

# A Hybrid Logarithmic Gradient Algorithm for Poisson Noise Removal in Medical Images

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## Abstract

This paper proposes a Poisson noise removal filter consisting of a modified gradient algorithm. Square of each pixel is subtracted from the center pixel of a 3x3 window. All gradient values are log added and then square root is taken. Bias reduction is done using log value of central pixel. The method is applied on Lena image and then on some biomedical images. Recovery results show that the proposed logarithmic gradient method is computationally simple and better visually. Proposed algorithm results are also better in terms of correlation, Structural Similarity (SSIM) index and Mean Square Error (MSE). The findings have potential for applications in biomedical image diagnostics.

**Keywords:** Biomedical Images, Poisson Noise, Image de-noising

## I. Introduction

Medical images are always with noise due the physical environment in which these images are obtained and processed. Hence noise removal remains as one of

the challenging and difficult tasks in medical and digital image processing where quality of recovered image is vital for medical diagnostic purposes. Several effective de-noising algorithms have been proposed over the past decades, and a review of these algorithms is reproduced recently in literature [1]. Some of common de-noising techniques are based on Gaussian and Wiener filtering, producing distinctively not very clear results in which the edges appear seemingly lost containing noise content [2].

The most general noise-removal model follows the additive property, generally known as Additive White Gaussian Noise (AWGN). In this model random noise from many sources summed up to have a kind of Gaussian distribution independent of the signal in actual image. The AWGN model is also used in channel modeling for estimating parameters of fading, nonlinearity and dispersion [3].

Let,  $Y_i$ , be noise less (or noise free) image and let  $\mathfrak{N}_i$  be noise following Gaussian distribution and having additive property [4], then noisy image  $U_i$  is as given as in (1).

$$U_i = Y_i + \mathfrak{N}_i \forall i \in I \quad (1)$$

On the other hand some noise elements exhibit the signal dependent nature and also multiplicative property, following the Poisson distribution, which frequently occurs in medical images. Let,  $Y_i$ , be noise less (or noise free image) and,  $\mathfrak{N}_i$ , be noise following Poisson distribution and having multiplicative nature [4], then noisy image  $U_i$  is as given in (2).

$$U_i = Y_i \cdot \mathfrak{N}_i \forall i \in I \quad (2)$$

Images generated by devices using photon-counting processes like in X-rays, Positron Emission Tomography (PET) [5], Single Photon Emission Computed Tomography (SPECT), are having the problem of getting corrupted by Poisson noise [6, 7] rather than by the Gaussian noise.

Poisson noise is an electronic noise that happens when limited quantity of energy containing particles, for example, electrons (or photons) in electronic circuit (or in optical) devices, is small in number, not enough to produce some detectable statistical variations in estimation. Poisson noise is also known as short or photon noise [7]. Nature of Poisson noise is multiplicative and also it has pixel-intensity dependency, meaning that statistic of contamination of dark pixels is less than bright pixels [8].

Let  $v = \{v_{i,j} : i, j = 1, \dots, N\}$  and  $\omega = \{\omega_{i,j} : i, j = 1, \dots, N\}$  define the original and noisy images respectively, where the noisy image contains Poisson

noise. With true image  $v$ , the likelihood for observing  $\omega$  is given by (3) as is given in [9]

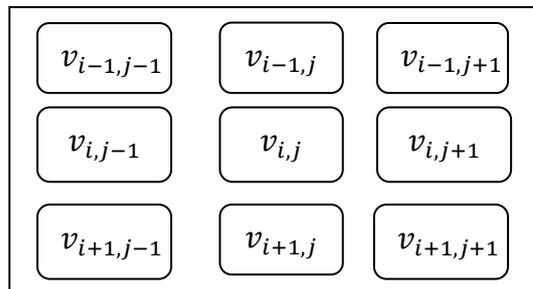
$$p(\omega/v) = \prod_{i,j=1}^N \frac{e^{-v_i v_{i,j}} v_{i,j}^{\omega_{i,j}}}{\omega_{i,j}!} \quad (3)$$

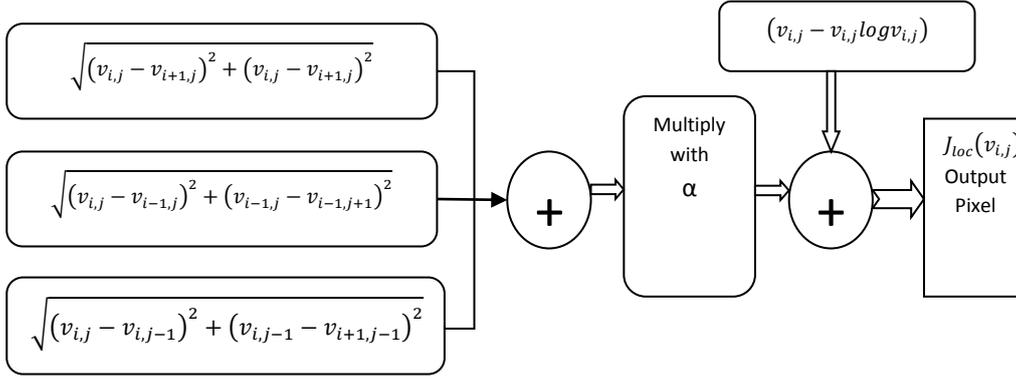
In this work we propose a modified version of coordinate descent method for Poisson noise removal in medical images.

## II. Coordinate Descent Method

Coordinate Descent Method is one of simplest and easiest optimization methods. For finding a local minimum fine search is performed along coordinate direction by considering current point as reference and search is performed in all iterations. Coordinate Descent Method follows the basic idea of minimization of multivariable function, taking one variable (coordinate) at a time at the outset of which direction for another coordinate is set, moving onward the whole process over the remaining coordinates cyclically, thus optimizes target function through a sequence of one-coordinate optimizations. For any current point,  $v_{i,j}$ , the cost function of the Coordinate Descent Method is given as is given in [8].

$$J_{loc}^1(v_{i,j}) = \alpha[\sqrt{(v_{i,j} - v_{i+1,j})^2 + (v_{i,j} - v_{i+1,j})^2} + \sqrt{(v_{i,j} - v_{i-1,j})^2 + (v_{i-1,j} - v_{i-1,j+1})^2} + \sqrt{(v_{i,j} - v_{i,j-1})^2 + (v_{i,j-1} - v_{i+1,j-1})^2}] + (v_{i,j} - v_{i,j} \log v_{i,j}) \quad (4)$$



**Figure1 (a):** Pixel Position in 3x3 windows

**Figure 1 (b):** Coordinate Descent Method Diagrammatical Representation  
 Figure 1(a) shows the pixel position in 3x3 window while Figure 1(b) shows diagrammatical presentation of the Coordinate Descent.

## II. Proposed Technique

The coordinate descent method in this proposed work is borrowed and tuned it to what is called a Hybrid Logarithmic Gradient method, using which the first pixel (called central pixel) being de-noised is considered as the center of a 3x3 window. Square of neighbors of the center pixel in all direction are subtracted from the center pixel. All gradients are log added and the square root is taken. Bias reduction is done using the log of the center pixel. Thus weight for the central pixel is acquired which is the multiplied with noisy central pixel and the result is further smoothed out using Median filter.

Moreover, the Hybrid Logarithmic Gradient differs from Step Descent Method in ways that de-noising the pixel, which is always a central pixel of a 3x3 window, remains the reference pixel. However, in Step Descent Method, for de-noising the pixel, which is central pixel of a 3x3 window, the reference pixel changes within a 3x3 window.

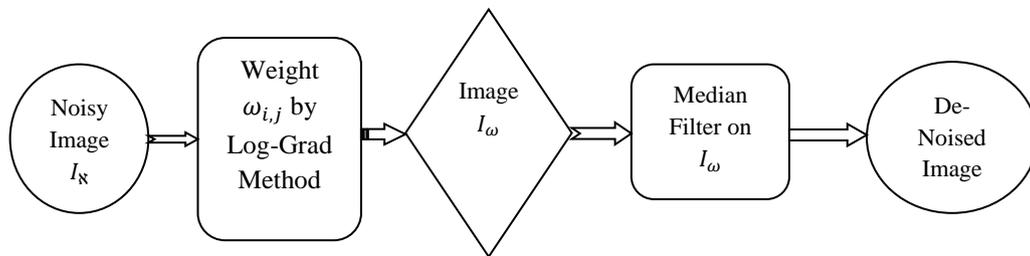
The proposed modified Log-Gradient method can be expressed mathematically as under

$$\omega_{i,j} = \sqrt{\log\{(P_{i,j} - P_{i+1,j}^2) + (P_{i,j} - P_{i-1,j}^2) + (P_{i,j} - P_{i,j+1}^2) + (P_{i,j} - P_{i,j-1}^2)\} - \log(P_{i,j})} \quad (5)$$

Let  $I_N$  be the noisy image and  $P_{i,j}$  define the current pixel, so weight for current image is computed by above Equation (5). After getting weights for whole image by Log-Gradient method, the new image  $I_\omega$  is formed by mean of equation Moreover,

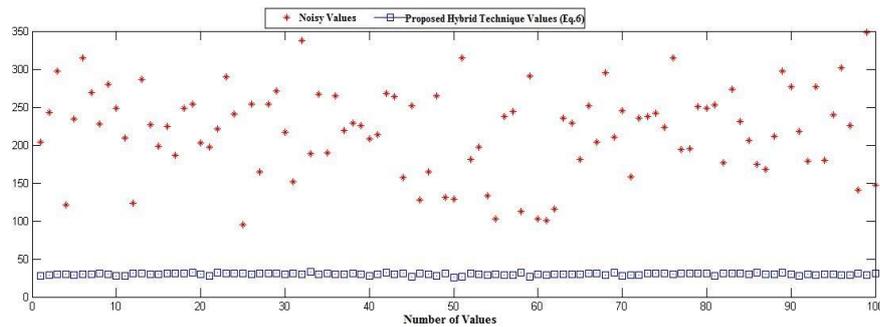
$$I_\omega = I_N * \exp(\omega_{i,j}) \quad (6)$$

after computing  $I_\omega$  which contains low noise than noisy image Median filter is applied on  $I_\omega$  for getting completely de-noised image. Figure 2 illustrates the block diagram for de-noising of general and medical images corrupted by Poisson noise.



**Figure 2 (a):** Complete De-noising Procedure for Proposed Technique

The above equation (Eq. 6) is plotted as shown in Figure 2(b), showing how the proposed hybrid technique looks like. The matrix of random numbers of 10x10 order is taken and Poisson noise is included. After that proposed hybrid technique (in the form of Eq.6) is applied on it, it is clear that noise is removed considerably.

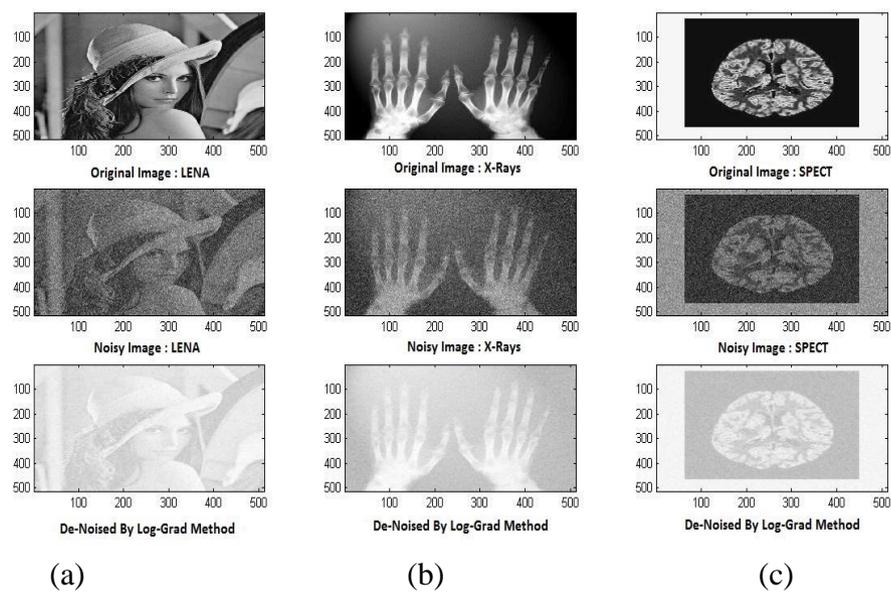


**Figure 2 (b):** The proposed technique is being compared with noisy values

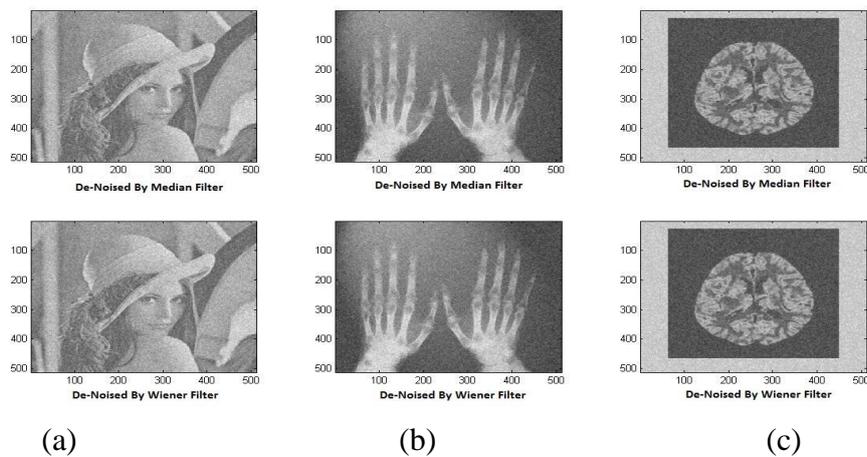
### III. Simulation and Results

Log-Grad method with Median filter is tested on several case-study images of LENA, X-Rays and SPECT contaminated with Poisson noise. The efficiency of

proposed technique using hybrid-gradient followed by filter is compared with results obtained from using: (a) Median filter directly applied on noisy images, and (b) Wiener filter de-noising results. The efficiency of proposed technique is judged both in the case of Median filter alone and Wiener filter in terms of visual quality, correlation, MSSIM and RMSE. Simulation results for LENA, X-Rays and SPECT are given as plots shown in Figure 2, 3, 4, 5, 6, while its specific parameters are reproduced in tabular form as given in Table 1, Table 2, and Table 3.



**Figure 3:** Original images, Noisy Images, Images de-noised by Proposed Technique



**Figure 4:** Images De-noised by Median Filter and Wiener Filter

Original images of LENA, X-Rays and SPECT are shown in the top row, while the noisy images are placed in the middle row, and images de-noised by proposed technique are shown in the bottom row of Figure 3 (a), (b) and (c) respectively. The same three target images de-noised by Median and Wiener filters are as shown in Figure 4 (a), (b) and (c) respectively.

Correlation, Mean Structural Similarity (MSSIM) index and Root Mean Square Error (RMSE) results of proposed technique are shown as coexistent graphical plots with those obtained from Median and Wiener filters in Figure 5, Figure 6 and Figure 7 respectively. The SSIM index showing similarity between two images, and can be viewed as a quality measure of one of the images being compared, provided the other image is assumed to be of perfect quality.

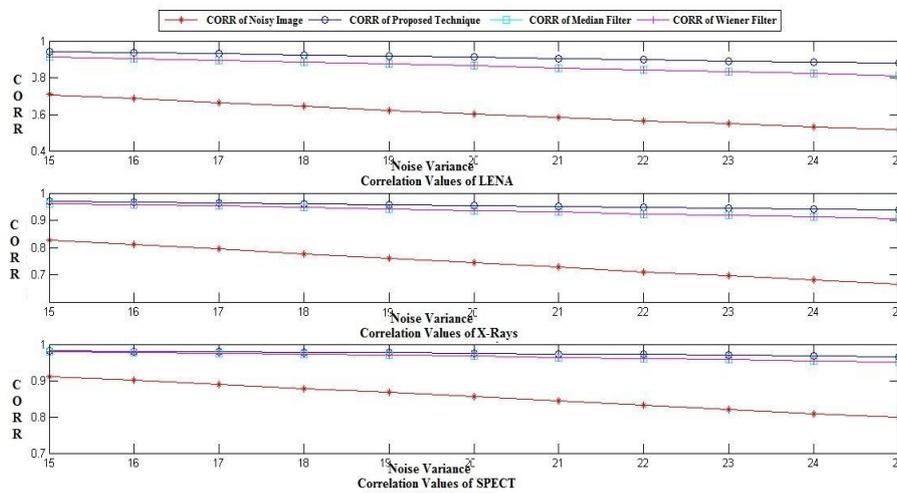


Figure 5: Correlation Graphs of log-grad, Median Filter and Wiener Filter

#### IV. Discussion of Results

Correlation graph of Lena image indicates that the correlation value of image de-noised by proposed technique is significantly higher than those obtained by the Median and Wiener filter techniques. Furthermore, correlation values of image de-noised by proposed technique have significantly better value for highly noisy image than those by the techniques of Median and Wiener filters.

The performance of Median and Wiener filters decreases for higher noise values while MSSIM values of proposed technique remains steady from very low to high level of noise. On the other hand, performance of Median and Wiener filters in term of MSSIM decreases with raise in level of noise. However the tabular and graphical result shows that efficiency of proposed technique in terms of RMSE remaining almost in steady state. Efficiency of Median and Wiener filters decreases

with raise in noise. Visual results show that the image de-noised by proposed technique have clear and good look than their counterpart images de-noised by Median and Wiener filters. As image de-noised by Median and Wiener filters exhibit the noise even for image corrupted by moderate level noise.

Correlation graph of X-Rays image shows that the correlation value of image de-noised by proposed technique is significantly higher than Median and Wiener filter values. The study of the results of MSSIM and RMSE shows that efficiency of proposed technique is significantly better than those by the techniques of Median and Wiener filters. Overall efficiency of proposed technique remains significantly better for every level of noise. Visual results shows that the X-rays image de-noised by proposed technique have clear and good look than image de-noised by Median and Wiener filters. As image de-noised by Median and Wiener filters exhibit the noise even for image corrupted by moderate level of noise.

**Table 1: Correlation Results**

	Correlation of			
	Noisy Image	Proposed Technique	Median Filter	Wiener Filter
LENA	0.6022	0.9156	0.8674	0.8674
SPECT	0.7412	0.9551	0.9345	0.9345
X-Rays	0.8553	0.9747	0.9667	0.9667

Table 1 contains the values of correlation of Noisy images of LENA, SPECT and X-Rays at an intermediate noise level and also the correlation values of de-noised images, which are de-noised by Proposed Technique, Median filter and Wiener filter. The results clearly show that performance of Proposed Technique is significantly much better than the Median and Wiener filters. However, taking the difference of correlation results of noisy images and images de-noised by proposed technique, the result of LENA image is much better from SPECT and X-Rays.

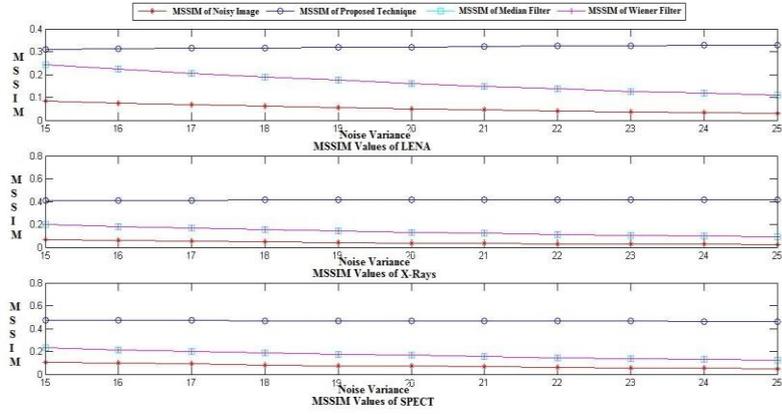
**Table 2: MSSIM Results**

	MSSIM of			
	Noisy Image	Proposed Technique	Median Filter	Wiener Filter
LENA	0.0498	0.3203	0.1600	0.1600
SPECT	0.0699	0.4671	0.1648	0.1648
X-Rays	0.0381	0.4140	0.1324	0.1324

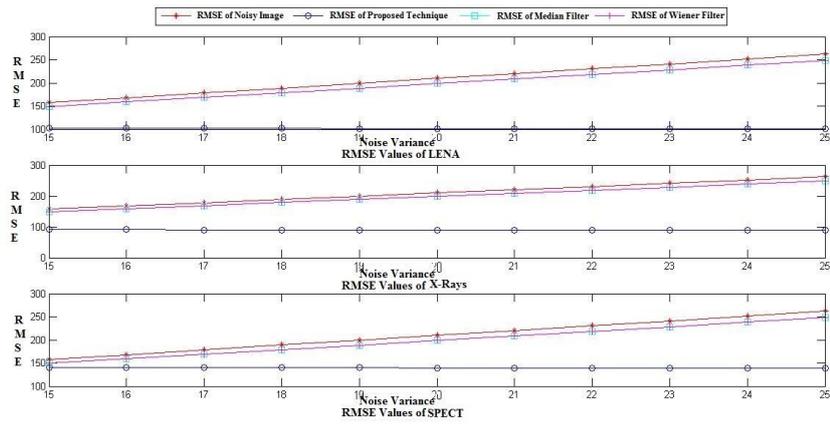
Table 2 contains the values of MSSIM of Noisy images of LENA, SPECT and X-Rays at an intermediate noise level and also the correlation values of de-noised images, which are de-noised by Proposed Technique, Median filter and Wiener filter.

The results clearly show that in MSSIM case the performance of Proposed Technique is also significantly much better than the Median and Wiener filters. However, taking the difference of MSSIM results of noisy images and images de-noised by proposed technique, the result of SPECT image is much better from LENA and X-Rays.

While correlation graph of SPECT image shows that the efficiency of proposed technique which is, though, marginally better than Median and Wiener filters for moderate level of noise but have significantly better efficiency than Median and Wiener filters for high level of noise. The Study of results of MSSIM and RMSE shows that the efficiency of proposed technique is significantly much better than the results by the Median and Wiener filters. However, the visual result of proposed technique is efficiently clean and clearer than Median and Wiener filters for SPECT image.



**Figure 6: MSSIM Graphs of Proposed Technique, Median Filter and Wiener Filter**



**Figure 7: RMSE Graphs of Proposed Technique, Median Filter and Wiener Filter**

**Table 3: RMSE Results**

	RMSE of			
	Noisy Image	Proposed Technique	Median Filter	Wiener Filter
LENA	209.6533	101.3023	198.6631	198.6631
SPECT	209.7474	139.6346	198.9054	198.9054
X-Rays	209.3712	89.7705	198.3503	198.3503

## V. Conclusion

This paper has proposed a new method for Poisson noise removal used for filtering biomedical images of noise. The method is based on the summation of logarithmic gradient values of 3x3 windows, which are formed for each pixel. Bias reduction is done using the logarithmic values of the central pixel. The method is demonstrated for different Poisson noise levels on Lena image and then on different medical images like X-rays, PET and SPECT images. The performance of the resulting images is evaluated by comparing its correlation MSSIM and MSE results with those derived using by the Median and Wiener filters. The quality of noise removal is shown in terms of tabulated index parameters. The utility of the suggested technique lies in processing biomedical images for better diagnostic purposes.

## References

- [1]. A.Buades, B. Coll, and J. M. Morel, "A Review of Image De-noising Algorithms with a new one." *Multi-scale Modeling and Simulation (SIAM Interdisciplinary Journal)*, 4.2(2005): 490-530.
- [2]. Avindra Fernando, "Image De-noising with the Non-local Means Algorithm", University of Kansas December (2010).
- [3]. A.Buades, B. Coll and J. Morel, "A non-local Algorithm for image denoising" IEEE International Conference on Computer Vision and Pattern Recognition, (2005).
- [4]. V. Naga Prudhvi Raj, T. Venkateswarlu, "De-noising of Poisson and Rician Noise from Medical Images using Variance Stabilization and Multi-scale Transforms" *International Journal of Computer Applications on* 57.21(2012): 0975-8887.
- [5]. J. M. Ollinger and J. A. Fessler, "Positron Emission Tomography", *IEEE Signal Process. Mag.*, 14.1(1997): 43-55.
- [6]. A. Aldroubi and M. Unser, "A Review of Wavelets in Biomedical Applications", *Proceedings of the IEEE*, 84.4(1996): 626-638.
- [7]. M. Rangaraj Rangayyan, "Biomedical Signal Analysis: A Case Study Approach", *IEEE Process. Mag.*, 19.4(2002).

- [8]. H. Chan and KeChen, “Multilevel Algorithm for a Poisson Noise Removal Model with Total-Variation Regularization”.
- [9]. Rodrigues, I. Sanches and J. Bioucas-Dias, “De-Noising of Medical images Corrupted By Poisson Noise”, Image Processing, IEEE International Conference (2008):1756 – 1759.

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