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Nontrivial Solutions for Nonlinear Higher Order Multi-Point Boundary Value Problem on Time Scales with All Derivatives

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Abstract

In this paper, we consider the existence of nontrivial solutions for nonlinear higher order multi-point boundary value problem on time scales with all derivatives. In the case where a nonlinearity may change sign and contains all derivatives, several sufficient conditions for the existence of nontrivial solution are obtained by using Leray-Schauder nonlinear alternative under certain growth conditions on the nonlinearity. As an application, some examples to demonstrate our results are given.

Mathematics Subject Classification: 34B16

Keywords: Time scale; Positive solutions; Higher order multi-point boundary value problem; Leray- Schauder nonlinear alternative

1 Introduction

A time scale $\mathbf{T}$ is a nonempty closed subset of R. We make the blanket assumption that 0, T are point in $\mathbf{T}$. By an internal $(0, T)$, we always mean the intersection of the real internal $(0, T)$ with the given time scale, that is (0, T)$\bigcap \mathbf{T}$. The theory of dynamical systems on time scales is undergoing rapid development as it provides a unifying structure for the study of differential equations in the continuous case and the study of finite difference equations in

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the discrete case; Here, two-point boundary-value problems have been extensively studied; see [2, 4, 9-12, 14, 16-19] and the references therein.

In [2], Anderson discussed the following dynamic equation on time scales:

$$\begin{cases} u^{\Delta\nabla}(t) + a(t)f(u(t)) = 0, t \in (0, T), \\ u(0) = 0,\ \alpha u(\eta) = u(T). \end{cases}$$

He obtained some results for the existence of one positive solution of the problem based on the limits $f_0 = \lim\limits_{u\to 0^+} \dfrac{f(u)}{u}$ and $f_\infty = \lim\limits_{u\to\infty} \dfrac{f(u)}{u}$ as well as existence of at least three positive solutions.

In [19], Sun considered the following third-order two-point boundary value problem on time scales:

$$\begin{cases} u^{\Delta\Delta\Delta}(t) + f(t, u(t), u^{\Delta\Delta}(t)) = 0, t \in [a, \sigma(b)], \\ u(a) = A, \quad u(\sigma^b) = B, \quad u^{\Delta\Delta}(a) = C, \end{cases}$$

where $a, b \in \mathbf{T}$ and $a < b$. Some existence criteria of solution and positive solution are established by using Leray-Schauder fixed point theorem.

All the above works were done under the assumption that the nonlinear term is nonnegative. However, little work has been done to the existence of positive solutions for boundary value problem with nonlinear term f being allowed to change sign. Especially, to date no paper has appeared in the literature which discusses the nonlinear higher order multi-point boundary value problem on time scales with all derivatives when nonlinearity in the differential equation may change sign.

In this paper, we are concerned with the existence of nontrivial solution of the following higher order multi-point boundary value problem on time scales with all derivatives:

$$\begin{cases} u^{\Delta^n}(t) + f(t, u(t), u^{\Delta}(t) \cdots, u^{\Delta^{n-2}}(t), u^{\Delta^{n-1}}(t)) = 0, \quad t \in (0, T), \\ u(0) = u^{\Delta}(0) = \cdots = u^{\Delta^{n-2}}(0) = 0, \quad u^{\Delta^{n-1}}(T) = \sum\limits_{i=1}^{m-2} k_i u^{\Delta^{n-1}}(\xi_i), \end{cases} \tag{1.1}$$

where $\xi_i \in (0, \rho(T)),\ k_i \geq 0, 0 < \sum_{i=1}^{m-2} k_i < 1, f \in C_{ld}([0, T] \times R^n, R), R = (-\infty, +\infty)$. By using Leray- Schauder nonlinear alternative, we study the existence of nontrivial solutions of multi-point boundary value problem (1.1). The interesting point of this paper is the nonlinear term f with all derivatives may change sign.

The aim of this paper is to establish some simple criteria for the existence of nontrivial solution of the boundary value problem (1.1). Our results are new and different from those of [2, 11, 12, 14]. Particularly, we do not require any monotonicity and nonnegative on f, which was essential for the technique used in [2, 11, 12, 14]. And our conditions are given in integral expression, they are easy to check.

We will always suppose that the following conditions are satisfied throughout this paper.

(H_1) $f \in C_{ld}([0,T] \times R^n, R), R = (-\infty, +\infty)$;

(H_2) $\xi_i \in (0, \rho(T))$, $k_i > 0, 0 < \sum_{i=1}^{m-2} k_i < 1$.

2 Preliminaries and Lemmas

For convenience, we list the following definitions which can be found in [1, 3, 5, 6].

Definition 2.1. A time scale $\mathbf{T}$ is a nonempty closed subset of real numbers R. For $t < \sup \mathbf{T}$ and $r > \inf \mathbf{T}$, define the forward jump operator σ and backward jump operator ρ, respectively,

$$\sigma(t) = \inf\{\tau \in \mathbf{T} \mid \tau > t\} \in \mathbf{T},$$

$$\rho(r) = \sup\{\tau \in \mathbf{T} \mid \tau < r\} \in \mathbf{T}.$$

for all $t, r \in \mathbf{T}$. If $\sigma(t) > t$, t is said to be right scattered, and if $\rho(r) < r$, r is said to be left scattered; if $\sigma(t) = t$, t is said to be right dense, and if $\rho(r) = r$, r is said to be left dense.

Definition 2.2. Fix $t \in \mathbf{T}$. Let $f : \mathbf{T} \longrightarrow R$. the delta derivative of f at the point t is defined to be the number $f^{\Delta}(t)$ (provided it exists), with the property that, for each $\epsilon > 0$, there is a neighborhood U of t such that

$$|f(\sigma(t)) - f(s) - f^{\Delta}(t)(\sigma(t) - s)| \leq \epsilon|\sigma(t) - s|,$$

for all $s \in U$. Define $f^{\Delta^n}(t)$ to be the delta derivative of $f^{\Delta^{n-1}}(t)$, i.e., $f^{\Delta^n}(t) = (f^{\Delta^{n-1}}(t))^{\Delta}$.

Definition 2.3. A function f is left-dense continuous (i.e. ld-continuous), if f is continuous at each left-dense point in $\mathbf{T}$ and its right-sided limit exists at each right-dense point in $\mathbf{T}$. If $F^{\Delta}(t) = f(t)$, then define the delta integral by

$$\int_a^t f(s)\Delta s = F(t) - F(a).$$

For the rest of this article, we denote the set of left-dense continuous functions from $[0,T] \times R$ to R and $[0,T]$ to R by $C_{ld}([0,T] \times R, R)$ and $C_{ld}([0,T], R)$, respectively.

Let $C_{ld}([0,T], R)$ be endowed with the ordering $x \leq y$ if $x(t) \leq y(t)$ for all $t \in [0,T]$, and $\|u\| = \max_{t \in [0,T]} |u(t)|$ is defined as usual by maximum norm. The $C_{ld}([0,T], R)$ is a Banach space.

Lemma 2.1. *Supposed (H_2) holds, if $y \in C_{ld}([0,T],R)$, then the problem*

$$v^{\Delta}(t) + y(t) = 0, \quad t \in (0,T), \tag{2.1}$$

$$v(T) = \sum_{i=1}^{m-2} k_i v(\xi_i), \tag{2.2}$$

has unique solution:

$$v(t) = -\int_0^t y(s)\Delta s + \frac{1}{1-\sum_{i=1}^{m-2} k_i}\int_0^T y(s)\Delta s - \frac{1}{1-\sum_{i=1}^{m-2} k_i}\sum_{i=1}^{m-2} k_i \int_0^{\xi_i} y(s)\Delta s.$$

Proof. From (2.1), we have

$$v(t) = -\int_0^t y(s)\Delta s + c. \tag{2.4}$$

By (2.2), it follows that

$$c = \frac{1}{1-\sum_{i=1}^{m-2} k_i}\int_0^T y(s)\Delta s - \frac{1}{1-\sum_{i=1}^{m-2} k_i}\sum_{i=1}^{m-2} k_i \int_0^{\xi_i} y(s)\Delta s.$$

So, we get

$$v(t) = -\int_0^t y(s)\Delta s + \frac{1}{1-\sum_{i=1}^{m-2} k_i}\int_0^T y(s)\Delta s - \frac{1}{1-\sum_{i=1}^{m-2} k_i}\sum_{i=1}^{m-2} k_i \int_0^{\xi_i} y(s)\Delta s.$$

Lemma 2.2. *For $y \in C_{ld}([0,T],R)$ and $y(t) \geq 0$, if $v(t)$ is the solution of (2.1) and (2.2), then we have $v(t) \geq 0$, $v^{\Delta}(t) < 0$.*

Lemma 2.3. *The Green's function for the following problem:*

$$-v^{\Delta}(t) = 0, \quad t \in (0,T), \tag{2.5}$$

$$v(T) = \sum_{i=1}^{m-2} k_i v(\xi_i), \tag{2.6}$$

is given as

$$G(t,s)=\begin{cases}\dfrac{\sum_{i=1}^{j-1}k_i}{1-\sum_{i=1}^{m-2}k_i}, & s\le t,\quad \xi_{j-1}<s\le \xi_j, j=1,2,\cdots,m-1,\\[2ex] \dfrac{1-\sum_{i=j}^{m-2}k_i}{1-\sum_{i=1}^{m-2}k_i}, & s>t,\quad \xi_{j-1}<s\le \xi_j, j=1,2,\cdots,m-1,\end{cases}$$

where $\sum_{i=l}^{l'}k_i=0$, *for* $l'<l$.

Proof. For $0\le t\le \xi_1$, the unique solution of (2.1) and (2.2) can be expressed as

$$v(t)=\int_t^{\xi_1}y(s)\Delta s+\sum_{j=2}^{m-2}\int_{\xi_{j-1}}^{\xi_j}\frac{1-\sum_{i=j}^{m-2}k_i}{1-\sum_{i=1}^{m-2}k_i}y(s)\Delta s+\int_{\xi_{m-2}}^{T}\frac{1}{1-\sum_{i=1}^{m-2}k_i}y(s)\Delta s.$$

For $\xi_{l-1}\le t\le \xi_l, l=2,3,\cdots,m-2$, the unique solution of (2.1) and (2.2) can be expressed as

$$v(t)=\sum_{j=2}^{l-2}\int_{\xi_{j-1}}^{\xi_j}\frac{\sum_{i=1}^{j-1}k_i}{1-\sum_{i=1}^{m-2}k_i}y(s)\Delta s+\int_{\xi_{l-1}}^{t}\frac{\sum_{i=1}^{l-1}k_i}{1-\sum_{i=1}^{m-2}k_i}y(s)\Delta s+\int_t^{\xi_l}\frac{1-\sum_{i=l}^{m-2}k_i}{1-\sum_{i=1}^{m-2}k_i}y(s)\Delta s$$

$$+\sum_{j=l+1}^{m-2}\int_{\xi_{j-1}}^{\xi_j}\frac{1-\sum_{i=j}^{m-2}k_i}{1-\sum_{i=1}^{m-2}k_i}y(s)\Delta s+\int_{\xi_{m-2}}^{T}\frac{1}{1-\sum_{i=1}^{m-2}k_i}y(s)\Delta s.$$

For $\xi_{m-2}\le t\le T$, the unique solution of (2.1) and (2.2) can be given in the form

$$v(t)=\sum_{j=2}^{m-2}\int_{\xi_{j-1}}^{\xi_j}\frac{\sum_{i=1}^{j-1}k_i}{1-\sum_{i=1}^{m-2}k_i}y(s)\Delta s+\int_{\xi_{m-2}}^{t}\frac{\sum_{i=1}^{m-2}k_i}{1-\sum_{i=1}^{m-2}k_i}y(s)\Delta s+\int_t^{T}\frac{1}{1-\sum_{i=1}^{m-2}k_i}y(s)\Delta s.$$

Therefore, the unique solution of (2.1) and (2.2) can be expressed as

$$v(t) = \int_0^T G(t,s)y(s)\Delta s.$$

Let $u^{\Delta^{n-1}}(t) = v(t), t \in [0,T]$, by conditions $u(0) = u^{\Delta}(0) = \cdots = u^{\Delta^{n-2}}(0) = 0$, we get

$$u^{\Delta^{n-k-2}}(t) = \int_0^t \frac{(t-s)^k}{k!} v(s)\Delta s, \quad k = 0,1,2,\cdots,n-2.$$

Define an operator $A_k : C_{ld}([0,T],R) \to C_{ld}([0,T],R)$ by

$$A_k v(t) = \int_0^t \frac{(t-s)^k}{k!} v(s)\Delta s, \quad k = 0,1,2,\cdots,n-2.$$

Lemma 2.4 *The boundary value problem (1.1) has a nontrivial solution if and only if the following boundary value problem*

$$\begin{cases} v^{\Delta}(t) + f(t, A_{n-2}v(t), A_{n-3}v(t)\cdots, A_0 v(t), v(t)) = 0, \quad t \in (0,T), \\ v(T) = \sum\limits_{i=1}^{m-2} k_i v(\xi_i), \end{cases} \tag{2.7}$$

has a nontrivial solution

Proof. In fact, by definition of the operator A_k, if u is a solution of (1.1), then $v = u^{\Delta^{n-1}}$ is a solutio of (2.7). Conversely, if v is a solution of (2.7), then $u = A_{n-2}v$ is a solution of (1.1).

Lemma 2.5.(see [8]) *Let X be a real Banach space and Ω be a bounded open subset of X,$0 \in \Omega$, $F : \overline{\Omega} \longrightarrow X$ be a completely continuous operator. Then either there exist $x \in \partial\Omega$, $\lambda > 1$ such that $F(x) = \lambda x$, or there exists a fixed point $x^* \in \overline{\Omega}$.*

3 Main Results

Our main results are the following theorems.

Theorem 3.1. *Suppose $f(t,0,\cdots 0) \not\equiv 0$, $t \in [0,T]$, and there exist nonnegative functions $p_1,\cdots,p_n$ $q \in L^1[0,T]$ such that*

$$|f(t,u_1,\cdots,u_n)| \le \sum_{i=1}^n p_i(t)|u_i| + q(t), \quad a.e.\ (t,u_1,\cdots,u_n) \in [0,T]\times R^n, \tag{3.1}$$

and

$$K \max_{0\le t\le T} \int_0^T G(t,s) \sum_{i=1}^n p_i(s)\Delta s < 1. \tag{3.2}$$

where $K = \dfrac{T^{n-1}}{(n-2)!} + \dfrac{T^{n-2}}{(n-3)!} + \cdots + \dfrac{T}{0!} + 1.$

Then, the boundary value problem (1.1)*has at least one nontrivial solution* $u^* \in C_{ld}([0,T], R)$.

Proof. Let

$$A = \max_{0\le t\le T} \int_0^T G(t,s)q(s)\Delta s,$$

$$B = K \max_{0\le t\le T} \int_0^T G(t,s) \sum_{i=1}^n p_i(s)\Delta s.$$

By hypothesis $B < 1$. Since $f(t,0,\cdots,0) \not\equiv 0$, there exists $[m,n] \subset [0,T]$, then $\min\limits_{m\le t\le n} |f(t,0,\cdots,0)| > 0$. On the other hand, from the condition $q(t) \ge |f(t,0,\cdots,0)|$, *a.e.* $t \in [0,T]$, we know that $A > 0$.

Let $d = A(1-B)^{-1}$, $\Omega_d = \{u \in C_{ld}([0,T], R) : \|u\| < d\}$. For $0 \le t \le T$, by Lemma 2.1 and Lemma 2.3, we define an operator $F : C_{ld}([0,T]) \to C_{ld}([0,T])$ by

$$Fv(t) = \int_0^T G(t,s) f(s, A_{n-2}v(s), A_{n-3}v(s)\cdots, A_0v(s), v(s))\Delta s.$$

Obviously, the boundary value problem (1.1) has a nontrivial solution if and only if the operator F has a fixed point. Using the Arzela-Ascoli theorem, we can conclude that $F : C_{ld}([0,T], R) \longrightarrow C_{ld}([0,T], R)$ is a completely continuous operator.

Noticing that $|A_k v(t)| = \left|\int_0^t \dfrac{(t-s)^k}{k!} v(s)\Delta s\right| \le \dfrac{T^{k+1}}{k!}||v||$. By hypothesis (3.1), we have

$$\begin{aligned}
&\Big|f(s, A_{n-2}v(s), A_{n-3}v(s)\cdots, A_0v(s), v(s))\Big| \\
&\le p_1(s)|A_{n-2}v(s)| + \cdots + p_{n-1}(s)|A_0v(s)| + p_n(s)|v(s)| + q(s) \\
&= K[p_1(s) + \cdots + p_n(s)]||v|| + q(s),
\end{aligned}$$

where $K = \dfrac{T^{n-1}}{(n-2)!} + \dfrac{T^{n-2}}{(n-3)!} + \cdots + \dfrac{T}{0!} + 1.$

Suppose $u \in \partial\Omega_d$, $\lambda > 1$ such that $Fu = \lambda u$ then

$$\begin{aligned}
\lambda d &= \lambda\|u\| = \|Fu\| = \max_{0\le t\le T} |(Fu)(t)| \\
&= \max_{0\le t\le T}\left|\int_0^T G(t,s) f(s, A_{n-2}v(s), A_{n-3}v(s)\cdots, A_0v(s), v(s))\Delta s\right| \\
&\le ||v||K \max_{0\le t\le T} \int_0^T G(t,s)\sum_{i=1}^n p_i(s)\Delta s + \max_{0\le t\le T}\int_0^T G(t,s)q(s)\Delta s \\
&= dB + A.
\end{aligned}$$

Therefore

$$(\lambda-1)d \le A-(1-B)d = A-A=0,$$

which contradicts $\lambda>1$. By Lemma 2.5, F has a fixed point $u^*\in\overline{\Omega}_d$. Noting $f(t,0,\cdots,0)\not\equiv 0$, the boundary value problem (1.1) has at least one nontrivial solution $u^*\in C_{ld}([0,T],R)$. This completes the proof.

Theorem 3.2. *Suppose $f(t,0,\cdots,0)\not\equiv 0$, $t\in[0,T]$, and there exist nonnegative functions $p_i(s)\in L^1[0,1]$ such that*

$$\begin{aligned}&|f(t,u_1,\cdots,u_n)-f(t,v_1,\cdots,v_n)|\\ &\le \sum_{i=1}^{n} p_i(t)|u_i-v_i|,\ a.e.\ (t,u_1,\cdots,u_n),\ (t,v_1,\cdots,v_n)\in[0,T]\times R^n,\end{aligned}\tag{3.5}$$

and (3.2) holds. Then the boundary value problem (1.1) *has at least one nontrivial solution $u^*\in C_{ld}([0,T],R)$.*

Proof. In fact, if $v_1=v_2=\cdots=v_n\equiv 0$, then we have

$$|f(t,u_1,\cdots,u_n)|\le\sum_{i=1}^{n}p_i(t)|u_i|+|f(t,0,\cdots,0)|.$$

From the proof of Theorem 3.1, we can know the boundary value problem (1.1) has at least one nontrivial solution $u^*\in C_{ld}([0,T],R)$.

But in this case, we prefer to concentrate unique of nontrivial solution for the boundary value problem (1.1). We shall show that the operator F is a contraction. In fact, by (3.5), a similar method to Theorem 3.1, we have

$$\begin{aligned}&\Big|f(s,A_{n-2}u(s),A_{n-3}u(s)\cdots,A_0u(s),u(s))-f(s,A_{n-2}v(s),A_{n-3}v(s)\cdots,A_0v(s),v(s))\Big|\\ &\le p_1(s)|A_{n-2}u(s)-A_{n-2}v(s)|+\cdots+p_{n-1}(s)|A_0u(s)-A_0v(s)|+p_n(s)|u(s)-v(s)|\\ &\le K[p_1(s)+\cdots+p_n(s)]||u-v||.\end{aligned}$$

Then,

$$\begin{aligned}||Fu-Fv|| &\le \max_{0\le t\le T}\int_0^T G(t,s)\Big|f(s,A_{n-2}u(s),A_{n-3}u(s)\cdots,A_0u(s),u(s))\\ &\quad -f(s,A_{n-2}v(s),A_{n-3}v(s)\cdots,A_0v(s),v(s))\Big|\Delta s\\ &\le K\max_{0\le t\le T}\int_0^T G(t,s)[p_1(s)+\cdots+p_n(s)]\Delta s||u-v||.\end{aligned}$$

So (3.2) implies that F is indeed a contraction. Finally we use the Banach fixed point theorem to deduce the existence of a unique nontrivial solution to the boundary value problem (1.1).

4. Some examples

In the section, in order to illustrate our results, we consider some examples. We only study the case $\mathbf{T} = R$ and $(0, T) = (0, 1)$.

Example 4.1. Consider the following the third-order three-point boundary value problem

$$u''' + (t - t^3)\sin u + t^3 u' + (t^4 - t)u'' + t^3 - 2\sin t = 0, \quad t \in (0, 1), \tag{4.1}$$

$$u(0) = u'(0) = 0, \quad u''(1) = \frac{1}{2}u''(\frac{1}{2}), \tag{4.2}$$

Set $k_1 = \frac{1}{2}, \xi_1 = \frac{1}{2},\ n = 3,\ f(t, u_1, u_2, u_3) = (t - t^3)\sin u_1 + t^3 u_2 + (t^4 - t)u_3 + t^3 + \sin t,\ p_1(t) = t - t^3,\ p_2(t) = t^3, p_3(t) = t^4 - t,\ q(t) = t^3 + 2\sin t$. By computing, we have $K = 3 > 0$, $G(t, s) \equiv 1$. Then it is easy to prove that

$$|f(t, u_1, u_2, u_3)| \leq p_1(t)|u_1| + p_2(t)|u_2| + p_3(t)|u_3| + q(t), \quad a.e.\ (t, u_1, u_2, u_3) \in [0, 1] \times R^3,$$

and

$$B = K \max_{0 \leq t \leq T} \int_0^1 G(t, s) \sum_{i=1}^{n} p_i(s)\Delta s = 3\int_0^1 s^4 ds = \frac{3}{5} < 1.$$

By Theorem 3.1, the boundary value problem(4.1)-(4.2) has at least one nontrivial solution $u^* \in C_{ld}([0, 1], R)$.

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