Phasing of Traffic Lights at a Road Junction

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Abstract

A method for solving of a special category of traffic problems has been presented in this paper. The compatibility graph corresponding to the problem and circular arc graphs have been introduced. Compatibility graph corresponding to the problem, spanning subgraph and circular arc graphs then utilized to reduce our problem to the solution of LP problems. Illustrative examples are included to demonstrate the validity and applicability of the technique.

Keywords: LP problem, circular arc graph, maximal clique

1 Introduction

Recent progress in mathematics specially in its applications caused in observable progress of graph theory, in a way that it is now a proper tool for researches in different fields like Encoding theory, Electrical networks, Operation research and other fields. In this paper we review the applications of circular arc graphs in solving a special category of traffic problems. The paper is organized as follows: in section 2 we describe the required definitions and preliminary. The proposed method has been presented in form of example in section 3. In section 4 we solved an example of the problem with offering method.

2 Preliminaries

Consider a finite family of non-empty sets. The intersection graph of this
family is obtained by representing each set by a vertex, two vertices being connected by an edge if and only if the corresponding sets intersect. Intersection graphs have received much attention in the study of algorithmic graph theory and their applications. Well-known special classes of intersection graphs include interval graphs, chordal graphs, circular arc graphs, and so on.

**Definition 2.1** A circular arc graph is the intersection graph of a family of arcs on a circle. We say that these arcs are a circular arc representation of the graph.

**Definition 2.2** A clique of graph $G$ is a complete subgraph of $G$.

**Definition 2.3** A clique of graph $G$ is a maximal clique of $G$ if it is not properly contained in another clique of $G$.

### 3 Problem statement

The problem is to install traffic lights at a road junction in such a way that traffic flows smoothly and efficiently at the junction. We take a specific example and explain how our problem could be tackled. Figure 1 displays the various traffic streams, namely $a$, $b$, $c$, $d$.

![Figure 1: Traffic streams](image)

Certain traffic streams may be termed compatible if their simultaneous flow would not result in any accidents. For instance, in Figure 1, streams $a$ and $b$ are compatible, whereas $a$ and $b$ are not. The phasing of lights should be such that when the green lights are on for two streams, they should be compatible. We suppose that the total time for the completion of green and red lights during one cycle is two minutes. We form a graph $G$ whose vertex set consist of the traffic streams in question and we make two vertices of $G$ adjacent if, and only if, the corresponding streams are compatible. This graph is the compatibility graph corresponding to the problem in question. The compatibility graph of Figure 1 is shown in Figure 2.
We take a circle and assume that its perimeter corresponds to the total cycle period, namely 120 seconds. We may think that the duration when a given traffic stream gets a green light corresponds to an arc of this circle. Hence, two such arcs of the circle can overlap only if the corresponding streams are compatible. The resulting circular arc graph may not be the compatibility graph because we do not demand that two arcs intersect whenever correspond to compatible flows. (There may be two compatible streams but they need not get a green light at the same time). However, the intersection graph $H$ of this circular arc graph will be a spanning subgraph of the compatibility graph.

So we have to take all spanning subgraph of $G$ in to account and choose from them the spanning subgraph that has the most maximal clique. The proper graph $H$ for the above example is shown in Figure 3.

The efficiency of our phasing may be measured by minimizing the total red light time during a traffic cycle, that is the total waiting time for all the traffic streams during a cycle. For the sake of concreteness, we may assume that at the time of starting, all lights are red. The maximal clique of $H$ are $k_1 = \{a,b,c\}$ and $k_2 = \{b,d\}$. Each clique $k_i$, $1 \leq i \leq 2$, corresponds to a phase during which all streams in that clique receive green lights. In phase 1, traffic streams $a,b$ and $c$ receive a green light; in phase 2, $b$ and $d$ receive a green light. Suppose we assign to each phase $k_i$ a duration $d_i$. Our aim is to determine the $d_i$ s ($\geq 0$) so that the total waiting time is minimum. Further, we may assume that the minimum green light time for any stream is 20 seconds. Traffic stream $a$ gets a red light when the phase $k_2$ receive a green light. Hence $a$'s total red light time is $d_2$. Similarly, the total red light times of traffic streams $c$, and $d$, respectively, are $d_2$, and $d_3$. Therefore, the total red light time of all the streams in one cycle is $Z = 2d_2 + d_1$. Our aim is to minimize $Z$ subject to $d_i \geq 0, 1 \leq i \leq 2$ and $d_2 \geq 20, d_1 \geq 20, d_1 + d_2 \geq 20, d_1 + d_2 = 120$. The optimal solution to this problem is $d_1 = 100, d_2 = 20, \min Z = 140$. The phasing that corresponds to this least value would then be the best phasing of the traffic lights.
4 Example

The Figure 4 displays the various traffic streams, namely $a, b, \ldots, g$. We apply the proposed method for this example. The compatibility graph corresponding to the problem is shown in Figure 5.

The proper graph (the subgraph of $G$ that has the most maximal clique) $H$ for this example is shown in Figure 6.

The maximal clique of $H$ are $k_1 = \{a, b, d\}$, $k_2 = \{a, c, d\}$, $k_3 = \{d, e\}$ and $k_4 = \{e, f, g\}$. Our aim is to minimize $Z = 4d_1 + 4d_2 + 4d_3 + 3d_4$ subject to $d_i \geq 0, 1 \leq i \leq 4$ and $d_1 + d_2 \geq 20, d_1 \geq 20, d_2 \geq 20, d_1 + d_2 + d_3 \geq 20, d_3 + d_4 \geq 20, d_4 \geq 20$, and $d_1 + d_2 + d_3 + d_4 = 120$. The optimal solution to this problem is $d_1 = 80, d_2 = 20, d_3 = 0, d_4 = 20$ and min $Z = 460$ (in seconds).
Figure 4: Traffic streams

Figure 5: Graph G (Compatibility graph)
5 Consolution

In this paper we present an application of circular arc graphs to the problem of phasing of traffic lights. The compatibility graph corresponding to the problem and circular arc graphs have been introduced. Illustrative example is included to demonstrate the validity and applicability of the technique.

References


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