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Triple Positive Solutions of Four-Point Singular Boundary Value Problems for p-Laplacian Dynamic Equations on Time Scales

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Abstract

Let **T** be a time scale. We study the existence of three positive solutions for the nonlinear four-point singular boundary value problem with p-Laplacian operator on time scales. Some new results are obtained for three positive solutions by applying Leggett-Williams fixed-point theorem in a cone, which is different from the previous.

Mathematics Subject Classification: 34B16

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1 Introduction

A time scale **T** is a nonempty closed subset of R. We make the blanket assumption that 0, T are point in **T**. By an internal $(0, T)$, we always mean the intersection of the real internal $(0, T)$ with the given time scale, that is $(0, T) \bigcap \mathbf{T}$. The theory of dynamical systems on time scales is undergoing rapid development as it provides a unifying structure for the study of differential equations in the continuous case and the study of finite difference equations in the discrete case; Here, two-point boundary-value problems have been extensively studied; see [1, 2, 3, 4, 5] and the references therein. However, there are not much concerning the p-Laplacian problems on time scales.

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In this paper, we study the existence of three positive solutions for the following nonlinear four-point singular boundary value problem with p-Laplacian operator on time scales:

$$\begin{cases} (\phi_p(u^{\triangle}))^{\nabla} + a(t)f(u(t)) = 0, \ t \in (0,T), \\ \alpha\phi_p(u(0)) - \beta\phi_p(u^{\triangle}(\xi)) = 0, \ \gamma\phi_p(u(T)) + \delta\phi_p(u^{\triangle}(\eta)) = 0, \end{cases} \tag{1.1}$$

where $\phi_p(s)$ is a p-Laplacian operator, i.e. $\phi_p(s) = |s|^{p-2}s, \ p > 1, \ \phi_q = \phi_p^{-1}$, $\frac{1}{p} + \frac{1}{q} = 1, \ \xi, \eta \in (0,T)$ is prescribed and $\xi < \eta, \ a : (0,1) \to [0,\infty), \ \alpha > 0, \ \beta \geq 0, \ \gamma > 0, \ \delta \geq 0$, and

(H_1) $f \in C([0,+\infty),[0,+\infty))$;

(H_2) $a : (0,T) \to [0,+\infty)$ is left dense continuous (i.e.,$a(t) \in C_{ld}((0,T),[0,+\infty))$ and there exists $t_0 \in (0,T)$, such that

$$a(t_0) > 0, \ 0 < \int_0^T a(s)\,\nabla s < +\infty.$$

For convenience, we list here the following definitions which are needed later.

A time scale $\mathbf{T}$ is an arbitrary nonempty closed subset of real numbers R. The operators σ and ρ from $\mathbf{T}$ to $\mathbf{T}$ which defined by [1-5],

$$\sigma(t) = \inf\{\tau \in \mathbf{T} \mid \tau > t\} \in \mathbf{T}, \ \rho(t) = \sup\{\tau \in \mathbf{T} \mid \tau < t\} \in \mathbf{T}.$$

are called the forward jump operator and the backward jump operator, respectively.

The point $t \in \mathbf{T}$ is left-dense, left-scattered, right-dense, right-scattered if $\rho(t) = t, \ \rho(t) < t, \ \sigma(t) = t, \ \sigma(t) > t$, respectively. If $\mathbf{T}$ has a right scattered minimum m, define $\mathbf{T}_k = \mathbf{T} - \{m\}$; otherwise set $\mathbf{T}_k = \mathbf{T}$. If $\mathbf{T}$ has a left scattered maximum M, define $\mathbf{T}^k = \mathbf{T} - \{M\}$; otherwise set $\mathbf{T}^k = \mathbf{T}$.

Let $f : \mathbf{T} \to R$ and $t \in \mathbf{T}^k$ (assume t is not left-scattered if $t = \sup \mathbf{T}$), then the delta derivative of f at the point t is defined to be the number $f^{\Delta}(t)$ (provided it exists) with the property that for each $\epsilon > 0$ there is a neighborhood U of t such that

$$|f(\sigma(t)) - f(s) - f^{\Delta}(t)(\sigma(t) - s)| \leq |\sigma(t) - s|, \quad \text{for all } s \in U.$$

Similarly, for $t \in \mathbf{T}$ (assume t is not right-scattered if $t = \inf \mathbf{T}$), the nabla derivative of f at the point t is defined in [1] to be the number $f^{\nabla}(t)$ (provided it exists) with the property that for each $\epsilon > 0$ there is a neighborhood U of t such that

$$|f(\rho(t)) - f(s) - f^{\nabla}(t)(\rho(t) - s)| \leq |\rho(t) - s|, \quad \text{for all } s \in U.$$

If $\mathbf{T} = R$, then $x^{\Delta}(t) = x^{\nabla}(t) = x'(t)$. If $\mathbf{T} = Z$, then $x^{\Delta}(t) = x(t+1) - x(t)$ is the forward difference operator while $x^{\nabla}(t) = x(t) - x(t-1)$ is the backward difference operator.

2 Preliminaries and Lemmas

In this section, we give some definitions and preliminaries.
Definition 2.1. Let E be a real Banach space. A nonempty closed set $P \subset E$ is said to be a cone provided that

(i) $u \in P$, $a \geq 0$ implies $au \in P$; and

(ii) $u, \ -u \in P$ implies $u = 0$.

Definition 2.2. Given a cone P in a real Banach space E, a functional $\alpha : P \to [0, \infty)$ is said to be nonnegative continuous concave on P, provided $\alpha(tx + (1-t)y) \geq t\alpha(x) + (1-t)\alpha(y)$, for all $x, \ y \in P$ with $t \in [0, 1]$.

Let $a, \ b, \ r > 0$ be constants with P and α as defined above, we note

$$P_r = \{y \in P| \ \ ||y|| < r\}, \quad P\{\alpha, a, b\} = \{y \in P| \ \ \alpha(y) \geq a, \quad ||y|| \leq b\}.$$

Our main tool of this paper is the following the fixed point theorem due to Leggett-Williams (see[6, 7]).
Theorem 2.1(Leggett-Williams). Assume E be a real Banach space, $P \subset E$ be a cone. Let $A : \overline{P}_c \to \overline{P}_c$ be completely continuous and α be a nonnegative continuous concave functional on P such that $\alpha(y) \leq ||y||$, for $y \in \overline{P}_c$. Suppose that there exist $0 < a < b < d \leq c$ such that

(C_1) $\{y \in P(\alpha, \ b, \ d)| \ \ \alpha(y) > b\} \neq \emptyset$ and $\alpha(Ay) > b$, for all $y \in P(\alpha, \ b, \ d)$;

(C_2) $||Ay|| < a$, for all $||y|| \leq a$;

(C_3) $\alpha(Ay) > b$ for all $y \in P(\alpha, \ b, \ c)$ with $||Ay|| > d$.

Then A has at least three fixed points $y_1, \ y_2, \ y_3$ satisfying

$$||y_1|| < a, \qquad b < \alpha(y_2),$$

and

$$||y_3|| > a, \quad \alpha(y_3) < b.$$

Let $E = C_{ld}([0, \ T], \ R)$ which is a Banach space with the maximum norm $\|u\| = \max_{t \in [0, \ T]} |u(t)|$. And define the cone $P \subset E$ by

$$P = \{u \in E : u(t) \geq 0, \quad u(t) \text{ is concave function}, t \in [0, \ T]\}.$$

We can easily get the following lemmas.

Lemma 2.1. Suppose condition (H_2) holds, then there exists a constant $\mu \in (0, \ \frac{1}{2})$ satisfies

$$0 < \int_{\mu}^{T-\mu} a(t) \nabla t < \infty.$$

Furthermore, the function

$$y_1(t) = \phi_q\left(\int_\mu^t a(t)\nabla t\right) + \phi_q\left(\int_t^{T-\mu} a(t)\nabla t\right),\ t \in [\mu,\ T-\mu].$$

is positive continuous functions on $[\mu,\ T-\mu]$, therefore $y_1(t)$ has minimum on $[\mu,\ T-\mu]$, hence we suppose that there exists $L_1 > 0$ such that $L_1 = \min_{t\in[\mu,\ T-\mu]} y_1(t)$.

Lemma 2.2. Let $u \in P$ and μ of Lemma 2.1, then

$$u(t) \geq \mu\|u\|,\ t \in [\mu,\ T-\mu].$$

The proof of Lemma 2.2 is similar to the proof of Lemma 2.2 in [9], so we omit the details.

Now, we define a mapping $A : P \to E$ given by

$$(Au)(t) = \begin{cases} \phi_q\left(\frac{\beta}{\alpha}\int_\xi^\sigma a(r)f(u(r))\nabla r\right) + \int_0^t \phi_q\left(\int_s^\sigma a(r)f(u(r))\nabla r\right)\Delta s, & 0 \leq t \leq \sigma, \\ \phi_q\left(\frac{\delta}{\gamma}\int_\sigma^\eta a(r)f(u(r))\nabla r\right) + \int_t^T \phi_q\left(\int_\sigma^s a(r)f(u(r))\nabla r\right)\Delta s, & \sigma \leq t \leq T. \end{cases}$$

Because of

$$(Au)^\Delta(t) = \begin{cases} \phi_q\left(\int_t^\sigma a(r)f((u)(r))\nabla r\right) \geq 0, & 0 \leq t \leq \sigma, \\ -\phi_q\left(\int_\sigma^t a(r)f((u)(r))\nabla r\right) \leq 0, & \sigma \leq t \leq T, \end{cases}$$

the operator A is monotone decreasing continuous and $(Au)^\Delta(\sigma) = 0$, and for any $u \in P$, we have

$$(\phi_q(Au)^\Delta)^\nabla(t) = -a(t)f((u)(t), \quad a.e.\ t \in (0,1),$$

and $(Au)(\sigma) = \|A(u)\|$. Therefore we have $A(P) \subset P$.

Lemma 2.3. $A : P \to P$ is completely continuous.

Proof. Suppose $D \subset P$ is a bounded set, Let $M > 0$ such that $\|u\| \leq M,\ u \in D$. For any $u \in D$, we have

$$\begin{aligned} &\|Au\| = (Au)(\sigma) \\ &= \phi_q\left(\frac{\beta}{\alpha}\int_\xi^\sigma a(r)f(u(r))\nabla r\right) + \int_0^\sigma \phi_q\left(\int_s^\sigma a(r)f(u(r))\nabla r\right)\Delta s, \\ &\leq \left[\phi_q\left(\frac{\beta}{\alpha}\int_0^\sigma a(r)\nabla r\right) + \int_0^\sigma \phi_q\left(\int_s^\sigma a(r)\nabla r\right)\Delta s\right]\phi_q\left(\sup_{u\in D} f(u)\right). \end{aligned}$$

Then $A(D)$ is bounded.

Furthermore it is easy see by Arzela-ascoli theorem and Lebesgue dominated convergence theorem that $A : P \to P$ is completely continuous. The proof is complete.

3 Main Results

For notational convenience, we introduce the following constants:

$$\lambda = \mu L_1, Q = (T + \phi_q\left(\frac{\beta}{\alpha}\right))\phi_q\left(\int_0^T a(r)\nabla r\right).$$

Finally, we define the nonnegative, continuous concave functional $\alpha : P \to [0, \infty)$ by

$$\alpha(u) = \frac{1}{2}[u(\mu) + u(T - \mu)].$$

We observe here that, for every $u \in P$, $\alpha(u) \leq ||u||$.

Our main result of this paper is as fellows:

Theorem 3.1. Suppose that conditions (H_1), (H_2) hold. Let $a < \mu b < b < \dfrac{b}{\mu} = d \leq c$ and assume that the following conditions are satisfied

(A_1): $f(u) < \phi_p(\dfrac{a}{Q})$, for $0 \leq u \leq a$;

(A_2): there exists a number $q > d$ such that $f(u) < \phi_p(\dfrac{q}{Q})$, for $0 \leq u \leq q$;

(A_3): $f(u) \geq \phi_p(\dfrac{2b}{\lambda})$, for $\mu b \leq u \leq d$.

Then, the boundary value problem (1.1) has at least three positive solutions u_1, u_2 and u_3 such that

$$||u_1|| < a, \quad b < \alpha(u_2),$$

and

$$||u_3|| > a, \quad \alpha(u_3) < b.$$

Proof . By Lemma 2.3, we can know $A : P \to P$ is completely continuous. Now we show that the conditions of Theorem 2.1 are satisfied. We first assert that if there exists a positive number c where $c > d$ such that $A(\overline{P}_c) \subset \overline{P}_c$. Suppose that condition (A_2) holds, if $u \in \overline{P}_q$, then

$$\begin{aligned}
&||Au|| = (Au)(\sigma)\\
&= \phi_q\left(\tfrac{\beta}{\alpha}\int_\xi^\sigma a(r)f((u)(r))\nabla r\right) + \int_0^\sigma \phi_q\left(\int_s^\sigma a(r)f((u)(r))\nabla r\right)\Delta s\\
&\leq \phi_q\left(\tfrac{\beta}{\alpha}\int_0^T a(r)f((u)(r))\nabla r\right) + \int_0^T \phi_q\left(\int_0^T a(r)f((u)(r))\nabla r\right)\Delta s\\
&= \left[T + \phi_q\left(\tfrac{\beta}{\alpha}\right)\right]\phi_q\left(\int_0^T a(r)f((u)(r))\nabla r\right) \leq \left[T + \phi_q\left(\tfrac{\beta}{\alpha}\right)\right]\phi_q\left(\int_0^T a(r)\nabla r\right)\tfrac{q}{Q}\\
&= q.
\end{aligned}$$

Thus $Au \in \overline{P}_q$. Therefore, taking $c = q$, we have $A(\overline{P}_c) \subset \overline{P}_c$. Especially, if $u \in \overline{P}_a$, then assumption (A_1) implies $f(u) < \phi_p(\dfrac{a}{Q})$, for $0 \leq u \leq a$, from which we obtained above, we have $A : \overline{P}_a \to P_a$.

To fulfil condition (C_1) of Theorem 2.1, let $u(t) \equiv \frac{b+d}{2}$, then $u \in P$, $||u|| = \frac{b+d}{2}$ and $\alpha(u) = \frac{b+d}{2} > b$. That is $\{u \in P(\alpha,\ b,\ d) |\ \ \alpha(u) > b\} \neq \emptyset$. Moreover, if $u \in P(\alpha,\ b,\ d)$, then $\alpha(u) \geq b$, so $b \leq ||u|| \leq d$. By Lemma 2.2 we have $\mu b \leq \mu||u|| \leq u(t) \leq d$, for $t \in [\mu, T-\mu]$. From assumption (A_3) implies $f(u) \geq \phi_p(\frac{2b}{\lambda})$, for $\mu b \leq u \leq d$

We shall discuss it from three perspectives.

(i) If $\sigma \in [\mu,\ T-\mu]$, we have

$$\begin{aligned}
&2\alpha(A(u)) = (Au)(\mu) + (Au)(T-\mu)\\
&\geq \int_0^\mu \phi_q\left(\int_s^\sigma a(r)f((u)(r))\nabla r\right)\Delta s + \int_{T-\mu}^T \phi_q\left(\int_\sigma^s a(r)f((u)(r))\nabla r\right)\Delta s\\
&\geq \mu\left(\phi_q(\int_\mu^\sigma a(r)\nabla r) + \phi_q(\int_\sigma^{T-\mu} a(r)\nabla r)\right)\frac{2b}{\lambda}\\
&\geq b > b.
\end{aligned}$$

(ii) If $\sigma \in (T-\mu,\ T]$, we have

$$\begin{aligned}
&\alpha(A(u)) = \tfrac{1}{2}((Au)(\mu) + (Au)(T-\mu)) \geq (A(u))(\mu)\\
&\geq \int_0^\mu \phi_q\left(\int_s^\sigma a(r)f((u)(r))\nabla r\right)\Delta s \geq \int_0^\mu \phi_q\left(\int_\mu^{T-\mu} a(r)f((u)(r))\nabla r\right)\Delta s\\
&\geq \mu\phi_q\left(\int_\mu^{T-\mu} a(r)\nabla r\right)\frac{2b}{\lambda} \geq 2b > b.
\end{aligned}$$

(iii) If $\sigma \in [0,\ \mu)$, we have

$$\begin{aligned}
&\alpha(A(u)) = \tfrac{1}{2}((Au)(\mu) + A(u)(T-\mu)) \geq (A(u))(T-\mu)\\
&\geq \int_{T-\mu}^T \phi_q\left(\int_\sigma^{T-\mu} a(r)f((u)(r))\nabla r\right)\Delta s \geq \int_{T-\mu}^T \phi_q\left(\int_\mu^{T-\mu} a(r)f((u)(r))\nabla r\right)\Delta s\\
&\geq \mu\phi_q\left(\int_\mu^{T-\mu} a(r)\nabla r\right)\frac{2b}{\lambda} \geq 2b > b.
\end{aligned}$$

Therefore, condition (C_1) of Theorem 2.1 is satisfied.

Finally, we address condition (C_3) of Theorem 2.1. For this we choose $u \in P(\alpha,\ b,\ c)$ with $||Au|| > d$. Then from Lemma 2.2, we have

$$\alpha(Au) = \frac{1}{2}[(Au)(\mu) + (Au)(T-\mu)] \geq \mu||Au|| > \mu d = b.$$

Hence, condition (C_3) of Theorem 2.1 holds.

To sum up, all the hypotheses of Theorem 2.1 are satisfied. Hence A has at least three fixed points, i.e., the boundary value problem (1.1) has at least three positive solutions u_1, u_2 and u_3 such that

$$||u_1|| < a, \quad b < \alpha(u_2),$$

and

$$||u_3|| > a, \quad \alpha(u_3) < b.$$

Theorem 3.2. Suppose that conditions (H_1), (H_2) hold. Let $a < \mu b < b < \frac{b}{\mu} = d \leq c$ and assume that the following conditions are satisfied

(B_1): $f(u) < \phi_p(\frac{a}{Q})$, for $0 \leq u \leq a$;

(B_2): $\lim_{u\to\infty} \frac{f(u)}{\phi_p(u)} < \phi_p\left(\frac{1}{Q}\right)$;

(B_3): $f(u) \geq \phi_p(\frac{2b}{\lambda})$, for $\mu b \leq u \leq d$.

Then, the boundary value problem (1.1) has at least three positive solutions $u_1,\ u_2$ and u_3 such that

$$||u_1|| < a, \quad b < \alpha(u_2),$$

and

$$||u_3|| > a, \quad \alpha(u_3) < b.$$

Proof . By Lemma 2.3 and Lemma 2.4, we can know $A : P \to P$ is completely continuous. Now we show that the conditions of Theorem 2.1 are satisfied. We only prove that there exists a positive number c where $c > d$ such that $A(\overline{P}_c) \subset \overline{P}_c$ when condition (B_2) holds. Suppose that condition (B_2) holds, then there exist $L > 0$ and $\varepsilon < \phi_p\left(\frac{1}{Q}\right)$ such that

$$\frac{f(u)}{\phi_p(u)} < \varepsilon \quad \text{for} \quad u > L.$$

Set $M = \{f(u)|\ \ 0 \leq u \leq L\}$, then

$$f(u) \leq M + \varepsilon\phi_p(u) \quad \text{for} \quad u \geq 0.$$

We choose c such that

$$\phi_p(c) > \max\left\{\phi_p(d),\ M\left(\phi_p(\frac{1}{Q} - \varepsilon)\right)^{-1}\right\}.$$

If $u \in \overline{P}_c$, then

$$\begin{aligned}
&||Au|| = (Au)(\sigma) \\
&= \phi_q\left(\tfrac{\beta}{\alpha}\textstyle\int_\xi^\sigma a(r)f((u)(r))\nabla r\right) + \int_0^\sigma \phi_q\left(\int_s^\sigma a(r)f((u)(r))\nabla r\right)\Delta s \\
&\leq \phi_q\left(\tfrac{\beta}{\alpha}\textstyle\int_0^T a(r)f((u)(r))\nabla r\right) + \int_0^T \phi_q\left(\int_0^T a(r)f((u)(r))\nabla r\right)\Delta s \\
&= \left[T + \phi_q\left(\tfrac{\beta}{\alpha}\right)\right]\phi_q\left(\textstyle\int_0^T a(r)f((u)(r))\nabla r\right) \\
&\leq \left[T + \phi_q\left(\tfrac{\beta}{\alpha}\right)\right]\phi_q\left(\textstyle\int_0^T a(r)\left(\phi_p(c) \times (\phi_p(\frac{1}{Q}) - \varepsilon) + \varepsilon\phi_p(c)\right)\nabla r\right) \\
&\leq \left[T + \phi_q\left(\tfrac{\beta}{\alpha}\right)\right]\phi_q\left(\textstyle\int_0^T a(r)\nabla r\right)\tfrac{c}{Q} \\
&= c.
\end{aligned}$$

Thus $Au \in \overline{P}_c$. Therefore, we have $A(\overline{P}_c) \subset \overline{P}_c$. Especially, if $u \in \overline{P}_a$, then assumption (A_1) implies $f(u) < \phi_p(\frac{a}{Q})$, for $0 \le u \le a$, from which we obtained above, we have $A : \overline{P}_a \to P_a$. The rest proof of Theorem 3.2 is similar to Theorem 3.1, we omit it.

From Theorem 3.1, we see that, when assumption like (A_1), (A_2) and (A_3) are imposed appropriately on f, we can establish the existence of an arbitrary odd number of positive solutions of boundary value problem (1.1).

Theorem 3.3. Suppose that conditions (H_1), (H_2) hold. Let $a_1 < \mu b_1 < b_1 < \frac{b_1}{\mu} = d_1 \le a_2 < \mu b_2 < b_2 < \frac{b_2}{\mu} = d_2 \le \cdots < a_n, n \in N$ and assume that the following conditions are satisfied

(D_1): $f(u) < \phi_p(\frac{a_i}{Q})$, for $0 \le u \le a_i,\ i = 1,\ 2,\ \cdots n$;

(D_2): $f(u) \ge \phi_p(\frac{2b_i}{\lambda})$, for $\mu b_i \le u \le d_i,\ i = 1,\ 2,\ \cdots n$.

Then, the boundary value problem (1.1) has at least $2n-1$ positive solutions.

Proof . When $n = 1$, it follows from condition (D_1) that $A : \overline{P}_{a_1} \to P_{a_1}$. Therefore there exists at least one fixed point $u_1 \in \overline{P}_{a_1}$ by the Schauder fixed point theorem. When $n = 2$, it is that Theorem 3.1 holds (with $d_1 = \frac{b_1}{\mu}, c_1 = a_2$), so we can get at least three positive solutions $u_1,\ u_2$ and u_3 such that$||u_1|| < a, \quad b < \alpha(u_2)$, and $||u_3|| > a, \quad \alpha(u_3) < b$. Following the identical fashion, by the induction method we immediately complete the proof.

4 Application

In this section , we give an example as an application to illustrate our results.

Example. Consider the following singular boundary value problem (SBVP) with p-Laplacian

$$\begin{cases} (\phi_p(u^\Delta))^\nabla + a(t)f(u) = 0, \quad 0 < t < \frac{3}{2}, \\ 4\phi_p(u(0)) - \phi_p(u^\Delta(\frac{1}{4})) = 0, \quad \phi_p(u(\frac{3}{2})) + \delta\phi_p(u^\Delta(\frac{1}{2})) = 0, \end{cases} \tag{4.1}$$

where

$$\beta = \gamma = 1, \quad \alpha = 4, \quad p = \frac{3}{2}, \quad \delta \ge 0, \quad \xi = \frac{1}{4}, \quad \eta = \frac{1}{2}, \quad T = \frac{3}{2}, \quad a(t) = \frac{1}{4}t^{-\frac{1}{2}},$$

and

$$f(u) = \begin{cases} 1, & 0 \le u \le 2, \\ 4u + 11, & 2 \le u \le 9, \\ 47, & u \ge 9. \end{cases}$$

Obviously, $q = 3$, we choose $\mu = \frac{1}{4}$, then it is easy to see by calculating that

$$L_1 = \min_{t\in[\mu,T-\mu]} y_1(t) = 3 - \sqrt{5},$$

$$Q = \frac{25\sqrt{6}}{64}.$$

Let $a = 2,\ b = 9,\ c = 6250$. Then we have

$$2 < \frac{9}{4} < 9 < 36 \leq 6250,$$

$$f(u) = 1 < \phi_p(\frac{a}{Q}) = \sqrt{\frac{2 \times 32\sqrt{6}}{75}}, \quad \text{for} \quad 0 \leq u \leq 2,$$

$$f(u) \leq 47 < \phi_p(\frac{c}{Q}) = \sqrt{\frac{6250 \times 32\sqrt{6}}{75}}, \quad \text{for} \quad 0 \leq u \leq 6250,$$

$$f(u) \geq 20 > \phi_p(\frac{2b}{\lambda}) = \sqrt{\frac{2 \times 9 \times 4}{3 - \sqrt{5}}}, \quad \text{for} \quad \frac{9}{4} \leq u \leq 36.$$

Therefore, by Theorem 3.1 we can know SBVP (4.1) has at least three positive solutions u_1, u_2 and u_3 satisfying

$$||u_1|| < 2, \quad 9 < \alpha(u_2),$$

and

$$||u_3|| > 2, \quad \alpha(u_3) < 6.$$

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