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Research on the Priority Method of Interval Fuzzy Preference Relation[1]

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Abstract

The priority method of interval fuzzy preference relation is studied. A linear programming(LP) model based on multiplicative transitivity is constructed to derive interval weights from consistent or inconsistent interval fuzzy preference relation. By solving only one LP model, the proposed method can get the interval weights from an interval fuzzy preference relation, which can reduce the amount of computation. Some numerical examples are illustrated to show the effectiveness and potential practicality of the proposed method.

Keywords: Interval fuzzy preference relation; Consistency; Multiplicative transitivity

1 Introduction

Due to the complexity and uncertainty involved in real world decision problems, interval fuzzy preference relation[6,10] is very suitable for expressing the decision maker preference information. Some authors have paid attention to the priority method of the interval fuzzy preference relation[1-4,12], but they require the solution of (2n+1) linear programming(LP) models or

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transform the interval fuzzy preference relation into interval multiplicative preference relation, which lost decision information.

This paper give a simple method to get the priority from interval fuzzy preference relation. Section2 establishes one linear programming model based on multiplicative transitivity to derive interval priority weights from consistent or inconsistent interval fuzzy preference relation. Section 3 provides two numerical examples to show the potential applications and validity of the proposed method. Finally, in section 4, we conclude the paper and give some remarks.

2 A linear programming model based on multiplicative transitivity

Definition 1[8]. A fuzzy preference relation $\overline{R}$ on the set $X = \{x_1, x_2, \cdots, x_n\}$ is represented by a complementary matrix $\overline{R} = (\overline{r}_{ij})_{n\times n} \subset X \times X$ with $\overline{r}_{ij} \geq 0$, $\overline{r}_{ij} + \overline{r}_{ji} = 1$, $\overline{r}_{ij} = 0.5$, for all $i, j = 1, 2, \cdots, n$.
Definition2[7,11]. A fuzzy preference relation $\overline{R} = (\overline{r}_{ij})_{n\times n}$ is called a multiplicative consistent fuzzy preference relation, if the following additive transitivity is satisfied $\overline{r}_{ik}\overline{r}_{kj}\overline{r}_{ji} = \overline{r}_{ki}\overline{r}_{jk}\overline{r}_{ij}$, for all $i, j = 1, 2, \cdots, n$.

Then the multiplicative consistent fuzzy preference relation $\overline{R}$ can be given by[6]: $\overline{r}_{ij} = w_i/(w_i + w_j)$ for all $i, j = 1, 2, \cdots, n$, where w_i reflects the importance degree of x_i and satisfies $w_i \geq 0, i = 1, 2, \cdots, n, \sum_{i=1}^{n} w_i = 1$.
Definition 3[9]. Let $\widetilde{w}_i = [w_i^-, w_i^+]$ and $\widetilde{w}_j = [w_j^-, w_j^+]$ be two any interval weights, where $0 \leq w_i^- \leq w_i^+ \leq 1$ and $0 \leq w_j^- \leq w_j^+ \leq 1$, then the degree of possibility of $\widetilde{w}_i \geq \widetilde{w}_j$ is defined as $p(\widetilde{w}_i \geq \widetilde{w}_j) = \frac{\max(0, w_i^+ - w_j^-) - \max(0, w_i^- - w_j^+)}{w_i^+ - w_i^- + w_j^+ - w_j^-}$.

Consider a multiple criteria decision making problem with a finite set of n criteria, and let $X = (x_1, x_2, \cdots, x_n)$ be the set of criteria. A decision maker compares each pair of criteria in X , and provides his/her interval preference degree $r_{ij} = [r_{ij}^-, r_{ij}^+]$ of the criterion x_i over x_j. All these interval preference degrees $r_{ij}(i, j = 1, 2, \cdots, n)$ compose an interval fuzzy preference relation $\overline{R} = (\overline{r}_{ij})_{n\times n} \subset X \times X$ with $r_{ij} = [r_{ij}^-, r_{ij}^+]$, $r_{ij}^- + r_{ji}^+ = r_{ij}^+ + r_{ji}^- = 1$, $r_{ij}^+ \geq r_{ij}^- \geq 0$, $r_{jj}^+ = r_{ii}^- = 0.5$, for all $i, j = 1, 2, \cdots, n$.

By definition 2, if the interval fuzzy preference relation is the precise comparison, then there should exists a normalized interval weight vector $W = (\widetilde{w}_1, \widetilde{w}_2, \cdots, \widetilde{w}_n) = ([w_1^-, w_1^+], \cdots, [w_n^-, w_n^+])^T$, which is close to R in the sense that $r_{ij} = [r_{ij}^-, r_{ij}^+] \approx \widetilde{w}_i/(\widetilde{w}_i + \widetilde{w}_j)$ for all $i, j = 1, 2, \cdots, n, i \neq j$. According to [5], the interval weight vector W is said to be normalized if and only $w_i^- + \sum_{j=1, j\neq i} w_i^+ \geq 1, w_i^- + \sum_{j=1, j\neq i} w_i^+ \leq 1, i = 1, 2, \cdots, n$.

As is known, if the interval fuzzy preference relation R is the precise comparison about the interval weight vector W, then R must be able to be written as $R = \left(\left[\frac{w_i^-}{w_i^-+w_j^+}, \frac{w_i^+}{w_i^++w_j^-}\right]\right)_{n\times n}$. Let

$$R^- = \left(\frac{w_i^-}{w_i^-+w_j^+}\right)_{n\times n}, R^+ = \left(\frac{w_i^+}{w_i^++w_j^-}\right)_{n\times n}, B^- = (r_{ij}^-)_{n\times n}, B^+ = \left(r_{ij}^+\right)_{n\times n},$$
$$\lambda^- = (n-1-\sum_{j=2}^n r_{1j}^-, n-1-\sum_{j=1,j\neq 2}^n l_{2j}, \cdots, n-1-\sum_{j=1,j\neq n}^n r_{nj}^-),$$
$$\lambda^+ = (n-1-\sum_{j=2}^n r_{1j}^+, n-1-\sum_{j=1,j\neq 2}^n l_{2j}, \cdots, n-1-\sum_{j=1,j\neq n}^n r_{nj}^+).$$

It is easy to prove that $B^-W^+ = \lambda^-w^-, B^+W^- = \lambda^+w^+$, where $W^- = (w_1^-, w_2^-, \cdots, w_n^-)^{\mathrm{T}}$ and $W^+ = (w_1^+, w_2^+, \cdots, w_n^+)^{\mathrm{T}}$. Due to the presence of subjectivity and uncertainty, the equtions may not hold precisely. Based on such an analysis, we introduce the deviation vector $\Xi^+ = (\varepsilon_1^+, \varepsilon_2^+, \cdots, \varepsilon_n^+)^T$, $\Xi^- = (\varepsilon_1^-, \varepsilon_2^-, \cdots, \varepsilon_n^-)^T, \Gamma^+ = (\gamma_1^+, \gamma_2^+, \cdots, \gamma_n^+)^T, \Gamma^- = (\gamma_1^-, \gamma_2^-, \cdots, \gamma_n^-)^T$, which satisfy $B^-W^+ - \lambda^-w^- - \Xi^+ + \Xi^- = 0$, $B^+W^- - \lambda^+w^+ - \Gamma^+ + \Gamma^- = 0$, where $\Xi^+, \Xi^-, \Gamma^+, \Gamma^- \geq 0$. It is most desirable that the deviation variables should be kept as small as possible, which leads to the following model to be constructed:

$$\min J = \sum_{i=1}^n (\varepsilon_i^+ + \varepsilon_i^- + \gamma_i^+ + \gamma_i^-) = e^T(\Xi^+ + \Xi^- + \Gamma^+ + \Gamma^-)$$
$$s.t. \begin{cases} B^-W^+ - \lambda^-W^- - \Xi^+ + \Xi^- = 0, \\ B^+W^- - \lambda^+W^+ - \Gamma^+ + \Gamma^- = 0, \\ w_i^- + \sum_{j=1,j\neq i} w_j^+ \geq 1, i = 1, 2, \cdots, n, \\ w_i^- + \sum_{j=1,j\neq i} w_j^+ \leq 1, i = 1, 2, \cdots, n, \\ W^+ - W^- \geq 0, \\ W^+, W^- \geq 0, \\ W^-, W^+, \Xi^+, \Xi^-, \Gamma^+, \Gamma^- \geq 0. \end{cases} \tag{1}$$

3 Case illusions

Example 1[12]. Consider a multiple criteria decision making problem, there are four criteria . A decision maker compares each pair of criteria x_i and x_j, and provides his/her interval preference degree r_{ij} of the criterion x_i over x_j , and then constructs the following interval fuzzy preference relation:

$$R = \begin{bmatrix} [0.5, 0.5] & [0.3, 0.4] & [0.5, 0.7] & [0.4, 0.5] \\ [0.6, 0.7] & [0.5, 0.5] & [0.6, 0.8] & [0.2, 0.6] \\ [0.3, 0.5] & [0.2, 0.4] & [0.5, 0.5] & [0.4, 0.8] \\ [0.5, 0.6] & [0.4, 0.8] & [0.2, 0.6] & [0.5, 0.5] \end{bmatrix}.$$

By model(1), we get $W = ([0.2044, 0.2044], [0.2156, 0.3897], [0.1383, 0.2444], [0.1616, 0.3781])$, using definition 3, we have the ranking: $\widetilde{w}_2 \geq \widetilde{w}_4 \geq \widetilde{w}_1 \geq \widetilde{w}_3$. If we replace the elements $r_{12} = [0.3, 0.4]$ and $r_{21} = [0.6, 0.7]$ of R in Example 1 with a pair of new elements $r'_{12} = [0.1, 0.2]$ and $r'_{21} = [0.8, 0.9]$,By model(1), we have $W = ([0.1522, 0.1522], [0.2415, 0.5016], [0.1397, 0.2379],$

$[0.1707, 0.3684]$), using definition 3, we have the ranking: $\widetilde{w}_2 \geq \widetilde{w}_4 \geq \widetilde{w}_3 \geq \widetilde{w}_1$.

From Example1, we can conclude that the ranking divided by model(1) is similar to the ranking in literature[12], but the proposed methods are much more simple than the methods in literature[12]. If we utilize the practical case in literature[12] involving the assessment of a set of agroecological regions in Hubei Province, China, to illustrate the proposed method. Then, By model (1), we can get $W = ([0.2291, 0.3591], [0.1437, 0.2129], [0.1116, 0.1647], [0.1815, 0.2555], [0.1067, 0.1521], [0.0423, 0.0969])$, by definition 3, we can conclude that the proposed model gets the same ranking in literature[12]: $\widetilde{w}_1 \geq \widetilde{w}_4 \geq \widetilde{w}_2 \geq \widetilde{w}_3 \geq \widetilde{w}_5 \geq \widetilde{w}_6$.

4 Concluding remarks

In this paper, one linear programming model is constructed to derive interval weights from consistent or inconsistent interval fuzzy preference relations. Several numerical examples are illustrated to show that the proposed methods can reduce the amount of computation.

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