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A Study on Intuitionistic L-Fuzzy Subgroups

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Abstract

In this paper, we made an attempt to study the algebraic nature of intuitionistic L-fuzzy subgroups and its properties.

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INTRODUCTION:

Ever since the introduction of fuzzy sets by L.A.ZADEH [6] several researchers explored on the generalization of the notion of fuzzy set. The concept of intuitionistic fuzzy set was introduced by K.T.ATANASSOV [2] as a generalization of the notion of fuzzy set. KOG.A & BALKANAY.E [4] defined θ-Euclidean L-fuzzy ideals of rings. We study some of the concepts of intuitionistic L- fuzzy subgroups.

1. PRELIMINARIES:

1.1. Definition:

Let X be a non-empty set and L = (L, $\leq$,$\vee$, $\wedge$) be a completely distributive lattice, which has least and greatest elements say, 0 and 1 respectively. A L-fuzzy subset A of X is a function $A : X \rightarrow L$.

1.2. Definition:

Let (L, $\leq$) be a complete lattice with an involutive order reversing operation $N : L \rightarrow L$. An intuitionistic L-fuzzy set (ILFS) A in X is defined as an object of the form $A = \{< x, \mu_A(x), \nu_A(x) > / x \in X\}$, where $\mu_A : X \rightarrow L$ and $\nu_A : X \rightarrow L$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ satisfying $\mu_A(x) \leq N(\nu_A(x))$.

1.3 Definition: Let G be a group. A L-fuzzy subset A of G is said to be a L-**fuzzy subgroup** of G (LFSG) if $A(xy) \geq A(x) \wedge A(y)$ and $A(x^{-1}) \geq A(x)$, for all x and $y \in G$.

1.4 Definition:

Let G be a group. A L-fuzzy subset A of G is said to be an **anti-L-fuzzy subgroup** of G (ALFSG) if $A(xy) \leq A(x) \vee A(y)$ and $A(x^{-1}) \leq A(x)$, for all x and $y \in G$.

1.5 Definition:

Let G be a group. An intuitionistic L-fuzzy subset A of G is said to be an **intuitionistic L-fuzzy subgroup**(ILFSG) of G if

(i) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$,
(ii) $\mu_A(x^{-1}) \geq \mu_A(x)$,
(iii) $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$,
(iv) $\nu_A(x^{-1}) \leq \nu_A(x)$, for all x and $y \in G$.

SOME PROPERTIES:

1.1 Proposition:

If A is an intuitionistic L-fuzzy subgroup of a group G, then $\mu_A(x^{-1}) = \mu_A(x)$ and $\nu_A(x^{-1}) = \nu_A(x)$, $\mu_A(x) \leq \mu_A(e)$ and $\nu_A(x) \geq \nu_A(e)$, for $x \in G$ and $e \in G$.

Proof:

Let $x \in G$ and $e \in G$. Now, $\mu_A(x) = \mu_A((x^{-1})^{-1}) \geq \mu_A(x^{-1}) \geq \mu_A(x)$. Therefore, $\mu_A(x) = \mu_A(x^{-1})$.And, $\nu_A(x) = \nu_A((x^{-1})^{-1}) \leq \nu_A(x^{-1}) \leq \nu_A(x)$. Therefore, $\nu_A(x^{-1}) = \nu_A(x)$.

Now, $\mu_A(e) = \mu_A(xx^{-1}) \geq \mu_A(x) \wedge \mu_A(x^{-1}) = \mu_A(x)$. Therefore, $\mu_A(e) \geq \mu_A(x)$. And, $\nu_A(e) = \nu_A(xx^{-1}) \leq \nu_A(x) \vee \nu_A(x^{-1}) = \nu_A(x)$. Therefore, $\nu_A(e) \leq \nu_A(x)$.

1.2 Proposition:

If A is an intuitionistic L-fuzzy subgroup of a group G, then

(i) $\mu_A(xy^{-1}) = \mu_A(e)$ gives $\mu_A(x) = \mu_A(y)$.

(ii) $\nu_A(xy^{-1}) = \nu_A(e)$ gives $\nu_A(x) = \nu_A(y)$, for x and $e \in G$.

Proof: Let x and $e \in G$.
Now, $\mu_A(x) = \mu_A(xy^{-1}y) \geq \mu_A(xy^{-1}) \wedge \mu_A(y) = \mu_A(e) \wedge \mu_A(y) = \mu_A(y)$ $= \mu_A(yx^{-1}x) \geq \mu_A(yx^{-1}) \wedge \mu_A(x) = \mu_A(e) \wedge \mu_A(x) = \mu_A(x)$.
Therefore, $\mu_A(x) = \mu_A(y)$. And, $\nu_A(x) = \nu_A(xy^{-1}y) \leq \nu_A(xy^{-1}) \vee \nu_A(y)$ $= \nu_A(e) \vee \nu_A(y) = \nu_A(y) = \nu_A(yx^{-1}x) \leq \nu_A(yx^{-1}) \vee \nu_A(x) = \nu_A(e) \vee \nu_A(x)$ $= \nu_A(x)$. Therefore, $\nu_A(x) = \nu_A(y)$.

1.3 Proposition: A is an intuitionistic L-fuzzy subgroup of a group G iff $\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(xy^{-1}) \leq \nu_A(x) \vee \nu_A(y)$, for all x and $y \in G$.

Proof: Assume that A is an intuitionistic L-fuzzy subgroup of a group G.
We have, $\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$. Therefore, $\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$. And, $\nu_A(xy^{-1}) \leq \nu_A(x) \vee \nu_A(y^{-1}) \leq \nu_A(x) \vee \nu_A(y)$. Therefore, $\nu_A(xy^{-1}) \leq \nu_A(x) \vee \nu_A(y)$. Conversely, if $\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(xy^{-1}) \leq \nu_A(x) \vee \nu_A(y)$. Replace y by x, then $\mu_A(x) \leq \mu_A(e)$ and $\nu_A(x) \geq \nu_A(e), \forall$ x and $y \in G$. Now, $\mu_A(x^{-1}) = \mu_A(ex^{-1}) \geq \mu_A(e) \wedge \mu_A(x) = \mu_A(x)$. Therefore, $\mu_A(x^{-1}) \geq \mu_A(x)$. It follows that, $\mu_A(xy) = \mu_A(x(y^{-1})^{-1}) \geq \mu_A(x) \wedge \mu_A(y^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$. Therefore, $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$. And, $\nu_A(x^{-1}) = \nu_A(ex^{-1}) \leq \nu_A(e) \vee \nu_A(x) = \nu_A(x)$. Therefore, $\nu_A(x^{-1}) \leq \nu_A(x)$. Then $\nu_A(xy) = \nu_A(x(y^{-1})^{-1}) \leq \nu_A(x) \vee \nu_A(y^{-1}) \leq \nu_A(x) \vee \nu_A(y)$. Therefore $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$. Hence A is an intuitionistic L-fuzzy subgroup of a group G.

1.4 Proposition: Let A be an intuitionistic L-fuzzy subset of a group G. If $\mu_A(e) = 1$and $\nu_A(e) = 0$ and $\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(xy^{-1}) \leq \nu_A(x) \vee \nu_A(y)$, then A is an intuitionistic L-fuzzy subgroup of G.

Proof: We verify the axioms of an intuitionistic L-fuzzy subgroup of a group G. (i) $\mu_A(x^{-1}) = \mu_A(ex^{-1}) \geq \mu_A(e) \wedge \mu_A(x) = 1 \wedge \mu_A(x) = \mu_A(x)$.
Therefore, $\mu_A(x^{-1}) \geq \mu_A(x)$.
(ii) $\nu_A(x^{-1}) = \nu_A(ex^{-1}) \leq \nu_A(e) \vee \nu_A(x) = 0 \vee \nu_A(x) = \nu_A(x)$.
Therefore, $\nu_A(x^{-1}) \leq \nu_A(x)$.
(iii) $\mu_A(xy) = \mu_A(x(y^{-1})^{-1}) \geq \mu_A(x) \wedge \mu_A(y^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$. Therefore, $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$. (iv) $\nu_A(xy) = \nu_A(x(y^{-1})^{-1}) \leq \nu_A(x) \vee \nu_A(y^{-1}) \leq \nu_A(x) \vee \nu_A(y)$. Therefore, $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$. Hence A is an intuitionistic L-fuzzy subgroup of G.

1.5 Proposition: If A is an intuitionistic L-fuzzy subgroup of a group G, then H ={ x / x$\in$G : $\mu_A(x) = 1$, $\nu_A(x) = 0$ }is either empty or is a subgroup of G.

Proof: If no element satisfies this condition, then H is empty.
If x and y$\in$H, then $\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y^{-1}) \geq \mu_A(x) \wedge \mu_A(y) = 1 \wedge 1 = 1$. Therefore, $\mu_A(xy^{-1}) = 1$. And, $\nu_A(xy^{-1}) \leq \nu_A(x) \vee \nu_A(y^{-1}) = \nu_A(x) \vee \nu_A(y) = 0 \vee 0 = 0$. Therefore, $\nu_A(xy^{-1}) = 0$. We get $xy^{-1} \in$H. Therefore, H is a subgroup of G. Hence H is either empty or is a subgroup of G.

1.6 Proposition: If A is an intuitionistic L-fuzzy subgroup of a group G, then H = { < x, $\mu_A(x)$ > : $0 < \mu_A(x) \leq 1$ and $\nu_A(x) = 0$ }is either empty or is a L-fuzzy subgroup of G.

Proof: If no element satisfies this condition, then H is empty.
If A is an intuitionistic L-fuzzy subgroup of a group G , then $\nu_A(xy^{-1}) \leq \nu_A(x) \vee \nu_A(y^{-1}) \leq \nu_A(x) \vee \nu_A(y) = 0 \vee 0 = 0$. Therefore, $\nu_A(xy^{-1}) = 0$.
And, $\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$.
Therefore, $\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$. Hence H is L-fuzzy subgroup of G. Therefore H is either empty or is L-fuzzy subgroup of G.

1.7 Proposition: If A is an intuitionistic L-fuzzy subgroup of a group G, then H = {< x, $\mu_A(x)$ >: $0 < \mu_A(x) \leq 1$} is a L-fuzzy subgroup of G.

Proof: If A is an intuitionistic L-fuzzy subgroup of a group G, then $\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$. Therefore, $\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$. Hence H is a L-fuzzy subgroup of G.

1.8 Proposition: If A is an intuitionistic L-fuzzy subgroup of a group G, then H = {< x, $\nu_A(x)$ >: $0 < \nu_A(x) \leq 1$} is an anti-L-fuzzy subgroup of G.

Proof: If A is an intuitionistic L-fuzzy subgroup of a group G, then $\nu_A(xy^{-1}) \leq \nu_A(x) \vee \nu_A(y^{-1}) \leq \nu_A(x) \vee \nu_A(y)$. Therefore, $\nu_A(xy^{-1}) \leq \nu_A(x) \vee \nu_A(y)$. Hence H is an anti-L-fuzzy subgroup of G.

1.9 Proposition: If A is an intuitionistic L-fuzzy subgroup of a group G, then H = { x$\in$G : $\mu_A(x) = \mu_A(e)$ and $\nu_A(x) = \nu_A(e)$ }is a subgroup of G.

Proof: Given H = { x$\in$G : $\mu_A(x) = \mu_A(e)$ and $\nu_A(x) = \nu_A(e)$ }.Let x$\in$G. By Proposition 1.1, we have $\mu_A(x^{-1}) = \mu_A(x) = \mu_A(e)$ and $\nu_A(x^{-1}) = \nu_A(x) = \nu_A(e)$. Therefore, $\nu_A(x^{-1}) = \nu_A(e)$. Therefore, $\mu_A(x^{-1}) = \mu_A(e)$ and $\nu_A(x^{-1}) = \nu_A(e)$. Hence, $x^{-1} \in$ H, Now, $\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$
$= \mu_A(e) \wedge \mu_A(e) = \mu_A(e)$. Therefore, $\mu_A(xy^{-1}) \geq \mu_A(e)$ ----------- (1).
And, $\mu_A(e) = \mu_A((xy^{-1})(xy^{-1})^{-1}) \geq \mu_A(xy^{-1}) \wedge \mu_A(xy^{-1})^{-1} \geq \mu_A(xy^{-1}) \wedge \mu_A(xy^{-1}) = \mu_A(xy^{-1})$. Therefore, $\mu_A(e) \geq \mu_A(xy^{-1})$----------- (2).

From (1) and (2), we get $\mu_A(e) = \mu_A(xy^{-1})$. Now, $\nu_A(xy^{-1}) \leq \nu_A(x) \vee \nu_A(y^{-1}) \leq \nu_A(x) \vee \nu_A(y) = \nu_A(e) \vee \nu_A(e) = \nu_A(e)$.
Therefore, $\nu_A(xy^{-1}) \leq \nu_A(e)$ ---- (3).
And, $\nu_A(e) = \nu_A((xy^{-1})(xy^{-1})^{-1}) \leq \nu_A(xy^{-1}) \vee \nu_A(xy^{-1})^{-1}$
$\leq \nu_A(xy^{-1}) \vee \nu_A(xy^{-1}) = \nu_A(xy^{-1})$. Therefore, $\nu_A(e) \leq \nu_A(xy^{-1})$------ (4).
From (3) and (4), we get $\nu_A(e) = \nu_A(xy^{-1})$. Hence $\mu_A(e) = \mu_A(xy^{-1})$ and $\nu_A(e) = \nu_A(xy^{-1})$. Therefore, $xy^{-1} \in H$.
Hence H is a subgroup of G.

1.10 Proposition: If A is an intuitionistic L-fuzzy subgroup of a group G, then $H = \{ < x, \mu_A(x) > : \mu_A(x) = \mu_A(e)$ and $\nu_A(x) = \nu_A(e) \}$ is a L-fuzzy subgroup of G.

Proof: Given that A is an intuitionistic L-fuzzy subgroup of a group G.
Let x, y and $e \in G$. By Proposition 1.9, $H = \{ x : \mu_A(x) = \mu_A(e)$ and $\nu_A(x) = \nu_A(e) \}$ is a subgroup of G. Therefore, $xy^{-1} \in H$.
Now, $\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$.
Therefore, $\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$. Hence H is a L-fuzzy subgroup of G.

1.11 Proposition: If A is an intuitionistic L-fuzzy subgroup of a group G, then $H = \{ < x, \nu_A(x) > : \mu_A(x) = \mu_A(e)$ and $\nu_A(x) = \nu_A(e) \}$ is an anti-L-fuzzy subgroup of G.

Proof: Given that A is an intuitionistic L-fuzzy subgroup of a group G.
Let x, y and $e \in G$. By Proposition 1.9, $H = \{ x : \mu_A(x) = \mu_A(e)$ and $\nu_A(x) = \nu_A(e) \}$ is a subgroup of G. Therefore, $xy^{-1} \in H$.
Now, $\nu_A(xy^{-1}) \leq \nu_A(x) \vee \nu_A(y^{-1}) \leq \nu_A(x) \vee \nu_A(y)$.
Therefore, $\nu_A(xy^{-1}) \leq \nu_A(x) \vee \nu_A(y)$.
Hence H is an anti-L-fuzzy subgroup of G.

1.12 Proposition:
If A is an intuitionistic L-fuzzy subgroup of a group G, then
(i) if $\mu_A(xy^{-1}) = 1$, then $\mu_A(x) = \mu_A(y)$.
(ii) if $\nu_A(xy^{-1}) = 0$, then $\nu_A(x) = \nu_A(y)$ for x and $y \in G$.

Proof: Let x and $y \in G$. (i) Now, $\mu_A(x) = \mu_A(xy^{-1}y) \geq \mu_A(xy^{-1}) \wedge \mu_A(y)$
$= 1 \wedge \mu_A(y) = \mu_A(y) = \mu_A(y^{-1}) = \mu_A(x^{-1}xy^{-1}) \geq \mu_A(x^{-1}) \wedge \mu_A(xy^{-1}) = \mu_A(x^{-1}) \wedge 1$
$= \mu_A(x^{-1}) = \mu_A(x)$. Therefore, $\mu_A(x) = \mu_A(y)$.
(ii) Now, $\nu_A(x) = \nu_A(xy^{-1}y) \leq \nu_A(xy^{-1}) \vee \nu_A(y) = 0 \vee \nu_A(y) = \nu_A(y) = \nu_A(y^{-1})$
$= \nu_A(x^{-1}xy^{-1}) \leq \nu_A(x^{-1}) \vee \nu_A(xy^{-1}) = \nu_A(x^{-1}) \vee 0 = \nu_A(x^{-1}) = \nu_A(x)$.
Therefore, $\nu_A(x) = \nu_A(y)$.

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