Reflection and Refraction of Quasi P and SV Waves at the Interface of Fibre-Reinforced Media

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Abstract

The reflection and refraction of plane waves at the interface of two fibre reinforced media is discussed. The phase velocities of quasi-P waves and quasi-SV are obtained in terms of components of propagation vector. It has been assumed that in anisotropic medium the direction of particle motion is neither parallel nor perpendicular to the direction of propagation. The expressions for reflection and refraction coefficients are obtained. Numerical results of reflection and refraction coefficients are presented graphically and compared with the isotropic case.

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Introduction

Fibre reinforced composite materials have become very attractive in many engineering applications recently due to their superiority over other structural materials in applications requiring high strength and stiffness and lightweight components. Consequently the characterization of their mechanical behaviour is of particular importance for structural design using these materials.

Effect of earthquake on artificial structures is of prime importance to engineers and architects. During an earthquake and similar disturbances a structure is excited into a more or less violent, with resulting oscillatory stresses, which depend both upon the ground vibration and physical properties of the structure. Most concrete construction includes steel reinforcing, at least nominally. So, wave propagation in a reinforced medium plays a very important role in civil engineering and geophysics.

The problem of reflection and refraction of elastic waves have been discussed by several authors. Without going into the details of the research work in this field we mention the papers by Knott [1899], Gutenberg [1944], Auld [1973], Achenbach [1976], Keith and Crampin [1977, 1977a, 1977b], Tolstoy [1982], Norris [1983], Pal and Chattopadhyay [1984], Borejko [1996], Ogden and Sotirropoulos [1997, 1998], Chattopadhyay and Rogerson [2001].

Crampin and Taylor [1971] studied surface wave propagation in examples of unlayered and multilayered anisotropic media, which is examined numerically with a program using as extension of the Thompson-Haskell matrix formulation. They studied some examples of surface wave propagation in anisotropic media to interpret a possible geophysical structure. Crampin [1975] showed that the surface waves have distinct particle motion when propagating in a structure having a layer of anisotropic material with certain symmetry relations.

The propagation of body waves in anisotropic media is fundamentally different from their propagation in isotropic media, although the differences may comparatively subtle and difficult to observe [Crampin (1975)]. In general, for any type of anisotropy, there are always three types of body waves propagating with three different velocities. Choosing the three components of displacement adequately, they are called quasi-P, quasi-SV and quasi-SH (qSH) waves. The velocities of these three waves change according to the type of symmetry present in the medium. Because of these properties, anisotropy is detected by observations of change in P-wave velocity along two perpendicular directions and by observations of S-wave splitting. For both effects it is not necessary that the whole medium be anisotropic; only a certain part of it need be [Udias (1999)]. Generally, the particle motion is neither purely longitudinal nor purely transverse. For this reason, the three types of body waves in an anisotropic medium are referred as qP, qSV and qSH instead of P, SV and SH.
In this paper we have studied the reflection and refraction of quasi-\(P\) waves and quasi-SV at the interface of two monoclinic media. The expressions for the phase velocities of quasi-\(P\) and quasi-SV waves have been obtained. Reflection and refraction coefficients have been computed and compared with the isotropic case. It has been observed that fibre-reinforced media plays a significant role in case of reflection and refraction and the effects are tremendous.

2 Formulation of the problem

The constitutive equations for fibre-reinforced linearly elastic medium whose preferred direction is that of \(\vec{a}\) are (Spencer [18] and Belfield et al [3])

\[
\tau_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu_T e_{ij} + \alpha (a_k a_m e_{km} \delta_{ij} + e_{kk} a_i a_j) + 2(\mu_L - \mu_T) (a_i a_k e_{kj} + a_j a_k e_{ki}) + \beta (a_k a_m e_{km} a_i a_j)
\]

where \(\tau_{ij}\) are components of stress, \(e_{ij}\) are components of infinitesimal strain, \(a_i\) are the components of \(\vec{a}\), all referred to cartesian coordinates. The vector \(\vec{a}\) may be a function of position. The coefficients \(\lambda, \mu_L, \mu_T, \alpha\) and \(\beta\) are elastic constants with dimension of stress.

If \(\vec{a}\) is so chosen that its components are \((1,0,0)\). The stress components (1) become

\[
\tau_{11} = (\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta)e_{11} + (\lambda + \alpha)e_{22} + (\lambda + \alpha)e_{33},
\]

\[
\tau_{22} = (\lambda + \alpha)e_{11} + (\lambda + 2\mu_T)e_{22} + \lambda e_{33},
\]

\[
\tau_{33} = (\lambda + \alpha)e_{11} + \lambda e_{22} + (\lambda + 2\mu_T)e_{33},
\]

\[
\tau_{12} = 2\mu_L e_{12}, \tau_{13} = 2\mu_L e_{13}, \tau_{23} = 2\mu_T e_{23}
\]

where \(2e_{ij} = u_{i,j} + u_{j,i}\) and \(u_i\) (\(i=1,2,3\)) are the displacement components.

We take the plane of symmetry of the fibre-reinforced medium as the \(x_1 x_2\)-plane and \(x_2\)-axis vertically upwards. For the plane wave propagation in \(x_1 x_2\)-plane, we have \(\frac{\partial}{\partial x_3} = 0\).

The non-vanishing equations of motion without body forces are

\[
\frac{\partial \tau_{11}}{\partial x_1} + \frac{\partial \tau_{12}}{\partial x_2} = \rho \frac{\partial^2 u_1}{\partial t^2},
\]

\[
\frac{\partial \tau_{21}}{\partial x_1} + \frac{\partial \tau_{22}}{\partial x_2} = \rho \frac{\partial^2 u_2}{\partial t^2},
\]
The stress equations of motion (3) with the help of (2) become

\[
(\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta) \frac{\partial^2 u_1}{\partial x_1^2} + \mu_L \frac{\partial^2 u_1}{\partial x_2^2} + (\lambda + \alpha + \mu_L) \frac{\partial^2 u_1}{\partial x_1 \partial x_2} = \rho \frac{\partial^2 u_1}{\partial t^2}
\]

(4)

\[
\mu_L \frac{\partial^2 u_2}{\partial x_1^2} + (\lambda + 2\mu_T) \frac{\partial^2 u_2}{\partial x_2^2} + (\lambda + \alpha + \mu_L) \frac{\partial^2 u_2}{\partial x_1 \partial x_2} = \rho \frac{\partial^2 u_2}{\partial t^2}
\]

(5)

\[
\mu_L \frac{\partial^2 u_3}{\partial x_1^2} + \mu_T \frac{\partial^2 u_3}{\partial x_2^2} = \rho \frac{\partial^2 u_3}{\partial t^2}
\]

(6)

From equations (4) to (6), it is obvious that qSH wave which is represented by \(u_3\) motion in eqn(6) is decoupled from \((u_1, u_2)\) motion representing qP and qSV waves. qSH may be considered as simply SH motion. We consider plane wave solution of equation (6) as

\[
u_3 = A \exp(ik(x_1p_1 + x_2p_2 - ct)).
\]

The phase velocity of qSH wave is

\[\rho c^2 = \mu_L p_1^2 + \mu_T p_2^2\]

(7)

where \(\vec{p} (p_1, p_2, 0)\) denote the unit propagation vector, \(c\) is the phase velocity and \(k\) is the wave number of the plane wave propagating in the \(x_1x_2\) -plane.

Let \(\vec{p} (p_1^{(n)}, p_2^{(n)}, 0)\) denote the unit propagation vector, \(c_n\) is the phase velocity and \(k_n\) is the wavenumber of plane waves propagating in the \(x_1x_2\) -plane. We consider plane wave solutions of equations (5) and (6) as

\[
\begin{pmatrix}
  u_1^{(n)} \\
  u_2^{(n)}
\end{pmatrix}
= A_n \begin{pmatrix}
  d_1^{(n)} \\
  d_2^{(n)}
\end{pmatrix}
\exp(\imath \eta_n)
\]

(8)

where

\[
\eta_n = k_n(x_1p_1^{(n)} + x_2p_2^{(n)} - c_n t)
\]

and \(\vec{d} (d_1, d_2, 0)\) is the unit displacement vector. Inserting expressions for \(u_1^{(n)}\) and \(u_2^{(n)}\) in the equations (4) and (5), we obtain

\[
(R_n - \rho c_n^2) d_1^{(n)} + S_n d_2^{(n)} = 0
\]

\[
S_n d_1^{(n)} + (T_n - \rho c_n^2) d_2^{(n)} = 0
\]

(9)

where

\[
R_n = (\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta)p_1^{(n)^2} + \mu_L p_2^{(n)^2},
\]

\[
S_n = \mu_L,
\]

\[
T_n = \mu_T.
\]
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\[ S_n = (\lambda + \alpha + \mu_L)p_1^{(n)} p_2^{(n)}, \]
\[ T_n = \mu_L p_1^{(n)^2} + (\lambda + 2\mu_T)p_2^{(n)^2}. \]  

From equations (9), we have

\[ \frac{d_1^{(n)}}{d_2^{(n)}} = \frac{S_n}{\rho c_n^2 - R_n} = \frac{\rho c_n^2 - T_n}{S_n}. \]  

From this equation \( \vec{d} \) may be calculated.

Eliminating \( d_1^{(n)} \) and \( d_2^{(n)} \) from (8) and (9), we obtain

\[ \rho^2 c_n^4 - (R_n + T_n)\rho c_n^2 + (R_n T_n - S_n^2) = 0. \]  

Solving (12), we obtain the velocities of quasi-P (qP) and quasi-SV (qSV) waves as

\[ \rho c_L^2 = \frac{(R_n + T_n) + \sqrt{(R_n - T_n)^2 + 4S_n^2}}{2} \]  

\[ \rho c_T^2 = \frac{(R_n + T_n) - \sqrt{(R_n - T_n)^2 + 4S_n^2}}{2} \]  

Equation (13) represents quasi-P (qP) wave velocities for \( n=0,1,3 \), and eqn.(14) represents for quasi-SV(qSV) wave velocities for \( n=2,4 \). From the equations (4) and (5), it may be deduced

\[ (\lambda + 2\alpha + 3\mu_L - 2\mu_T + \beta)d_1^{(n)^2}d_2^{(n)^2}p_1^{(n)}p_2^{(n)^2} + (\mu_L - \lambda - 2\mu_T)d_1^{(n)^2}d_2^{(n)^2} = 0. \]  

An incident wave which is pure shear or pure longitudinal may propagate only in certain specific direction. Longitudinal and transverse specific directions are found by putting \( \vec{d} = \vec{p} \) and \( \vec{d} \) perpendicular to \( \vec{p} \). In anisotropic case no such relations can be considered between the displacement vector and the propagation vector.

2.1 Incident qP waves

Consider homogeneous fibre-reinforced half-spaces occupying the regions \( x_2 \geq 0 \) (upper medium) and \( x_2 \leq 0 \) (lower medium). The plane of symmetry is taken as the \( x_1x_2 \)-plane. Plane qP wave is incident at the interface of boundary \( x_2 = 0 \). Incident qP wave will generate reflected qP, reflected qSV, refracted qP and refracted qSV waves. Let \( n=0,1,2,3,4 \) be assumed for incident qP, reflected qP, reflected qSV, refracted qP and refracted qSV respectively. The angle made by incident qP wave, reflected qP, reflected qSV, refracted qP and
refracted qSV waves with the normal to the interface are $\theta_0$, $\theta_1$, $\theta_2$, $\theta_3$, and $\theta_4$, respectively. We consider plane strain problem and hence

$$u_1 = u_1(x_1, x_2, t), \; u_2 = u_2(x_1, x_2, t), \; u_3 = 0.$$ 

In the plane $x_2 = 0$, the displacements and stresses of incident, reflected and refracted waves are

$$u^{(n)}_1 = A_n d_1^{(n)} \exp(i\eta_n), \quad u^{(n)}_2 = A_n d_2^{(n)} \exp(i\eta_n),$$

$$\tau^{(n)}_{22} = i A_n k_n [(\lambda + \alpha)d_1^{(n)} p_1^{(n)} + (\lambda + 2\mu_T)d_2^{(n)} p_2^{(n)}] \exp(i\eta_n),$$

$$\tau^{(n)}_{21} = i A_n k_n \mu L [d_1^{(n)} p_2^{(n)} + d_2^{(n)} p_1^{(n)}] \exp(i\eta_n),$$

where

$$\eta_n = k_n (x_1 p_1^{(n)} - c_n t).$$

For incident qp waves

$$p_1^{(0)} = \sin \theta_0, \; p_2^{(0)} = \cos \theta_0, \; c_0 = c_L.$$ 

For reflected qP waves

$$p_1^{(1)} = \sin \theta_1, \; p_2^{(1)} = -\cos \theta_1, \; c_1 = c_{L1}.$$ 

For reflected qSV waves

$$p_1^{(2)} = \sin \theta_2, \; p_2^{(2)} = -\cos \theta_2, \; c_2 = c_T.$$ 

For refracted qP waves

$$p_1^{(3)} = \sin \theta_3, \; p_2^{(3)} = \cos \theta_3, \; c_3 = c'_L.$$ 

For refracted qSV waves

$$p_1^{(4)} = \sin \theta_4, \; p_2^{(4)} = \cos \theta_4, \; c_4 = c'_T.$$ 

### 2.2 Incident qSV waves

This case is similar to the earlier case 2.1. In this case $n=0$ to be considered for incident quasi-SV wave. In the equation (18), $c_L$ to be replaced by $c_{T1}$. All the calculations are similar to incident qP waves.
3 Boundary conditions and solution of the problem

For physically most significant case of perfect contact, the displacements and stresses are continuous at the interface:

\[
\begin{align*}
    u_1^{(0)} + u_1^{(1)} + u_1^{(2)} &= u_1^{(3)} + u_1^{(4)}, \\
    u_2^{(0)} + u_2^{(1)} + u_2^{(2)} &= u_2^{(3)} + u_2^{(4)}, \\
    \tau_{22}^{(0)} + \tau_{22}^{(1)} + \tau_{22}^{(2)} &= \tau_{22}^{(3)} + \tau_{22}^{(4)}, \\
    \tau_{21}^{(0)} + \tau_{21}^{(1)} + \tau_{21}^{(2)} &= \tau_{21}^{(3)} + \tau_{21}^{(4)}.
\end{align*}
\]

Using the boundary conditions, we have the following four equations

\[
\begin{align*}
    A_0d_1^{(0)} \exp(i\eta_0) + A_1d_1^{(1)} \exp(i\eta_1) + A_2d_1^{(2)} \exp(i\eta_2) &= A_3d_1^{(3)} \exp(i\eta_3) + A_4d_1^{(4)} \exp(i\eta_4) \\
    A_0d_2^{(0)} \exp(i\eta_0) + A_1d_2^{(1)} \exp(i\eta_1) + A_2d_2^{(2)} \exp(i\eta_2) &= A_3d_2^{(3)} \exp(i\eta_3) + A_4d_2^{(4)} \exp(i\eta_4) \\
    ik_0A_0X_0 \exp(i\eta_0) + ik_1A_1X_1 \exp(i\eta_1) + ik_2A_2X_2 \exp(i\eta_2) &= ik_3A_3X_3 \exp(i\eta_3) + ik_4A_4X_4 \exp(i\eta_4) \\
    ik_0A_0Y_0 \exp(i\eta_0) + ik_1A_1Y_1 \exp(i\eta_1) + ik_2A_2Y_2 \exp(i\eta_2) &= ik_3A_3Y_3 \exp(i\eta_3) + ik_4A_4Y_4 \exp(i\eta_4)
\end{align*}
\]

where

\[
\begin{align*}
    X_0 &= (\lambda + \alpha)d_1^{(0)} \sin \theta_0 + (\lambda + 2\mu_T)d_2^{(0)} \cos \theta_0 \\
    X_1 &= (\lambda + \alpha)d_1^{(1)} \sin \theta_1 - (\lambda + 2\mu_T)d_2^{(1)} \cos \theta_1 \\
    X_2 &= (\lambda + \alpha)d_1^{(2)} \sin \theta_2 - (\lambda + 2\mu_T)d_2^{(2)} \cos \theta_2 \\
    X_3 &= (\lambda' + \alpha')d_1^{(3)} \sin \theta_3 + (\lambda' + 2\mu_T')d_2^{(3)} \cos \theta_3
\end{align*}
\]
\[ X_4 = (\lambda' + \alpha')d_1^{(4)} \sin \theta_4 + (\lambda' + 2\mu'r)d_2^{(4)} \cos \theta_4 \]

\[ Y_0 = \mu_L[d_1^{(0)} \cos \theta_0 + d_2^{(0)} \sin \theta_0], \]

\[ Y_1 = \mu_L[-d_1^{(1)} \cos \theta_1 + d_2^{(1)} \sin \theta_1], \]

\[ Y_2 = \mu_L[-d_1^{(2)} \cos \theta_2 + d_2^{(2)} \sin \theta_2], \]

\[ Y_3 = \mu'_L[d_1^{(3)} \cos \theta_3 + d_2^{(3)} \sin \theta_3], \]

\[ Y_4 = \mu'_L[d_1^{(4)} \cos \theta_4 + d_2^{(4)} \sin \theta_4], \] (31)

Equations (27) to (30) must be valid for all values of \( x_1 \) and \( t \). Therefore,

\[ \eta_0 = \eta_1 = \eta_2 = \eta_3 = \eta_4 \] (32)

which gives

\[ k_0 \sin \theta_0 = k_1 \sin \theta_1 = k_2 \sin \theta_2 = k_3 \sin \theta_3 = k_4 \sin \theta_4 = k \] (33)

and

\[ k_0c_L = k_1c_{L1} = k_2c_T = k_3c'_L = k_4c'_T = \omega \] (34)

where \( k \) is the apparent wave number and \( \omega \) the circular frequency. Solving the equations (27) to (30), the reflection and refraction coefficients may be obtained as

\[ \frac{A_1}{A_0} = \frac{D_1}{D_0}, \frac{A_2}{A_0} = \frac{D_2}{D_0}, \frac{A_3}{A_0} = \frac{D_3}{D_0}, \frac{A_4}{A_0} = \frac{D_4}{D_0}, \] (35)

The values of \( D_0, D_1, D_2, D_3, D_4 \) are as follows

\[ D_0 = \begin{vmatrix} -a_1 & -a_2 & a_3 & a_4 \\ -b_1 & -b_2 & b_3 & b_4 \\ -c_1 & -c_2 & c_3 & c_4 \\ -d_1 & -d_2 & d_3 & d_4 \end{vmatrix} \]

\[ D_1 = \begin{vmatrix} 1 & -a_2 & a_3 & a_4 \\ 1 & -b_2 & b_3 & b_4 \\ 1 & -c_2 & c_3 & c_4 \\ 1 & -d_2 & d_3 & d_4 \end{vmatrix} \]

\[ D_2 = \begin{vmatrix} -a_1 & 1 & a_3 & a_4 \\ -b_1 & 1 & b_3 & b_4 \\ -c_1 & 1 & c_3 & c_4 \\ -d_1 & 1 & d_3 & d_4 \end{vmatrix} \]
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\[
D_3 = \begin{vmatrix}
-a_1 & -a_2 & 1 & a_4 \\
-b_1 & -a_2 & 1 & b_4 \\
-c_1 & -c_2 & 1 & c_4 \\
-d_1 & -d_2 & 1 & d_4 \\
\end{vmatrix},
\]

\[
D_4 = \begin{vmatrix}
-a_1 & -a_2 & a_3 & 1 \\
-b_1 & -b_2 & b_3 & 1 \\
-c_1 & -c_2 & c_3 & 1 \\
-d_1 & -d_2 & d_3 & 1 \\
\end{vmatrix}
\]

where

\[
a_1 = \frac{d_1^{(1)}}{d_1^{(0)}}, a_2 = \frac{d_1^{(2)}}{d_1^{(0)}}, a_3 = \frac{d_1^{(3)}}{d_1^{(0)}}, a_4 = \frac{d_1^{(4)}}{d_1^{(0)}},
\]

\[
b_1 = \frac{d_2^{(1)}}{d_2^{(0)}}, b_2 = \frac{d_2^{(2)}}{d_2^{(0)}}, b_3 = \frac{d_2^{(3)}}{d_2^{(0)}}, b_4 = \frac{d_2^{(4)}}{d_2^{(0)}},
\]

\[
c_1 = \frac{k_1 X_1}{k_0 X_0}, c_2 = \frac{k_2 X_2}{k_0 X_0}, c_3 = \frac{k_3 X_3}{k_0 X_0}, c_4 = \frac{k_4 X_4}{k_0 X_0},
\]

\[
d_1 = \frac{k_1 Y_1}{k_0 Y_0}, d_2 = \frac{k_2 Y_2}{k_0 Y_0}, d_3 = \frac{k_3 Y_3}{k_0 Y_0}, d_4 = \frac{k_4 Y_4}{k_0 Y_0}.
\]

The values of the velocities and \(\frac{d^{(n)}}{d_2^{(0)}}\) can be calculated from (13), (14) and (11) for different values of \(n=0\) to 4. Quasi-P (qP) wave velocities are obtained by putting \(n=0,1,3\) and qSV wave velocities for \(n=2,4\).

### 4 Numerical Calculations and Discussions

Numerical calculations were performed for fibre-reinforced media and isotropic media. Three different cases are represented in graphs by the notations curve-1, curve-2 and curve-3.

**Case-1: (curve-1)**

In this case both the media are fibre-reinforced. The material constants for the upper and lower fibre-reinforced media are:

\[
\mu_T = 2.46GPa, \mu_L = 5.66GPa, \lambda = 5.65GPa, \beta = 220.90GPa,
\]

\[
\alpha = -1.28GPa, \rho = 7.8m/cm^3,
\]
and
\[ \mu_T = 3.5 \text{GPa}, \mu_L = 7.07 \text{GPa}, \lambda = 6.92 \text{GPa}, \beta = 133.49 \text{GPa}, \]
\[ \alpha = -0.49 \text{GPa}, \rho = 1.6 \text{gm/cm}^3, \]

Case-2: (curve-2)
The upper medium is isotropic and the lower medium is fibre-reinforced. The material constants for the upper isotropic medium are
\[ \mu_T = \mu_L = 25.0 \text{GPa}, \lambda = 30.0 \text{GPa}, \alpha = \beta = 0, \rho = 2.7 \text{gm/cm}^3, \]
and the material constants for the lower fibre-reinforced medium are
\[ \mu_T = 3.5 \text{GPa}, \mu_L = 7.07 \text{GPa}, \lambda = 6.92 \text{GPa}, \beta = 133.49 \text{GPa}, \]
\[ \alpha = -0.49 \text{GPa}, \rho = 1.6 \text{gm/cm}^3, \]

Case-3:(curve-3)
The material constants for upper and lower isotropic media are:
\[ \mu_T = \mu_L = 25.0 \text{GPa}, \lambda = 30.0 \text{GPa}, \alpha = \beta = 0, \rho = 2.7 \text{gm/cm}^3, \]
and
\[ \mu_T = \mu_L = 6.0 \text{GPa}, \lambda = 8.0 \text{GPa}, \alpha = \beta = 0, \rho = 2.3 \text{gm/cm}^3. \]

4.1 Incident $qP$ waves
Figure 1 shows that the reflection coefficients of $qP$ waves for different types of media (curves 1 to 3). In case of both the media are fibre-reinforced media (curve-1) the values of reflection coefficients are less compared to isotropic case (Curve-3) upto 25 degree then increases sharply upto 90 degree. The values of the curves 1 and 2 are negative upto 17 degree. Maximum increase of amplitude ratio of curve-1 is at 65 degree compared to isotropic case. Maximum decrease of amplitude ratio of curve-1 is at 10 degree compared to isotropic case. Curve-2 shows similar trend as curve-1.
Figure 2 shows that the reflection coefficients of $qSV$ waves. In this figure the values of curves 1 and 2 are negative from 8 degree to 67 degree.
Figure 3 shows the variation of refraction coefficients of $qP$ waves. Here all the values of refraction coefficients are positive.
Figure 4 shows the refraction coefficients of $qSV$ waves. In this case all the values of refraction coefficients of $qSV$ waves are negative for different types of media. In this case also the critical points exist for the curve-1 with curve-2 at 31 degrees, while the critical point of the curve-1 and 3 is at 22 degrees.
It is noted that the reflection and refraction coefficients are presented in the figures 1 to 4 upto 43 degrees.
4.2 Incident qSV waves

Reflection and refraction coefficients are presented graphically in figures 5 to 8. Figure 5 shows the reflection coefficients of qP waves. In this case the curves-1 and 2 are less in magnitude compared to isotropic case (curve-3). The existence of all the curves is upto 43 degrees. It is noted that all the values of reflection coefficients are negative in curves 1 and 2 whereas the reflection coefficients in case of isotropic media are all positive.

Figure 6 represents the reflection coefficients of qSV waves. In this case the magnitude of the curves 1 and 2 are more compared to isotropic case (curve-3). The values in case of curve-3 (isotropic case) are negative upto 32 degree, whereas all the values of curve-1 are positive.

Figure 7 represents the refraction coefficients of qP waves. In this case all the values of all the curves are positive.

Figure 8 shows the variations of refraction coefficients of qSV waves. In this case all the values of curves-1 and 2 are less compared to curve-3. Finally, we conclude that the material constants of fibre-reinforced media have a considerable effect on the reflection and refraction coefficients of qP and qSV waves in addition to the angle of incidence, shown by different values of amplitude ratios for different cases (case1, case 2 and case 3).

References


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Fig. 4. Reflected coefficient of qSV waves due to incident of qP waves

Fig. 5. Reflected coefficient of qP waves due to incident of qSV waves
Fig. 5. Reflection coefficient of qSV waves due to incident of qSV waves.
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**Figure 7**: Refracted coefficient of P waves due to incident of SV waves.

**Figure 8**: Refracted coefficient of SV waves due to incident of P waves.