Applied Mathematical Sciences, Vol. 3, 2009, no. 27, 1327 - 1340

Companies' Decisions for Profit Maximization: A Structural Model

Roy Cerqueti

Department of Economic and Financial Institutions
University of Macerata, Italy
roy.cerqueti@unimc.it

Giulia Rotundo

Department of Business, Technological, and Quantitative Studies
University of Tuscia, Viterbo, Italy
rotundo@unitus.it

Abstract

Huge analyses on firms data selected from public available databases accomplished the task to describe the size and growth of firms through interpolating functions. The structure and internal firms organization that lead to the optimal profit is a main matter of business studies and must take carefully into account internal work distribution and the subsequent productivity. Moreover factors external to firms, like as the evolution of markets and the availability of new technologies show their immediate bias on the wealth of the firms. In this paper a model is developed for a set of firms producing a single commodity. The shape of the productivity that leads to profit optimization is drawn and discussed. Furthermore the optimal time for the firm to renew its technology is established and consequences on the productivity are examined.

Mathematics Subject Classification: 90B30; 91B38

Keywords: Equilibrium model, Technology renewal, Optimization theory, Aggregate productivity, Firms size distribution

1 Introduction

Internal firms organization plays a key role in productivity and efficiency, due to many causes that must take into account wide variety of factors ranging from social dynamics of employers to the supposed managerial ability in the economy to the availability and cost of the instruments and commodities necessary for the job.

Theories driven by complex systems have shown that a hierarchical structure tree-like matches with data public available about firms [2] and that is able to absorb operational risk. Among the many factors that can improve efficiency and productivity we choose to focus on technology. The role of technology in firms is continuously growing. Competitive small as well as big firms must adequate their technology in order to survive on the market, independently on their size. An evident example is about the speed of the need of renewal of technology connected to the computer science: also small firms need the use of at least a computer and the use of new software requires the renewal of hardware. Even jobs based on human skills that improve through time, like law matter, can't avoid the retrieval of information through fast computer, fast database, and fast communication systems. In this paper we take into account firms that produce a single commodity and we explore the effects of the update of technology, looking for the optimal time for the renewal. Although the theory of Schumpeter on the importance of technological discontinuities in economic history we want to describe the effects that rise globally due to the spread of the technology through the whole set of firms, and the presence of many components makes the description through a single jump less suited to the problem. We rely on literature results about the distribution of the size of firms and of the distribution of the technology across firms. The first step is thus to model such features. External factors like as the occurrence of extreme or catastrophic events, like as hurricanes, earthquakes of or wars are not taken in account. The next section reviews empirical results available through the literature. In section 3 we set up the model and we draw the productivity function that leads to the optimal profit.in section 4 we determine the optimality solution w.r.t. the time and give some structural results.

2 Previous literature results

The following sections resume some results due to the analysis of raw data.

2.1 Distribution of firms depending on their size and growth rate

Several studies have been made about the detection of skew distribution of firms size and about the validity of Gibrat's law of proportionate effect for growth rate to explain the empirically observed distribution of firms. This law states that the expected increment to the firm size in each period is proportional to the current size of the firm, i.e. firm growth rates are uncorrelated and independent of size, and this leads to a growth rate log-normally distributed and thus highly right skewed. Thus small numbers of large firms coexist alongside larger numbers of smaller firms.

The studies about size and growth rate of firms differ for the hypotheses tested and for the data sets that were used. Most data were got from Census and COMPUSTAT data bases. Census data give more information about small firms. Although the position of individual firms in a size distribution does depend on the definition of size, the shape of the distribution does not and firms sizes in industrial countries are highly skew, such that small numbers of large firms coexist alongside larger numbers of smaller firms. Depending on the data set skewness has been shown either to be robust over time [4] or to grow during growing phases of the economy and to decrease during recessions [24], thus being an indicator of such economic cycles. A model is also proposed which offers a candidate explanation for the power-law relation between firm size and the variance of growth rates [52].

The detection of the distribution is important also for explaining differences of reaction of the market to external shocks: as an example in simulations [17] in the case of log-normally distributed data shocks are absorbed, whilst in the case of Pareto distribution shocks conduce to strong oscillations.

While in older studies [27, 28] the log-normal hypothesis received the most attention in recent papers the mainstream results indicates power law for firm size and laplace law for firms growth rates [17]. It can be shown that the logarithm of a power-law distributed variable obeys a laplace distribution, thus the laplace law for firms growth rate should be a consequence of power law for firm size and not succeed from log-normal one, thus invalidating the Gibrat hypotheses. However the weak form of Gibrat's law has been shown to be compatible with power law under further hypotheses. As an example the first model is the Simon's model [17] where the Gibrat's law is combined with an entry process to obtain a Levy distribution for firm's size. Particular assumptions like the one of the validity of the detailed balance, that states the time-reversal symmetry for the growth rate show that Gibrat's law and Pareto-Zipf's law hold for firms bigger than a fixed thresholds [23]. This property is not valid in general [31], but the behavior of biggest companies is important because determines the most part of economy.

The power law behavior seems to be common also to parameters that involve the most heavy countries. The results reported in [21] can be interpreted as the existence of a significant range of the world GDP distribution where countries share a common, size-independent average growth rate. Also particular hypotheses like entry and exit of companies from the market give results that contradicts the Gibrat's law. As an example in [2, 3, 49] the exponential distribution for the growth rate of firms has been found to hold for the 20 years 1974-1993 of COMPUSTAT publicy-traded United States manufacturing firms, whilst the variance of the growth rate should grow with the size of the firm. The fit of the log-normal distribution to size data is good close to the mean, but it performs less on the tails. In order to develop our model we need to fix a function for the distribution of firms depending on their size. We use the family of functions that include as a particular case the log-normal and that takes into account a power-law decay of tails in the general case [25], [26].

$$P(\ln S) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{|\ln S - \mu|^2}{2\sigma^2}} f(\ln S) \tag{1}$$

where μ and σ are the mean and standard deviation of $\ln(S)$ and $f(x)$ is given by:

$$f(x) = \begin{cases} 1 & \text{if } | x | \leq x_c \\ \exp\{-(\frac{|x|-x_c}{k})^\beta\} & \text{if } | x | > x_c \end{cases} \tag{2}$$

where x_c is some cut-off value where the switch of the regime starts. The constants k and β can be detected in order to get the best fit to data. Through Central Limit Theorem all distribution must approach to normal distribution. For log-normal distribution to approach normal distribution for larger steps, we should have $\beta = 2$.

2.2 The role of technology in productivity

The analyses performed along the literature involve several different measure [8], [36] for innovation technology, examine data about firms operating in different sectors of economy and belonging to several different countries. Technology diffusion patterns are not the same across the surveyed technologies [9], [12]. The reaction to innovation technology depends also on countries [9], [18], [19], [20], [38], [47], and the causality is explored through linear model or log regression models [35]. Whilst the data are widely inhomogeneous the result that emerges through all these papers is that there is a positive correlation between the adoption of a new technology and productivity [35]. As a matter of the fact establishments using advanced technologies exhibit high productivity [35]. However the direction of causality between productivity growth and the use of advanced technology was not completely explored, in fact the positive correlations could reflect either the independent effects of technology on performance, or the contributions of good managers who tend to adopt the best practices [7], [10], [46]. The comprehension of such cause could give strong indications about social policies. For example, policies to improve education and training might be relatively more important than those for supporting applied research and developement [13], [35], [38], [45] if the dominant source of enhanced performance is good management and a skilled workforce. In other cases to guarantee access to new technology could be enough. The age of

technology is not connected to the age of the firm adopting it: as an example new firms can not keep buying new technology in their first years because of budget constraints [40].
Information technology plays a crucial role across the whole system. Analyses of productivity performance show that the industries that got the largest productivity acceleration in the late 1990s were the producers and most intensive users of information technology [51].

2.3 Costs of renewal of technology

In order to model the costs of renewal we follow the approach purposed in [15]. The marginal cost at period t, free from renewal of technology, is given by $C\alpha^t$. Let $R(t)$ be the output rate. The parameter $0 \leq \alpha \leq 1$ measures the learning rate, thus the production costs over a single period without any innovation technology are given by $R(t)C\alpha^t$. If the firms switch to new technology in period T, there is a switching cost to pay that is assumed to be

$$K(T) = k + sC(\alpha^T - (\alpha\gamma)^T), \tag{3}$$

where k is the fixed portion of the switching cost, s is the parameter relating per unit cost savings to the market price for technology, $\alpha\gamma$ is a parameter indicating how well the marketplace reduces the manifacturing costs ($\alpha\gamma < \alpha$), The costs after the innovation at time T are given by $K(t) = RC\alpha\gamma^T\alpha^{t-T}$. The costs evolution before and after the innovation can be resumed by

$$K(t) = \begin{cases} R(t)C\alpha^t & t \leq T, \\ \\ R(t)C(\alpha\gamma)^T\alpha^{t-T} & t > T. \end{cases} \tag{4}$$

The above function can be smoothed by using the sigmoidal function:

$$\Theta_T(t) = \frac{T}{1 + e^{-\delta(t-T)}}$$

The δ parameter regulates the approximation of the sigmoidal function to the step function. Thus the cost function becomes:

$$K(t) = R(t)C\alpha^t\gamma^{\Theta_T(t)}. \tag{5}$$

3 The Model

In this section we set up the basic features of a model that takes into account a single commodity produced by a set of industries that experience innovation technology. The market size is defined by the exogenous parameter $M(t)$. Let us assume a linear demand function with slope σ. The output is sold at the market clearing price. The price per unit obtainable with an output rate of $R(t)$ is thus

$$M(t) - \sigma R(t).$$

The gross revenue $Y(t)$ resulting from this production is:

$$Y(R(t)) = R(t)(M(t) - \sigma R(t)).$$

The profit is given by the gross revenue minus the costs of the renewal of the technology $K(t)$:

$$\Pi(R(t), t) = Y(R(t)) - K(t). \tag{6}$$

The model looks for the maximum of profit with respect to the time in which the innovation is spread across the firms.

$$\max_t \Pi(R(t), t) = \max_t (Y(R(t)) - K(t)). \tag{7}$$

In order to proceed we need to specify the production rate, the costs of renewal and hence the profit.

3.1 The rate of output

The rate of output $R(t)$ depends on the number of production units $f(\cdot)$ with technology actually used at time t and by the productivity of these units $A(\cdot)$. We want to model that quantities by functions that rely on empirical results available through the literature. In particular we use the family of density functions (1) that describes the size of the firm as a proxy for the number of production units with fixed age [14]. In order to insert a measure of the age of the units we take into account that even if for big firms it is easier to adopt new technology also small firm can get the newest technology restricted to one sector. Moreover we assume that the innovation arises in an interval of time that is short if compared to the whole period, and that all the firms start to use the new technology and discard the old one. Also firms whose members improve their skills with the age, like law firms, must keep on with the technology. Thus as a first approximation we can assume that the innovation spreads across firms independently from their size and thus the number of production units can be obtained simply by integrating the density of firms'size. A function that takes into account these features is given by the density $f(S) := P(\ln S)$.
In order to model the productivity we assume that it depends on the size of the firm and on the age of the technology through two separate functions. The literature results allow to keep the distribution of the size of the firms independent from the time [2], [3], [49]. We explore the productivity function

depending on the size of the firms and the status of the technology that depends on the time t: $A(t,S)$. They are combined together in order to give the aggregate output

$$R(t) := \zeta \int_0^{\infty} \Big[A(t,S) \cdot f(S)\Big] \mathrm{d}S. \tag{8}$$

3.2 The profit

Now it is possible to define better the functions entering the problem:

$$\max_{R(t)} \Pi(R(t),t) = \max_{t} (R(t)M(t) - \sigma R^2(t) - R(t)C\alpha^t\gamma^{\Theta_T(t)}), \tag{9}$$

$$R(t) := \zeta \int_0^{\infty} \Big[A(t,S) \cdot f(S)\Big] \mathrm{d}S. \tag{10}$$

Let us proceed in two steps. At each time t we calculate the maximum profit Π^* with respect to $R(t)$ and we use the first order conditions in order to get the maximal output rate $R^*(t)$, $\forall t$. Then we use the optimal profit with respect to R in order to calculate the maximum over time.
The maximal output rate at time t is $R^*(t)$, such that

$$\phi(t) := \Pi(R^*(t),t). \tag{11}$$

By the first order condition we find the stationary point. We denote it as $R_0(t)$. We obtain

$$R_0(t) = \frac{M(t) - C\alpha^t\gamma^{\Theta_T(t)}}{2\sigma}.$$

The second order condition is equivalent to

$$\frac{\mathrm{d}}{\mathrm{d}t} R_0(t) > 0,$$

that means

$$-\frac{C}{2\sigma}\alpha^t\gamma^{\Theta_T(t)}\Big[\ln(\alpha) + \delta T \ln(\gamma)\frac{e^{-\delta(t-T)}}{(1+e^{-\delta(t-T)})^2}\Big] > 0,$$

that is always satisfied, since $\alpha, \gamma \in (0,1)$. Then $R_0(t) = R^*(t)$.
This allows for the detection of the optimal productivity functions. Thus

$$\zeta \int_0^{+\infty} \Big[A(t,S) \cdot f(S)\Big] \mathrm{d}S = \frac{M(t) - C\alpha^t\gamma^{\Theta_T(t)}}{2\sigma}. \tag{12}$$

This equation defines in an implicit way the optimal productivity function.
We can use this equation in order to give a characterization of the optimal productivity. Let us consider the integral (12) truncated at the size $\overline{s}$. We get

$$R_{\overline{s}}(t) := \zeta \int_0^{\overline{s}} A(t,S) f(S) \mathrm{d}S. \tag{13}$$

It is easy to prove that

$$\lim_{\overline{s}\to+\infty} R_{\overline{s}}(t) = R(t). \tag{14}$$

Moreover, for each $t > 0$, we have

$$\zeta \int_0^{\overline{s}} \Big(A(t,S)f(S) - \frac{M(t) - C\alpha^t \gamma^{\Theta_T(t)}}{2\sigma\overline{s}}\Big)\mathrm{d}S = 0, \tag{15}$$

and so

$$A(t,S)f(S) - \frac{M(t) - C\alpha^t \gamma^{\Theta_T(t)}}{2\sigma\overline{s}} = 0\ , \quad \forall S \in (0,\overline{s}). \tag{16}$$

The optimal productivity function over finite period is thus given by:

$$A(t,S) = \frac{M(t) - C\alpha^t \gamma^{\Theta_T(t)}}{2\sigma f(S)\overline{s}}\ , \quad 0 \le S \le \overline{s}, \quad t > 0. \tag{17}$$

Productivity can exhibit a wide variety of patterns depending on firms sectors. It is not always connected to the size of the firms because some management policies can drive companies to to grow in size without a following growth in productivity. In some situations the target of maximal growth in size not accompanied by the target of optimizing productivity led firms on the collapse. In this model we experience a set of companies for which the productivity is maximal for smallest firms describing well situations of locally concentrated firms with a small number of employers or at least none.
When allowing $\overline{s} \to +\infty$, then productivity grows again becoming maximal for huge firms that can rely over unlimited resources.
The worse productivity situation is experienced by middle size firms, that can bear the cost of non productive units that delay their growth.

4 Optimal profit

In this section we look for the optimal time for the switch to a new technology. It can be found by maximizing $\Pi^*(T)$ over the time. It takes into account a discount rate β^t and the discontinuity at the time T of the renewal of technology that gives the costs of the renewal. Getting continuous time, introducing the smoothing function there is not the need any more to keep separate summations and also by substituting $K(T)$ given by (3), $\Phi(t)$ becomes:

$$\Phi(T) = \int_0^\infty \phi(t)\beta^t \mathrm{d}t - K(T)\beta^T, \tag{18}$$

i.e.

$$\Phi(T) = \int_0^\infty \frac{(M(t) - C\alpha^t \gamma^{\frac{T}{1+e^{-\delta(t-T)}}})^2}{4\sigma}\beta^t \mathrm{d}t - [k + sC\alpha^T(1-\gamma^T)]\beta^T. \tag{19}$$

We need to study the region of the parameters entering the function for which the switch of technology produces a positive profit and for which it is concave, and admits a maximum point.

Remark 4.1 *We remark that in the limit $\delta \to \infty$ and for discrete times $\Phi(T)$ reduces to the function*

$$\Phi(T) = \sum_{t=0}^{T} \frac{(M(t) - C\alpha^t)^2}{4\sigma}\beta^t + \beta^T \sum_{t=T}^{\infty} \frac{(M(t) - C\gamma^T\alpha^t)^2}{4\sigma}\beta^t - K(T)\beta^T, \quad (20)$$

proposed in [14], which our model results an extension of.

4.1 Structural results

The behavior of this function depends on the parameters. First of all, f or the model to have economic sense $0 \leq \alpha\gamma \leq \alpha \leq 1$ and $\beta \leq 1$.

Theorem 4.2 *Assume that there exists $T^* > 0$ such that the following conditions hold.*

$$M(t) = C\alpha^t\gamma^{\frac{T^*}{1+e^{-\delta(t-T^*)}}}; \quad (21)$$

$$sC\alpha^{T^*}[-\ln(\alpha\beta) + \gamma^{T^*}\ln(\alpha\beta\gamma)] - k\ln\beta = 0; \quad (22)$$

$$\delta < \frac{2}{T}; \quad (23)$$

and, for $T \neq T^*$,

$$M(t) > C\alpha^t\gamma^{\frac{T}{1+e^{-\delta(t-T)}}}; \quad (24)$$

$$\alpha^T[\ln^2(\alpha\beta) - \gamma^T[\ln^2(\alpha\beta) + \ln\alpha\ln\beta + 2\ln\gamma\ln\beta + \ln^2\gamma] + \frac{k}{Cs}\ln^2\beta > 0. \quad (25)$$

Then it results

$$\Phi(T^*) = \max_{T>0} \Phi(T). \quad (26)$$

Proof. By assumptions (21)-(25), by imposing the first and second order conditions, we get the thesis.

Remark 4.3 *Theorem 4.2 states that, under a calibration on the parameters related to costs and productivity, the optimal profit can be reached.*

5 Conclusions

In this paper we started from empirical results given by the analysis of raw data performed through the literature in order to fix an economic model that allows to stand the condition for the optimal productivity function over the single period. The introduction of a smooth function of the optimal switch time to a more advanced technology allows to get a more tractable mathematical function that keeps the results of [14] as a particular case. Our purpose as a future work is to analyze the sensitivity to the skewness of the firm distribution and to change the truncation of the normal distribution following the approach of [17] instead of the one of [25]. Moreover some hypotheses over a hierarchical structure of firms can be introduced in order to get more information over the optimal productivity function. Multiperiodal model and the number of renewal optimal times as function of the parameters of the considered economy are also going to be analyzed.

References

[1] Amaral, L.A.N., Buldyrev, S.V., Havlin, S., Salinger, M.A., Stanley, H.E., Power Law Scaling for a System of Interacting Units with Complex Internal Structure, Physical Review Letters, **80**, No. 7, (1998), 1385-1388.

[2] Amaral, L. A. N., Buldyrev, S. V., Havlin, S., Leschhorn, H., Maass, P., Salinger , M. A., Stanley, H. E., Stanley, M. H. R., Scaling Behavior in Economics: I. Empirical Results for Company Growth, Journal de Physique I, **7**, (1997), 621-633.

[3] Amaral, L. A. N., Buldyrev, S. V., Havlin, S., Leschhorn, H., Maass, P., Salinger , M. A., Stanley, H. E., Stanley, M. H. R., Scaling Behavior in Economics: II. Modeling of Company Growth, Journal de Physique I, **7**, (1997), 635-650.

[4] Axtell, R.L., Zipf Distribution of U.S. Firm Sizes, Science, **293**, (2001), 1818-1820.

[5] Babovic, V., Keijzer, M., Stefansonn, M., Chaos Theory, Optimal Embedding and Evolutionary Algorithms, (2001), Preprint.

[6] Barbaroux, J.-M., Germinet, F., Tcheremchantsev, S., Generalized Fractal Dimensions: Equivalences and Basic Properties, J. Math. Pures Appl., **80**, No. 10, (2001), 977-1012.

[7] Basu, S:, Fernald, J. G., Aggregate productivity and aggregate technology, European Economic Review, **46**, No. 6, (2002), 963-91.

[8] Basu, S., Fernald, J. G., Kimball, M. S., Are technology improvements contractionary?, International finance discussion papers, Washington, D.C.: Board of Governors Federal Reserve System, No. 625, (1998).

[9] Basu, S., Fernald, J. G., Oulton, N., Srinivasan, S., The Case of the Missing Productivty Growth: Or, Does Information Technology Explain why Productivity Accelerated in the US but not the UK?, NBER Working Paper, No. W10010, (2003).

[10] Basu, S., Fernald, J. G., Shapiro, M. D., Productivity growth in the 1990s : technology, utilization, or adjustment?, NBER Working Paper, No. 8359, (2001).

[11] Basu, S., Weil, D N., Appropriate Technology and Growth, NBER Working Paper, No. W5865, (1996).

[12] Beede, D., Young, K., Patterns of Advanced Technology Adoption And Manufacturing Performance. An Overview, Business Economics, **33**, Issue 2, (1998), 43-48.

[13] Benabou, R., Human capital, technical change and the welfare state, Journal of European Economic Association, **1**, Issues 2-3, Papers and Proceedings of the Seventeenth Annual Congress of the European Economic Association, Venice, (2002), 522-532.

[14] Caballero, R. J., Hammour, M. L., The Cleansing Effect of Recessions, The American Economic Review, **84**, No. 5, (1994), 1350-1368.

[15] Chambers, C., Technological advancement, learning, and the adoption of new technology, European J. Oper. Res., **152**, No. 1, (2004), 226–247.

[16] De Fabritiis, G., Pammolli, F., Riccaboni, M., On size and growth of business firms, Physica A, **324**, Issue 1-2, (2003), 38-44.

[17] Delli Gatti, D., Di Guilmi, C., Gaffeo, E., Giulioni, G., Gallegati, M., Palestrini, A., A new approach to business fluctuations: eterogeneous interacting agents, scaling laws and financial fragility, Journal of Economic Behavior & Organization, forthcoming.

[18] Deraniyagala, S. , Adaptive Technology Strategies and Technical Efficiency: Evidence from the Sri Lankan Agricultural Machinery Industry, Journal of International Development, **12**, (2001), 1-13.

[19] Deraniyagala, S., The Impact of Technology Accumulation on Technical Efficiency: an Analysis of the Sri Lankan Clothing and Agricultural Machinery Industries, Preprint.

[20] Desnoyers, Y., The puzzle of U.S. productivity, Preprint.

[21] Di Guilmi, C., Gaffeo, E., Gallegati, M., Power Law Scaling in the World income Distribution, Economics Bulletin, **15**, No. 6, (2003), 1-7.

[22] Falconer, K., *Fractal Geometry: Mathematical Foundations and Applications*, (Wiley, New York, 2003).

[23] Fujiwara, Y., Di Guilmi, C., Aoyama, H., Gallegati, M., Souma, W., Do Pareto-Zipf and Gibrat laws hold true? An analysis with European firms, Physica A: Statistical Mechanics and its Applications, **335**, Issues 1-2, (2000), 197-216.

[24] Gaffeo, E., Gallegati, M., Palestrini, A., On the Size Distribution of Firms: Additional Evidence from the G7 Countries, Physica A: Statistical Mechanics and its Applications, **324**, Issues 1-2, (2003), 117-123.

[25] Gupta, H. M., Campanha, J. R., The gradually truncated Levy flight: stochastic process for complex systems, Physica A, **275**, Issue 3-4, (2000), 531-543.

[26] Gupta, H. M., Campanha, J. R., The gradually truncated Levy flight for systems with power-law distributions, Physica A, **268**, Issue 1-2, (1999), 231-239.

[27] Hart, P. E., Oulton, N., Growth and size of firms, The Economic Journal, **106**, (1996), 1242-1252.

[28] Hart, P. E., Oulton, N., Zipf and the size distribution of firms, Applied Economics Letters, **4**, (1997), 205-206.

[29] Havemann, F., Heinz, M., Wagner-Doebler, R., Firm-like Behavior of Journal? Scaling Properties of Their Output and Impact Growth Dynamics, Journal of the American Society for Information Science and Technology, to appear.

[30] Hsieh, A. D., Chaos and Non Linear Dynamics: Application to Financial Markets, Journal of Finance, **46**, (1991), 1833-1877.

[31] Kertesz J.; Kullmann L.; Zawadowski A.G.; Karadi R.; Kaski K., Correlations and response: absence of detailed balance on the stock market, Physica A, **324**, Issue 1, (2003), 74-80.

[32] Klages, R., Dorfman, J.R., Dynamical Crossover in Deterministic Diffusion, Physical Review E, **55**, No. 2, (1997), 1247-1250.

[33] Klages, R., Dorfman, J.R., Simple deterministic dynamical systems with fractal diffusion coefficients, Physical Review E, **59**, No. 5, (1999), 5361-5383.

[34] Mandelbrot, B., *The Fractal Geometry of nature* (W.H. Freeman and Company, S. Francisco, 1982).

[35] McGuckin, R. H., Streitwieser, M.L., Doms, M. E. The effects of technology use on productivity growth, CES (The Conference Board and Center for Economic Studies , U.S. Bureau of the Census), (1996), research papers No. 96-2.

[36] J. Melitz, Estimating Firm-Level Productivity in Differentiated Product Industries Marc, Preprint.

[37] Michelacci, C., Cross-Sectional Heterogeneity and the Persistence of Aggregate Fluctuations, (2004), Preprint.

[38] Milana, C., Zeli, A., The contribution of ICT to production efficiency in Italy: firm level evidence using data envelopment analysis and econometric estimations, STI Working Paper, No. 13, (2002).

[39] Peters, E., *Chaos and Order in the Capital markets: A new View of Cycles, Prices, and Market Volatility*, (A Wiley Finance Edition, 1996).

[40] Power, L., The Missing Link: Technology, Productivity, and Investment, The Review of Economics and Statistics, **80**, Issue 2, (1998), 300-313.

[41] Provenzale A., Smith L. A., Vio R., Murante G., Distinguishing between low-dimensional dynamics and randomness in measured time series, Physica D, **58**, (1992), 31-49.

[42] Ramsden, J.J.; Kiss-Haypál, Gy., Company size distribution in different countries, Physica A, **277**, Issue 1-2, (2000), 220-227.

[43] Rao, S., Ahmad, A., Horsman, W., Kaptein-Russell, Ph. The Importance of Innovation for Productivity, Centre for the Study of Living Standards, **2**, (2001), 11-18.

[44] Reitmann, W., Dimension Estimates for Invariant Sets of Dynamical Systems, appeared in *Ergodic theory, analysis, and efficient simulation of dynamical systems*, 585–615, (Springer, Berlin, 2001).

[45] Romijn, H., Technology Support for Small scale Industry in Developing Countries: a review of concepts and project practices, Oxford Development Studies, **29**, No. 1, (2001), 57-76.

[46] Rogers, M.,Productivity in Australian Enterprises: Evidence from the ABS Growth and Performance Survey, Working Paper of Melbourne Institut, No. 20, (1998).

[47] Sharpe, A., The Canada-US Manufacturing Productivity Gap: An Overview, paper presented at the CSLS session on the Canada-U.S. Manufacturing Productivity Gap at the annual meeting of the Canadian Economics Association, May 29-31, (1998), University of Ottawa, Ottawa, Ontario.

[48] Söderbom, M., Teal, F., Firm size and human capital as determinants of productivity and earnings, The Centre for the Study of African Economies Working Paper Series Year 2001, (2001), paper No. 147.

[49] Stanley, M. H. R., Amaral, L. A. N., Buldyrev, S. V., Havlin, S., Leschhorn, H., Maass, P., Salinger, M. A., Stanley, H. E., Scaling behaviour in the growth of companies, Letters to Nature, **379**, (1996), 804-806.

[50] Stanley, M. H. R., Buldyrev, S. V., Havlin, S., Mantegna, R. N., Salinger, M. A., Zipf plots and the size distribution of firms, Economics Letters, **49**, (1995), 453-457.

[51] Stiroh, J. K., Information Technology and the U.S. Productivity Revival: What Do the Industry Data Say?, Staff Reports No. 115, Federal Reserve Bank of New York, (2001).

[52] Sutton, J., The variance of corporate growth rates, Physica A, **324**, Issue 1, (2003), 45-48.

[53] Vicsek, M., Vicsek, T., Aggregation Models of Fractal Growth, Quarterly, **10**, No. 2, (1997), 153-178.

[54] Vandewalle, N., Ausloos, M., Fractals in Finance, in *Fractals and beyond, Complexity in the Sciences*, (M.M. Novak, Ed., World Scientific, Singapore, 1998) 355-356.

Received: November, 2008