

# Application of Homotopy-Perturbation Method for Solving Gas Dynamics Equation

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## Abstract

In this paper applies the He's homotopy perturbation method (HPM) to obtaining solution of nonlinear dynamic model. The nonlinear considered model is the Gas Dynamics equation.

**Mathematics Subject Classification:** 39B12, 35F25

**Keywords:** The homotopy-perturbation method; Gas Dynamics equation; Partial differential equation

## 1 Introduction

Nonlinear partial differential equations are useful in describing the various phenomena in disciplines. The Homotopy perturbation method, first proposed by He in 1998, was developed and improved by He [5, 6, 7, 8]. Homotopy perturbation method [4] is a novel and effective method, and can solve various nonlinear equations. This method has been successfully applied to solve many types of nonlinear problems [10, 2, 3, 5, 6].

The application of the homotopy-perturbation method (HPM) has been devoted by scientists and engineers, because this method is to continuously deform a difficult problems which is easier to solve.

In this paper we consider the nonlinear homogeneous gas dynamics equation

$$\frac{\partial u}{\partial t} + \frac{1}{2}(u^2)_x - u(1-u) = 0 \quad 0 \leq x \leq 1, t > 0, \quad (1)$$

with initial condition

$$u(x, 0) = g(x); \quad (2)$$

Evans and Bulut have solved this equation using Adomian decomposition method [1].

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## 2 Basic idea of the homotopy-perturbation method

In this section to illustrate the basic ideas of this method, we consider the following equation :

$$A(u) - f(r) = 0, \quad r \in \Omega, \quad (3)$$

with the boundary condition of:

$$B(u, \frac{\partial u}{\partial n}) = 0, \quad r \in \Gamma, \quad (4)$$

where  $A$  is a general differential operator,  $B$  is a boundary operator,  $f(r)$  is a known analytical function and  $\Gamma$  is the boundary of the domain  $\Omega$ .

$A$  can be divided into two parts which are  $L$  and  $N$ , where  $L$  is linear and  $N$  is nonlinear. Therefore Eq.(3) can be rewritten as follows:

$$L(u) + N(u) - f(r) = 0, \quad r \in \Omega, \quad (5)$$

Homotopy perturbation structure is shown as follows:

$$H(v, p) = (1 - p) * [L(v) - L(u_0)] + p [A(v) - f(r)] = 0, \quad (6)$$

In Eq.(6),  $p \in [0, 1]$  is an embedding parameter and is the first approximation that satisfies the boundary conditions. We can assume that the solution of Eq.(6) can be written as a power series in  $p$ , as following:

$$v = v_0 + p v_1 + p^2 v_2 + \dots, \quad (7)$$

and the best approximation is:

$$u = \lim_{p \rightarrow 1} v = v_0 + v_1 + v_2 + \dots \quad (8)$$

The convergence of series (7) is discussed in [9].

## 3 Experimental evaluation

In this section we consider the homogeneous differential

$$\frac{\partial u}{\partial t} + \frac{1}{2}(u^2)_x - u(1 - u) = 0 \quad 0 \leq x \leq 1, t > 0, \quad (9)$$

with specified conditions

$$u(x, 0) = g(x) = e^{-x}, \quad (10)$$

Substituting Eq.(9) into (6) and suppose  $u_0^* = u(x, 0) = e^{-x}$ , we have an equation system including  $(n + 1)$  equations to be simultaneously solved;  $n$  is the order of  $p$  in Eq.(7). Assuming  $n = 3$  the system is as follows:

$$p^0 : \quad L(u_0) - L(u_0^*) = 0, \quad (11)$$

$$p^1 : \quad L(u_1) + L(u_0^*) + \frac{1}{2}u_{0x}^2 - u_0 + u_0^2 = 0, \quad (12)$$

$$p^2 : \quad L(u_2) + (u_0u_1)_x + 2u_0u_1 - u_1 = 0, \quad (13)$$

$$p^3 : \quad L(u_3) + \frac{1}{2}(u_1^2)_x + (u_0u_2)_x + 2u_0u_2 + u_1^2 - u_2 = 0, \quad (14)$$

Solving differential Eqs. (11)-(14), the approximate solution of Eq.(9) can be readily obtained by

$$u(x) = \sum_{n=0}^{\infty} u_n, \quad (15)$$

So we have:

$$u(x, t) = e^{-x} \left( 1 + t + \frac{1}{2}t^2 + \frac{1}{6}t^3 + \dots \right) = e^{t-x}$$

It is obvious that a higher number of iterations makes  $u_n(x, t)$  converge to the exact solution  $e^{t-x}$ .

## 4 Conclusion

In this paper, the homotopy perturbation method (HPM) was successfully applied to study the homogeneous case of nonlinear gas dynamics equation with initial conditions. The results show that homotopy perturbation method is powerful and efficient techniques in finding exact and approximate solutions for nonlinear differential equations. *Mathematica* has been used for computations in this paper.

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**Received: April 26, 2008**