exp \left( -\frac{C}{2} + \frac{B^2}{8A} \right).
\]

References


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