Inventory Management of Time Dependent Deteriorating Items with Salvage Value

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Abstract

A mathematical model is developed for items which deteriorate with respect to time. The salvage value is incorporated to this deteriorated units. The sensitivity analysis is carried out to study the effect of salvage value and other parameters using numerical example.

Keywords: Deterioration Weibull distributions, Salvage value, Economic order Quantity (EOQ)

1. Introduction

Since long time, researchers are engaged in analyzing inventory models for deteriorating items such as volatile liquids, blood, medicines, electronic components, fashion goods, fruits and vegetables etc. Whitin (1957) studied deterioration at the end of the storage period, for example, for the fashion goods industry. Berrotoni (1962) observed that both the leakage failure of the dry batteries and life expectancy of ethical drugs could be expressed in terms of Weibull distribution, in discussing the problem of fitting empirical data to mathematical distributions. In both cases, the rate of deterioration increases with time or the longer the time remains unused, the higher rate at which they fail. Ghare and Schrader (1963) first formulated a mathematical model with a constant deterioration rate they classified the phenomena of inventory deterioration into three types, viz direct spoilage, physical depletion and deterioration. Covert and
Philip (1973) derived on EOQ model for items with Weibull distribution deterioration.

Since then Misra (1975), Shah (1977), Dave and Patel (1981), Holler Mak (1983), Heng et al. (1991), Hariga (1996) and Wee (1995) on deteriorating inventory systems. The review articles Rattat (1991), Shah & Shah (2000) and Goyal and Giri (2001) give a complete and up-to-date survey of published literature for the deteriorating inventory models. The most of the addressed articles assume that deterioration of units is the complete loss to the inventory system. In this paper an attempt has been made to develop a mathematical model for obtaining optimal purchase quantity for time dependent deteriorating items with some salvage value associated to the deteriorated units during the cycle time. The sensitivity analysis is carried out to study the effect of various parameters on the decision variables and objective function.

2. Assumptions and notations

The proposed model is derived under the following assumptions and notations:

- Demand of R units per time unit is deterministic and constant.
- The replenishment rate is infinite.
- The lead time is zero and shortages are not allowed.
- The purchase cost – C / units, the inventory holding cost – h / unit / time unit and the ordering cost- A / order are known during the cycle time under consideration.
- The rate of units in inventory follows the Weibull distribution function given by 
  \[ \theta(t) = \alpha \beta t^{(\beta - 1)} , \quad 0 \leq t \leq T \]  
  where \( \alpha (0 \leq \alpha \leq 1) \) denotes scale parameter, \( \beta (\beta \geq 1) \) denotes shape parameter and \( t (t > 0) \) is time to deterioration.
- The salvage value \( \gamma C (0 \leq \gamma < 1) \) is associated to deteriorated units during the cycle time.
- The deteriorated units can not be repaired or replaced during the period under review.

3 Mathematical Model

Let \( Q(t) \) be the on-hand inventory at any instant of time \( t \) \( (0 \leq t \leq T) \). The depletion of units inventory is due to demand and due to deterioration. The instantaneous state of \( Q(t) \) at any instant of time is governed by the differential equation.

\[
\frac{dQ(t)}{dt} + \theta(t)Q(t) = -R, \quad 0 \leq t \leq T
\]  

(3.1)
Inventory management

with initial condition \( Q(0) = Q \) and boundary condition \( Q(T) = 0 \), where \( 0(t) \) is given by equation (1.1). Taking series expansion and ignoring second and higher powers of \( \alpha \) (assuming \( \alpha \) very small), the solution of differential equation (2.1) using boundary condition \( Q(T) = 0 \) is

\[
Q(t) = R \left[ T - t + \frac{\alpha T}{\beta + 1} ((\beta - 1) + \beta t^\beta) + \frac{\alpha \beta T^{\beta+1}}{\beta + 1} \right]
\]

(3.2)

Using \( Q(0) = Q \), we get

\[
Q(t) = R \left[ T + \frac{\alpha T^{\beta+1}}{\beta + 1} \right]
\]

(3.3)

The number of units that deteriorated; \( D(T) \) during one cycle time is given by

\[
D(T) = Q - RT = \frac{R \alpha T^{\beta+1}}{\beta + 1}
\]

(3.4)

The average inventory, \( I_1(T) = \frac{1}{T} \int_0^T Q(t) \, dt \)

\[
= R \left[ \frac{T^2}{2} + \frac{\alpha \beta T^{\beta+2}}{(\beta + 1)(\beta + 2)} \right]
\]

(3.5)

The total cost per time unit, \( K(T) \) comprises of following cost:

a) Inventory holding cost per time unit IHC = \( hR \left[ \frac{T}{2} + \frac{\alpha \beta T^{\beta+1}}{(\beta + 1)(\beta + 2)} \right] \)

b) Ordering cost per order, OC = \( \frac{A}{T} \)

c) Cost due to deterioration per time unit, CD = \( \frac{C \gamma T^{\beta+1}}{\beta + 1} \)

d) Salvage value of deteriorated items per time unit, SV = \( \frac{\gamma CR \alpha T^{\beta+1}}{\beta + 1} \)

Thus, \( K(T) = IHC + OC + CD - SV \)

The necessary condition for \( K(T) \) to be minimum is \( \frac{\partial K(T)}{\partial T} = 0 \) and solving it for \( T \) by a suitable mathematical software. For obtained \( T \), \( K(T) \) is minimum only if

\[
\frac{\partial^2 K}{\partial T^2} = hR \left[ 1 + \frac{\alpha \beta T^{\beta+2} (\beta + 2)}{T^2 (\beta + 1)} - \frac{\alpha \beta T^{\beta+2}}{T^2 (\beta + 1)} \right] - 2hR \left[ \frac{T + \frac{\alpha \beta T^{\beta+2}}{T (\beta + 1)}}{T^2} \right] + 2h \]

\[
\left[ \frac{T^2}{2} + \frac{\alpha \beta T^{\beta+2}}{(\beta + 1)(\beta + 2)} \right] + \frac{C \gamma R \alpha T^{\beta+1} (\beta + 1)}{T^3} - \frac{3C \gamma R \alpha T^{\beta+1}}{T^3} + \frac{2C \gamma R \alpha T^{\beta+1}}{(\beta + 1)T^3} + \frac{2A}{T^3} - \]
\[
\gamma CR^\alpha T^{\beta + 1}(\beta + 1) + 3\gamma CR^\alpha T^{\beta + 1} + 2\gamma CR^\alpha T^{\beta + 1} > 0 \text{ for all } T > 0.
\]

**Special Cases:** When \(\gamma = 0\) and \(\beta = 1\) i.e. So salvage value is associated with deteriorated units, the derived model reduces to that of Ghare and Scharder (1963). When \(\alpha = 0\) and \(\beta = 1\), the model reduces to that of Aggarwal and Jaggi (1996).

### 4. Theorems

**Theorem 4.1:** The total cost is increasing function of scale parameter \(\alpha\).

**Proof:** Clearly,

\[
\frac{\partial K}{\partial \alpha} = \frac{hR^\alpha T^{\beta + 2}}{(\beta + 1)(\beta + 2)^T} + \frac{CRT^{\beta + 1}}{(\beta + 1)T} - \frac{\gamma CRT^{\beta + 1}}{(\beta + 1)T} > 0
\]

**Theorem 4.2:** The total cost is decreasing function of shape parameter \(\beta\).

**Proof:** Clearly,

\[
\frac{\partial K}{\partial \beta} = hR^\alpha T^{\beta + 2} + \frac{\alpha T^{\beta + 2}}{(\beta + 1)(\beta + 2)^T} - \frac{\alpha T^{\beta + 2}}{(\beta + 1)(\beta + 2)^T} - \frac{\alpha T^{\beta + 2}}{(\beta + 1)(\beta + 2)^T}
\]

\[
+ \frac{CRT^{\beta + 1}ln(T)}{(\beta + 1)^T} - \frac{CRT^{\beta + 1}ln(T)}{(\beta + 1)^T} - \frac{\gamma CRT^{\beta + 1}ln(T)}{(\beta + 1)^T} + \frac{\gamma CRT^{\beta + 1}ln(T)}{(\beta + 1)^T} < 0
\]

**Theorem 4.3:** The total cost is a decreasing function of salvage value \(\gamma\).

**Proof:** Clearly,

\[
\frac{\partial K}{\partial \gamma} = - \frac{CR^\alpha T^{\beta + 1}}{(\beta + 1)T} < 0
\]

### 5. Numerical Example

Consider an inventory system with the following parametric values in proper units:

\[ [R, C, h, A] = [10,000, 20, 2, 200] \]

**Table 1:** Sensitivity analysis of the scale parameter \(\alpha\)

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>(T)</th>
<th>(Q)</th>
<th>(D(T))</th>
<th>(TC)</th>
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<tr>
<td>0.10</td>
<td>0.1205</td>
<td>1206</td>
<td>16.73171</td>
<td>3167.65</td>
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<tr>
<td>0.15</td>
<td>0.1136</td>
<td>1138</td>
<td>22.9730</td>
<td>3312.32</td>
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<tr>
<td>0.20</td>
<td>0.1079</td>
<td>1082</td>
<td>28.3545</td>
<td>3445.57</td>
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<tr>
<td>0.25</td>
<td>0.1033</td>
<td>1036</td>
<td>33.2009</td>
<td>3569.67</td>
</tr>
</tbody>
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Table 2: Sensitivity analysis of the shape parameter $\beta$

<table>
<thead>
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<th>$\beta$</th>
<th>T</th>
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<th>D(T)</th>
<th>TC</th>
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</thead>
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</tr>
<tr>
<td>2.0</td>
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<td>1241</td>
<td>10.2506</td>
<td>3038.69</td>
</tr>
<tr>
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<td>1333</td>
<td>3.7001</td>
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<tr>
<td>3.0</td>
<td>0.1378</td>
<td>1379</td>
<td>1.3083</td>
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Table 3: Sensitivity analysis of the salvage value $\gamma$

<table>
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<th>$\gamma$</th>
<th>T</th>
<th>Q</th>
<th>D(T)</th>
<th>TC</th>
</tr>
</thead>
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</tr>
<tr>
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<td>1131</td>
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<td>1190</td>
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<td>0.7</td>
<td>0.1258</td>
<td>1262</td>
<td>35.6952</td>
<td>3065.84</td>
</tr>
</tbody>
</table>

6. Conclusions

The economic purchase quantity for time dependent deteriorated units with associated salvage value is analysed in this paper. It is observed that shape parameter $\beta$ increases number of units to be procured and decreases total cost of an inventory system. Increase in shape parameter $\alpha$ decreases procurement quantity and increases total cost of an inventory system. Increase in salvage value $\gamma$ decreases number of units to be procured and total cost of an inventory system.

References


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