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Prediction of Migration Path of a Colony of Bounded-Rational Species Foraging on Patchily Distributed Resources

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Abstract

A theoretical and conceptual model was formulated to determine, as well as to predict, possible migratory route of a colony of bounded-rational species. A network with connected patches was set to represent the environment, and the migration behavior of the colony was assumed to follow the Optimal Foraging Theory. Various dynamic environmental factors such as quality and quantity of available food, distance of the food sources, population growth rate of the colony, existence of competitors, and cost of transferring from one location to another can be considered in choosing the next residence of the species. The preferences of species were integrated using fuzzy Analytic Hierarchy Process. The predicted migratory route was obtained by successive Mixed-Integer Linear Goal Programming. The acquired result was treated only as a "satisficing" solution because of the species' cognitive limitations and possible conflicting preferences.

Keywords: migration, optimal foraging theory, multi-criteria decision making, linear goal programming

1 Introduction

There are several reasons why species migrate [1,4,9,11]. In most cases, they migrate to have the best possible living conditions. An individual or a community may move to a new location because of overpopulation and competition in their current habitat. Species, such as insects and humans, migrate in response to the depleting resource quantity. In this paper, we answered the question "What is the migratory route that the members of the colony should collectively choose to follow in order to obtain sustaining benefits with minimal cost?" by formulating a mathematical model. In determining the possible migration route, only a satisficing solution to our model was considered because of the bounded rationality of the species. A *satisficing solution* means that only the adequate criteria are satisfied, and this solution may not be optimal because of the existence of conflicting preferences. Species are *bounded rational* since they have limited knowledge about their environment.

Making decisions have always been a big part of daily life, especially when making choices among different alternatives. The determination of alternatives and their importance is a crucial and essential step [2,3,7,12]. This study will help in predicting the satisficing migration path of species by considering their preferences and needs for survival, given that the behavior of the species follows the model's assumptions. Moreover, it will give insights about the actual migration strategy of some species by measuring the deviation of the actual migration route of the species from the acquired results of the model. The degree of deviation depends on how "rational" the species are.

The necessary steps in the formulation of our theoretical and conceptual model are (1) determine and rank the preferences of species, (2) determine a satisficing migration route that would satisfy, or nearly satisfy, the colony's wants and needs considering the carrying capacity of the environment, (3) measure the underachievement of the colony's wants and needs, (4) determine how long the environment can provide enough food for the colony's survival, and (5) perform necessary sensitivity analysis after obtaining a solution.

The main assumptions of the model are the behavior of the colony follows the Optimal Foraging Theory and the species forage on patchily distributed food sources [8,13,14,15,16]. Optimal Foraging Theory implies that the species consider the costs and benefits of foraging in a certain patch or nearby patches. Also, the time period between migration activities was assumed to be discrete.

The needed input values are ranking of species' preferences, population growth rates, quantity of food in each food source, distance of patches from one another, cost of competition, cost of migration, quantity of food needed by the colony, and natural production and depletion rates of the food sources. Some, if not all, of these input information were assumed to be known to the colony. Obtaining these input information is feasible because some colonies do have scouts tasked to explore their surrounding environment. However, only the known food sources, new residence locations and competitors can be considered in the model.

2 Fuzzy Analytic Hierarchy Process

Analytic Hierarchy Process (AHP) is a multi-criteria decision making technique that is commonly used in ranking different alternatives. Saaty's scale [5,10,17] for the pair-wise comparison between alternatives is illustrated in Table 1. The comparison matrix showing the pair-wise comparison among alternatives is of the following form:

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{12} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

where a_{ij} is the relative importance of alternative i over alternative j quantified using Saaty's scale.

Table 1. Saaty's scale for pair-wise comparison.

Saaty's scale	Relative importance between factors
1	Equally important
3	Moderately important
5	Strongly important
7	Very strongly important
9	Extremely important
2,4,6,8	Intermediate values

To address the ambiguity in each preference made, the comparison matrix is transformed into a fuzzy pair-wise comparison matrix using the following relationship [6]:

$$r_{ij} = \frac{a_{ij}}{a_{ij} + 1}\,.$$

Table 2. Scale for fuzzy pair-wise comparison.

Fuzzy scale membership values (r_{ij})	Relative importance between factors
0.5	Equally Important
0.5≤0.55≤0.6	Slightly Important
0.6≤0.65≤0.7	Important
0.7≤0.75≤0.8	Strongly Important
0.8≤0.85≤0.9	Very Strongly Important
0.9≤0.95≤1.0	Extremely Important

The weights $w_1, w_2, ..., w_n$ are determined by the formula:

$$w_i = \frac{b_i}{\sum_{i=1}^{n} b_i} \qquad \text{where,} \quad b_i = \frac{1}{\left[\sum_{j=1}^{n} \frac{1}{r_{ij}}\right] - n}.$$

Some of the preferences may be inconsistent so it is suggested to measure these inconsistencies to be able to improve the judgments made. The consistency of the comparison matrix can be checked using the consistency ratio [10].

3 Model Formulation

A network is considered where the possible new residence location with connected food patches comprises the set of nodes. We will call the location of the residence as hive. There are four possible types of nodes: (1) old hive; (2) food source only; (3) possible new hive and a food source; and (4) possible new hive but not a food source, which is illustrated in Figure 1.

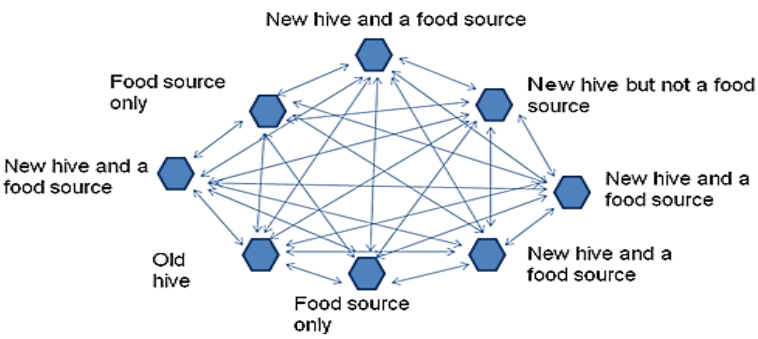

Figure 1. Network of the food sources and the possible new location

Several dynamic environmental factors can be considered in choosing the next hive location, such as preferences of the species. The preferences of the species with respect to the factors considered are incorporated in the model and are ranked using fuzzy Analytic Hierarchy Process (AHP). The population growth rate and food consumption of the migrating colony and the competitors can also be taken into account.

The actual distance between two nodes is converted into its equivalent perceived (inverse) distance value. In this paper, we used the negative exponential function $G = 2^{-\lambda}$ as the perceived distance function. The colony has its limitation on the distance it can travel. There is a maximum foraging distance on which they can only travel. In addition, the values used in the computations were non-dimensionalized and normalized using infinity-norm.

The solution to the mathematical program generated can be solved using Operations Research software.

Definition Variables and Parameters for time period *t*

$n \sim$ number of nodes

$m \sim$ number of factors or alternatives except the distance between nodes and the amount of food available at a node

$H \sim$ set of all possible new hive location

$J \sim$ set of nodes within foraging distance

$X_i \sim$ indicator of possible new hive location; this is our decision variable where

$$X_i = \begin{cases} 1, \text{if node } i \text{ is chosen} \\ 0, \text{otherwise} \end{cases}$$

$c_{ij} \sim$ normalized perceived (inverse) distance function value from node i to node j

$q_j \sim$ normalized total quantity of food available in node j

$F_{kj} \sim$ normalized value or amount of factor k at node j (excluding the distance between nodes and the amount of food available at a node)

$w_k \sim$ weights derived from fuzzy AHP assigned to factor k

$v_j \sim$ aggregate contribution of factors to the quality of node j, where

$$v_j = \sum_{k=1}^{m} w_k F_{kj}$$

$S \sim$ number of survivors of the migrating colony after a competition

$S_T \sim$ total number of survivors of all the competing colonies after the competition

$U_j \sim$ utility function value assigned to node j based from the cost of competition

$s_2 \sim$ the least number of survivors in order for a colony to be considered a winner in a competition

$s_1 \sim$ the most number of survivors in order for a colony to be considered a loser in a competition

$C_1 \sim$ cost in winning a competition (same dimension as $q_j v_j$)

$C_2 \sim$ cost in losing a competition (same dimension as $q_j v_j$)

$C_3 \sim$ cost in a tied competition (same dimension as $q_j v_j$)

$A \sim$ minimum quantity of food needed by the colony (same dimension as q_j)

$\beta \sim$ penalty weight for the underachievement of A

$\alpha \sim$ penalty weight for the cost of migration

$Z_i \sim$ costs of migrating to node i (aside from the factors included in the computation of v_j, this may include the risk of existence of predators near the possible hive location and the energy needed to transfer from the old hive to the new hive)

$\Phi_{i,t-1} \sim$ amount of remaining food at node i from the previous time period $t-1$

$R_j \sim$ expected total amount of food that the species can forage in node j considering competition

$d^+ \sim$ deviation variable corresponding to the overachievement of A

$d^- \sim$ deviation variable corresponding to the underachievement of A

The Mixed-Integer Linear Goal Program

The goal of the model is to maximize the benefit subject to the costs associated with the migration of the species. The aim is to maximize the Objective Function:

$$\sum_{i=1}^{n} X_i (Y_i - \alpha Z_i) - \beta d^-$$

The objective is subject to the following constraints.

Constraint 1:

$$\sum_{i=1}^{n} X_i = 1$$

where $X_i = 0$ if node i is not an element of H

Constraint 2: For node $i \in H$,

If there is no competition at node j,

$$Y_i = \sum_{j=1}^{n} c_{ij} q_j v_j$$

Else, if there is competition at node j,

$$Y_i = \sum_{j=1}^{n} c_{ij} U_j$$

where

$$U_j = P(S \ge s_2)[q_j v_j - C_1] - P(S \le s_1) C_2 + P(s_1 < S < s_2) \left[\frac{S q_j v_j}{S_T} - C_3 \right]$$

Constraint 3: For $j = 1,2,\dots,n$,

$$v_j = \sum_{k=1}^{m} w_k F_{kj}$$

Constraint 4:

$$\sum_{i \in H} \sum_{j \in J} (X_i c_{ij} R_j - d^+ + d^-) = A$$

where

$$R_j = \begin{cases} P(S \ge s_2) q_j + P(s_1 < S < s_2) \dfrac{S q_j}{S_T}, & \text{if there is a competitor at node } j \\ q_j, & \text{otherwise} \end{cases}$$

The first constraint restricts the species in choosing exactly one new hive location. The second constraint incorporates the factors considered in choosing the new location. The distance of a food source from the hive and the quantity of available food are assumed to be the most important factors in the species' decisions. In the third constraint, the preference of the species is being incorporated according to its importance. The possible compromise between the availability of food and the distance travelled by species during foraging is reflected on the fourth constraint. The non-negative variables d^+ and d^- are the deviational variables that represent the deviation above and below A (minimum quantity of food needed by the colony), respectively.

When there is known competition in the food source, corresponding cost is incorporated. If the colony wins the competition with the probability given by $P(S \geq s_2)$, then colony can take the entire food source on the location but will pay for the cost of competition. On the other hand, if the colony loses with probability given by $P(S \leq s_1)$, the colony will not get the food source and will pay for the cost of competition. But there could also be a possibility that the competing colonies will coexist, this means that they can share the food source on the location but both colony still need to pay for the cost of competition.

4 Prototype Example

Consider the network in Figure 2. The distances between nodes are given. Assume that the equivalent perceived distance function (G) of the actual distance (λ) is equal to $G = 2^{-\lambda}$. The perceived distance function value can also be interpreted as the energy needed during travel.

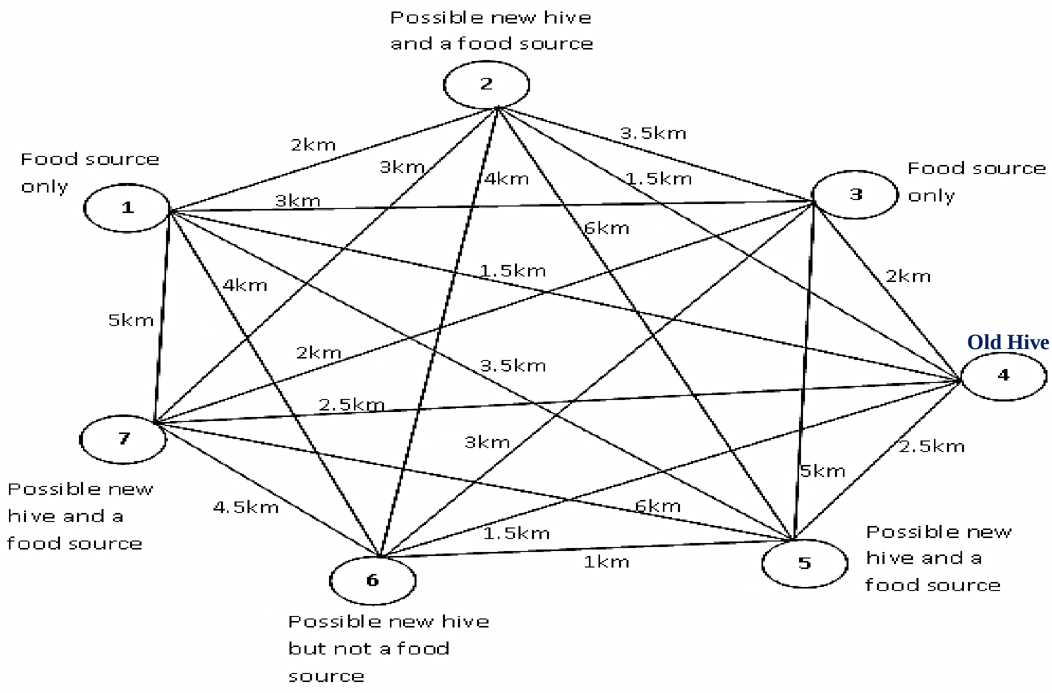

Figure 2. Network of possible locations and food sources

The available quantity of food in every node is given in Table 3. Let $A = 15$, $\alpha = 1$, $\beta = 1$ and the maximum flight distance equal to $4km$. The dimension of A should be the same as the dimension of q_j's. Assume that one food is good to sustain an individual member of the colony. The corresponding values for the cost of migration Z_i are given in Table 4. Different migrating costs were considered in every migration time period t because the energy needed by the colony to transfer from the old hive to the new hive varies. The dimension of Z_i is the same as the dimension of Y_i. In this example, we assumed the value of Z_i depends on the value of Y_i.

Table 3. Available amount of food per node.

Node	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Amount of food	6	3	11	0	4	0	7

Table 4. The cost of migration at a given time.

Time	**0**	**1**	**2**	**3**	**4**	**5**
$\mathbf{Z_i}$	45% Y_i	30% Y_i	35% Y_i	40% Y_i	50% Y_i	50% Y_i

There are three factors that were pair-wise compared. The factors are sucrose concentration, extent of smell and brightness of color of the food. The given pair-wise comparison matrix is shown in Table 5. These qualitative factors were quantified accordingly. The values of each factor at every node are given in Table 6.

Table 5. The pair-wise comparison matrix of the factors.

	Factor 1	**Factor 2**	**Factor 3**
Factor 1	1	6	4
Factor 2	1/6	1	1/3
Factor 3	1/4	3	1

Table 6. Value of Factor i at node j.

	Node 1	**Node 2**	**Node 3**	**Node 4**	**Node 5**	**Node 6**	**Node 7**
Factor 1	4	3	4	0	3	0	3
Factor 2	0	6	0	3	5	6	7
Factor 3	6	4	8	0	5	0	5

Assume that there is a competitor at node 5 and node 7. The corresponding utility function input values are given in Table 7. The dimensions of C_1, C_2 and C_3 are always the same as the dimension of $q_j v_j$'s.

Table 7. Utility Function Input Values.

	P(S≥30)	P(S≤20)	P(20<S<30)	C_1	C_2	C_3	S/S_T
Node 5	0.6	0.15	0.25	$0.4\,q_j v_j$	$2.5\,q_j v_j$	1.2 $q_j v_j$	0.6
Node 7	0.58	0.14	0.28	$0.3\,q_j v_j$	$1.5\,q_j v_j$	0.08 $q_j v_j$	0.6

Suppose that the natural production and depletion of the food source at time period t can be computed by considering the periodic formula: $q_{j,t} = \sigma \sin t + 0.8\Phi_{j,t-1}$, where $q_{j,t}$ is the natural production/depletion of the food source in node j at time period t, σ is the maximum production of the food source, and $\Phi_{j,t-1}$ is the remaining food in node j at time period $t-1$. Also, suppose that the population growth of the species follows the formula: $N_t = N_{t-1} + (0.2N_{t-1})$ where N_t is the colony's population at time period t and N_{t-1} is the colony's population at time period $t-1$.

After considering the given prototype example and conducting necessary calculations, the mathematical program for the model has been formulated. In the comparison made between the different factors considered using fuzzy AHP, the resulting weights are $w_1 = 0.71058$, $w_2 = 0.10067$ and $w_3 = 0.18875$.

For the time period $t = 0$, the resulting mathematical program is as follows:

$$\begin{cases} \text{Maximize} \\ 0.24981X_2 + 0.323942X_4 - 0.04359X_5 + 0.168403X_6 + 0.147623X_7 - d^- \\ \text{subject to:} \\ X_2 + X_4 + X_5 + X_6 + X_7 = 1 \\ 0.55697X_2 + 0.67162X_4 + 0.3564X_5 + 0.3125X_6 + 0.76009X_7 - d^- + d^+ = 1.36 \\ X_i \in \{0,1\},\ i = 2,4,5,6,7 \\ d^+, d^- \in [0,\infty) \end{cases}$$

For the time period $t = 1$, the resulting mathematical program is as follows:

$$\begin{cases} \text{Maximize} \\ 0.32915X_2 + 0.455083X_4 - 0.06877X_5 + 0.243584X_6 + 0.23287X_7 - d^- \\ \text{subject to:} \\ X_2 + X_4 + X_5 + X_6 + X_7 = 1 \\ 0.56589X_2 + 0.73606X_4 + 0.43270X_5 + 0.35093X_6 + 0.86931X_7 - d^- + d^+ = 1.64 \\ X_i \in \{0,1\},\ i = 2,4,5,6,7 \\ d^+, d^- \in [0,\infty) \end{cases}$$

For the time period $t = 2$, the resulting mathematical program is as follows:

$$\begin{cases} \text{Maximize} \\ 0.246509X_2 + 0.374003X_4 - 0.06588X_5 + 0.220907X_6 + 0.223517X_7 - d^- \\ \text{subject to:} \\ X_2 + X_4 + X_5 + X_6 + X_7 = 1 \\ 0.44726X_2 + 0.64351X_4 + 0.42394X_5 + 0.33897X_6 + 0.88053X_7 - d^- + d^+ = 1.96 \\ X_i \in \{0,1\},\ i = 2,4,5,6,7 \\ d^+, d^- \in [0, \infty) \end{cases}$$

For the time period $t = 3$, the resulting mathematical program is as follows:

$$\begin{cases} \text{Maximize} \\ 0.368114X_2 + 0.472137X_4 - 0.10719X_5 + 0.297321X_6 + 0.157841X_7 - d^- \\ \text{subject to:} \\ X_2 + X_4 + X_5 + X_6 + X_7 = 1 \\ 0.75495X_2 + 0.90221X_4 + 0.73383X_5 + 0.51385X_6 + 0.76357X_7 - d^- + d^+ = 2.36 \\ X_i \in \{0,1\},\ i = 2,4,5,6,7 \\ d^+, d^- \in [0, \infty) \end{cases}$$

For the time period $t = 4$, the resulting mathematical program is as follows:

$$\begin{cases} \text{Maximize} \\ 0.169049X_2 + 0.321784X_4 - 0.10899X_5 + 0.260222X_6 + 0.173674X_7 - d^- \\ \text{subject to:} \\ X_2 + X_4 + X_5 + X_6 + X_7 = 1 \\ 0.37405X_2 + 0.70992X_4 + 0.83763X_5 + 0.52435X_6 + 0.82831X_7 - d^- + d^+ = 2.83 \\ X_i \in \{0,1\},\ i = 2,4,5,6,7 \\ d^+, d^- \in [0, \infty) \end{cases}$$

For the time period $t = 5$, the resulting mathematical program is as follows:

$$\begin{cases} \text{Maximize} \\ 0.058483X_2 + 0.147054X_4 - 0.10899X_5 + 0.207822X_6 + 0.160277X_7 - d^- \\ \text{subject to:} \\ X_2 + X_4 + X_5 + X_6 + X_7 = 1 \\ 0.11227X_2 + 0.30721X_4 + 0.76006X_5 + 0.4075X_6 + 0.63014X_7 - d^- + d^+ = 3.39 \\ X_i \in \{0,1\},\ i = 2,4,5,6,7 \\ d^+, d^- \in [0, \infty) \end{cases}$$

In the objective functions, some values of Y_i's are negative, which is an implication of the competition existing in a node or of the high migration cost. The solution to the given prototype example is illustrated in Figure 3 and tabulated in Table 8.

Table 8. Solution to the given prototype example.

t	Preferred new hive (node)	Underachievement of food requirement	Objective Function Value
0	4	0.68840	-0.3666
1	4	0.90390	-0.4521
2	7	1.07950	-0.8586
3	4	1.45780	-0.9896
4	4	2.12010	-1.8006
5	7	2.75990	-2.6015

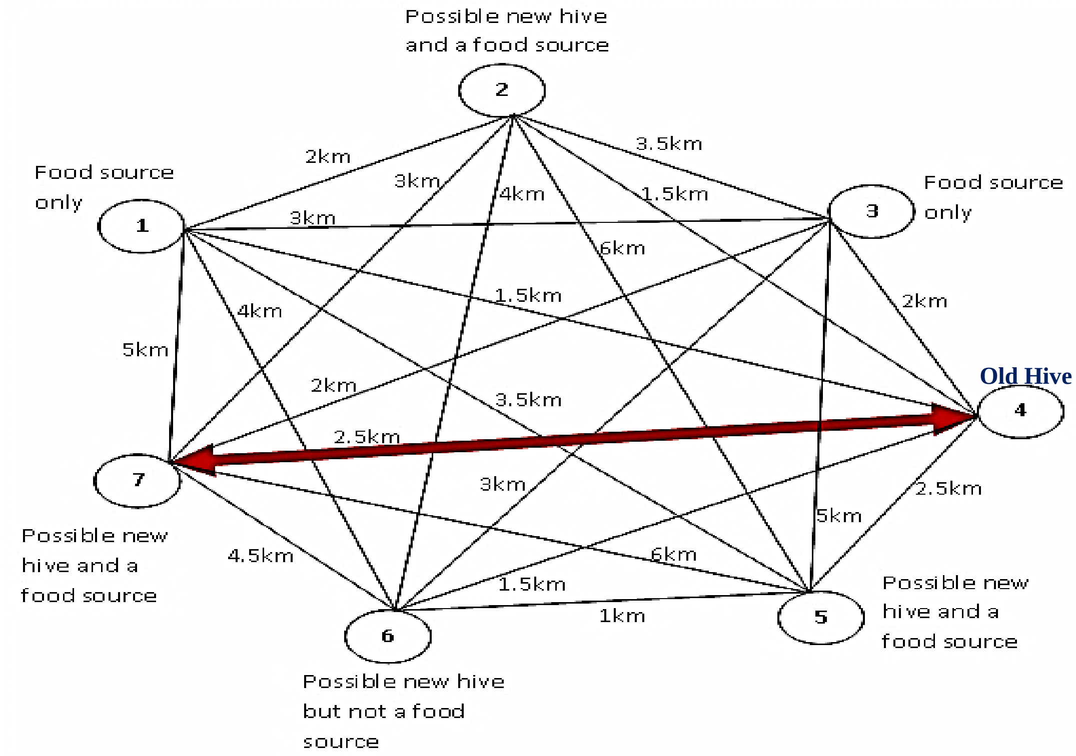

Figure 3. Visualization of the solution to the given prototype example

The result depicts that the colony will not transfer to a new hive location until the end of time period $t = 1$ because their present location (old hive, node 4) still gives the most satisficing benefits. But after time period $t = 1$, the colony will migrate to node 7, however, eventually the colony will migrate back to node

4 at $t = 3$ and $t = 4$. At $t = 5$, the colony will choose again node 7. After time period $t = 5$, the species could no longer find a patch where they can sustain the food requirement they needed. Though node 2 is a possible new hive and a food source, the colony cannot migrate to that location because it does not provide enough benefit for the colony.

Since the colony is considering the flight distance it can only travel as well as the distances of food sources, then it is possible that it cannot migrate to a specific new location. The negative value of the objective function value can be translated as the shortage on the desired benefits of species. The value of the deviational variable signifies the shortage from the minimum amount of food needed by a colony. These shortages could also be interpreted as the amount that the environment lacks in providing the colony's need. Sensitivity analysis can be done to observe variation in the optimal solution as input values vary.

5 Summary and Recommendation

The study aimed to determine a satisficing migration path of a colony of species foraging on patches. The study considered various factors such as the colony's collective preferences, availability of food, quality and quantity of food, distance of the food sources, population growth rate, existence of competition and cost of transferring from one location to another. The behavior of the colony was assumed to follow the Optimal Foraging Theory, where the colony considers the costs and benefits associated during migration. The preferences of species during migration were ranked using fuzzy Analytic Hierarchy Process (AHP). Successive Mixed-Integer Linear Goal Programming was used in making the necessary analysis and finding the satisficing solution. To illustrate the method for determining a specific migratory route for a certain colony of species, a prototype example was demonstrated.

In making the model, several things should be examined and reviewed. First and most importantly are the assumptions that are taken into account. Next thing is the listing of the factors and ranking it accordingly using the fuzzy AHP algorithm. In rating the preferences of species during migration, it is a must to check the consistency of the judgments made. Moreover, before doing the calculations, all values should have been non-dimensionalized and normalized.

It is recommended to add more factors and an actual observation or experimentation be done in order to test the validity of the model. The conceptual model formulated can be modified to fit a specific biological problem.

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