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Construction of Bounded Feedback by the Controllability Function Method

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Abstract

In this paper, we develop feedback control strategies for linear systems which are controllable. Our solution relies on a controllability functions construction. The state feedbacks obtained are bounded and ensure the global asymptotic stability. The time of stabilization is explicitly computed.

Keywords: controllability, stabilization, feedback, linear systems

1 Introduction

The stabilization of control systems is one of the most important in control theory. Many techniques have been developed during the last two decades to study the stabilizability of control systems and to design stabilizing feedback. One of the most popular technique of control design is the Lyapunov approach see for instance [2], [3], where this strategy of design is clearly exposed. The multiple advantages offered by this approach are well-known. Observe in particular that this approach yields a very efficient help for solving locally, and in some cases globally the problem of stabilizability for wide families of control systems. But sometimes, the construction of the Lyapunov function is very difficult and it is fair to say that there is a need for studying this problem.

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The stabilization of control systems has been observed in various systems, but seldom a link is made with the controllability function method.
However, the idea of the construction of bounded feedback by the controllability function method has led to one of the basis tools nowday for designing stabilizing feedbacks.
In [1], Korobov gives a repertory of one procedure which can be obtained to deal with various classes of systems by combining, maybe recursively, this particular controllability function design with other ones.

Consider the system

$$\dot{x} = f(x, u), \tag{1}$$

where $x \in \mathbb{R}^n$ and $u \in \Omega \subset \mathbb{R}^r$ (Ω is the neighbourhood of the orign).

. When is it possible to design a feedback law $u = \phi(x)$ such that the trajectory $x(t)$ of the closed-loop system

$$\dot{x} = f(x, \phi(x)), \tag{2}$$

which start at x_0 at the time $t_0 * = 0$ satisfies at any finite time $T(x_0*)$ the condition $x(T(x_0*)) = 0$, i.e., $\lim_{t \to T(x_0*)} *x(t)=0$?
In (1), the presence of u impedes the application of the controllability technique [1] to solve the problem.
The solution we propose here is based on a completely different approach and is appropriate only for a subclass of systems (1). This particular class allows us to design a controllability function. This construction utilizes tools and properties analogous to those employed to achieve stabilization in [2], [3]. Our result is a result of global asymptotic stability (GAS). Moreover, the class of feedback laws obtained are bounded.

Our work is organized as follows. In Section 2 is given the main result of our work. In Section 3 an example of canonical systems of this result is proposed. Section 4 solve some examples and Section 5 contains concluding remarks.

1.1 Notation and basic definitions

• Thoughout the paper, the symbol $\mathbb{R}$ denotes the set of real numbers and $\mathbb{R}^n$ the n-dimensional real linear space.
• x, y may represent columm vector of $\mathbb{R}^n$ and x^T a vector row.
• We denote by $\dot{x}$ the first derivative of a real-valued C^1 function $x(t) = (x_1 * (t), \ldots, x_n * (t))$.
• With capital letters, A, B, N, X we denote the matrices and A^T the transpose of the matrix A.

• s, t and θ are real scalar.
Some special notations will be defined.

2 Construction of the controllability function

Consider the particular case of (1)

$$\begin{cases} \dot{x} = Ax + Bu, \\ \\ x \in \mathbb{R}^n, u \in \Omega \subset \mathbb{R}^r, A \in \mathfrak{M}_{n,n} * (\mathbb{R}), B \in \mathfrak{M}_{n,r} * (\mathbb{R}), \end{cases} \tag{3}$$

where $\mathfrak{M}_{n,m}*(\mathbb{R})$ is the set of matrices with n rows and m columns. We assume that the pair (A, B) is controllable, i.e., $rank(B, AB, \ldots, A^{n-1}B) = n$.
Let $N(\theta)$ be the matrix

$$N(\theta) = \frac{1}{\theta} e^{-\frac{\alpha}{\beta} A \theta^{\frac{1}{\alpha}}} \int_0^\theta * \frac{\tau}{\beta} e^{\frac{\alpha}{\beta} A \tau^{\frac{1}{\alpha}}} BB^T e^{\frac{\alpha}{\beta} A^T \tau^{\frac{1}{\alpha)}}} d\tau e^{-\frac{\alpha}{\beta} A^T \theta^{\frac{1}{\alpha}}}, \tag{4}$$

with $\alpha > 0$, $\beta > 0$ and $\theta > 0$.

We will exploit the properties of the matrix $N(\theta)$ for designing the controllability function which facilitates the construction of the feedback law. More precisely, we have:

Lemma 1 *: For $\alpha > 0$, $\beta > 0$ and $\theta > 0$, $N(\theta)$ is symmetric positive definite and the solution of the differential equation*

$$\frac{dX}{d\theta} = \frac{1}{\beta} \theta^{\frac{1}{\alpha} - 1} \left[-AX - XA^T - \beta \theta^{-\frac{1}{\alpha}} X + BB^T \right]. \tag{5}$$

This Lemma is proved in Appendix A.

Remark 1 *: In the coordinate $t = 1 - \left(\frac{\tau}{\theta}\right)^{1/\alpha}$ (4) become*

$$N(\theta) = \frac{\alpha}{\beta} \theta^{\frac{1}{\alpha}} \int_0^1 *(1-t)^\alpha e^{-\frac{\alpha}{\beta} A \theta^{\frac{1}{\alpha}} t} BB^T e^{-\frac{\alpha}{\beta} A^T \theta^{\frac{1}{\alpha}} t} dt \tag{6}$$

and

$$\lim_{\theta \to 0+} * \frac{N(\theta)}{\frac{\alpha}{\beta} \theta^{\frac{1}{\alpha}}} = \frac{BB^T}{(\alpha + 1)}. \tag{7}$$

In the sequel the controllability function $\theta(x)$ is chosen to be the unique positive solution of

$$\theta(x) = \langle N^{-1}(\theta) x, x \rangle, \quad x \neq 0. \tag{8}$$

Moreover, about θ we prove in Appendix B:

Lemma 2 *: For each $\alpha > 0$, $\beta > 0$, $\theta = \theta(x)$ is of class C^1 and is the positive solution of the equation (8) for all $x \neq 0$.*

At this point of our design, we need the following preliminary result:

Lemma 3 *: The feedback*

$$u(x) = -\frac{1}{2}B^T N^{-1}(\theta)x, \tag{9}$$

where $\theta(x)$ is defined in (8) is Lipschitz on the compact set $\mathcal{K}(\rho, \rho_1*) \cap \mathcal{Q}$ with

$$\mathcal{Q} = \{x \in \mathbb{R}^n, \quad \theta(x) \leq c\}, \quad c > 0,$$

and

$$\mathcal{K}(\rho, \rho_1*) = \{x \in \mathbb{R}^n, \quad 0 < \rho \leq \|x\| \leq \rho_1*\}, \quad \rho, \; \rho_1* > 0.$$

:

If one computes the derivative of the feedback (9) with respect to x, one gets

$$\frac{\partial u(x)}{\partial x_i*} = -\frac{1}{2}B^T \left[\frac{\partial}{\partial x_i*}N^{-1}(\theta)x_i * + N^{-1}(\theta)\frac{\partial x}{\partial x_i*}\right], \quad i = 1, \ldots, n$$

Remark that

$$\begin{aligned} \frac{\partial}{\partial x_i*}N^{-1}(\theta) &= \frac{d}{d\theta}N^{-1}(\theta)\frac{\partial \theta}{\partial x_i*}, \\ &= -N^{-1}(\theta)\frac{dN(\theta)}{d\theta}N^{-1}(\theta)\frac{\partial \theta}{\partial x_i*} \end{aligned}$$

According to Lemma 2, $\dfrac{\partial \theta}{\partial x_i*}$, $\theta(x)$ and $\dfrac{\partial \theta}{\partial x}$ are continuous functions. It follows that the elements of the matrices $N^{-1}(\theta)$ and $\dfrac{d}{d\theta}N(\theta)$ are continuous functions on $\mathcal{K} \cap \mathcal{Q}$ in x which implies that $u(x)$ is Lipschitz continuous on $\mathcal{K} \cap \mathcal{Q}$.

△

Theorem 1 *: All solution $x(t, t_0*)$ of the closed-loop system (3)-(9) issue of the point x_0* at the time t_0* satisfies $\lim_{t \to T(x_0*)} *x(t) = 0$ where $T(x_0*) = \dfrac{\alpha}{\beta}\theta(x_0*)^{\frac{1}{\alpha}}$, i.e., the system (3) is globally asymptotically stabilizable by means of the feedback law (9).*

:
Consider as a candidate of Lyapunov function, the quadratic positive definite and proper function

$$\theta(x) = x^T N^{-1}(\theta) x$$

Let us evaluate its derivative along the trajectories of the closed-loop system (3)-(9). To this end, let the function

$$F(\theta, x) = \theta(x) - \langle N^{-1}(\theta)x, x\rangle$$

Use

$$dF = F'_\theta * \frac{\partial \theta}{\partial x} + F'_x * = 0,$$

where $F'_\theta * (\theta, x) = \dfrac{\partial F}{\partial \theta}$ and $F'_x * (\theta, x) = \dfrac{\partial F}{\partial x}$, one gets

$$\begin{aligned}
\dot\theta(x(t)) &= \left\langle \frac{\partial \theta}{\partial x},\ \dot x \right\rangle, \\
&= -\left\langle \frac{F'_x *}{F'_\theta *}, \dot x \right\rangle, \\
&= \frac{1}{F'_\theta *}\left[x^T \left\langle A^T N^{-1}(\theta) + N^{-1}(\theta)A - N^{-1}(\theta)BB^T N^{-1}(\theta)\right\rangle x\right]
\end{aligned}$$

From (5), we have

$$A^T N^{-1}(\theta) + N^{-1}(\theta)A - N^{-1}(\theta)BB^T N^{-1}(\theta) = \beta\theta^{1-\frac{1}{\alpha}}\left[\frac{d}{d\theta}N^{-1}(\theta) - \frac{1}{\theta}N^{-1}(\theta)\right]$$

and remark that

$$F'_\theta * (\theta, x) = x^T\left(\frac{N^{-1}(\theta)}{\theta} - \frac{d}{d\theta}N^{-1}(\theta)\right)x$$

one can deduce that

$$\dot\theta(x) = -\beta\theta(x)^{1-\frac{1}{\alpha}}$$

which is negative definite and thus all the conditions of the Theorem 1 of [1] are verified and this achieve the proof.

△

3 Canonical systems case

We assume that the pair (A, B) is in Brunovsky canonical form, i.e.,

- A is a block-diagonal matrix of the form

$$A = \begin{pmatrix} A_{k_1} & 0 & . & . & 0 \\ 0 & . & . & . & . \\ . & . & . & . & . \\ 0 & . & . & . & A_{k_r} \end{pmatrix}$$

where $A_{k_i}, 1 \leq i \leq r$, is a matrix in $\mathfrak{M}_{k_i,k_i}(\mathbb{R})$ given by

$$A_{k_i} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & . & . & . & 1 \\ 0 & . & . & . & 0 \end{pmatrix}$$

- B is a block-diagonal matrix of the form

$$B = \begin{pmatrix} b_{k_1} & 0 & . & . & 0 \\ 0 & . & . & . & . \\ . & . & . & . & . \\ 0 & . & . & 0 & b_{k_r} \end{pmatrix}$$

where $b_{k_i}, 1 \leq i \leq r$, is a column-vector in $\mathbb{R}^{k_i}$ given by

$$b_{k_i} = \begin{pmatrix} 0 \\ . \\ . \\ 0 \\ 1 \end{pmatrix}$$

The elements $[N(\theta)]_{i,j}*$ of the matrix $N(\theta)$ is given by

$$[N(\theta)]_{i,j} * = \frac{(-1)^{i+j} \left(\frac{\alpha}{\beta}\theta^{\frac{1}{\alpha}}\right)^{2n-i-j+1} (2n-i-j)!}{(n-i)!(n-j)!(\alpha+1)\dots(\alpha+2n-i-j+1)} \quad i,j = 1,\dots,n.$$

Using the above elements, a simple computation give

$$N(\theta) = \Delta_\theta * F(\alpha,\beta) * \Delta_\theta *, \tag{10}$$

where

$$\Delta_\theta * = diag(\theta^{\frac{(n-\frac{1}{2})}{\alpha}}, \theta^{\frac{(n-1)-\frac{1}{2})}{\alpha}}, \dots, \theta^{\frac{1-\frac{1}{2}}{\alpha}})$$

and $F(\alpha,\beta)$ a constant diagonal matrix which elements satisfies

$$[F(\alpha,\beta)]_{i,j} * = \frac{(-1)^{i+j} \left(\frac{\alpha}{\beta}\right)^{2n-i-j+1} (2n-i-j)!}{(n-i)!(n-j)!(\alpha+1)\dots(\alpha+2n-i-j+1)}$$

From (10), we have

$$N^{-1}(\theta) = \Delta^{-1}(\theta)F^{-1}(\alpha,\beta)\Delta_\theta^{-1} * . \tag{11}$$

With this decomposition the equation (4) become

$$\theta(x) = \langle \Delta_\theta^{-1} * F^{-1}(\alpha,\beta)\Delta_\theta^{-1} * x, x\rangle,$$

and θ satisfies the following equation

$$\theta^{1+\frac{(2n-1)}{\alpha}} = \sum_{i,j=1}^{n} * [F(\alpha,\beta)]_{i,j} * \theta^{\frac{(i+j-2)}{n}} x_i * x_j * . \tag{12}$$

Then one can state that

Corollary 1 *: If the matrices A and B are in the Brunovsky canonical form, then for any positive constant η, if $\theta(x)$ is the positive solution of (12), the system (3) is globally asymptotically stabilizable by means of the feedback law*

$$u(x) = -\frac{1}{2}\sum_{j=1}^{n} *F_{n,j} * (\alpha,\beta)\theta^{\frac{1}{\alpha}(-n+j-1)}x_j *, \tag{13}$$

which satisfies

$$\|u(x)\| \le \eta,\ \ \forall x \in R^n$$

4 Examples

Consider the system evolving in $\mathbb{R}$

$$\dot{x} = u \tag{14}$$

One may easily check that for any $\theta > 0$

$$N(\theta) = \frac{\alpha\theta^{\frac{1}{\alpha}}}{\beta(\alpha+1)},$$

and

$$N^{-1}(\theta) = \frac{\beta(\alpha+1)}{\alpha\theta^{\frac{1}{\alpha}}}$$

With $N^{-1}(\theta)$ it is immediat that

$$\theta(x) = \left[\frac{\alpha}{\beta}(\alpha+1)x^2\right]^{\dfrac{\alpha}{\alpha+1}}$$

is the solution of the equation

$$\theta(x) = \frac{\beta(\alpha+1)x^2}{\alpha\theta^{\frac{1}{\alpha}}}, \quad for\ \ x \neq 0$$

Following Theorem 1, the feedback

$$u(x) = -\frac{\beta(\alpha+1)x}{2\alpha\theta^{\frac{1}{\alpha}}}, \tag{15}$$

globally stabilizes system (14).
In particular if $\alpha = \beta = 1$ and $\eta = 1$ we have $\theta(x) = \sqrt{2}|x|$ and $u(x) = -\sqrt{2}sgn(x)$.

Consider the planar system

$$\ddot{x} = u, \tag{16}$$

By setting $x = x_1*$ and $\dot{x} = x_2*$ (16) become

$$\begin{cases} \dot{x}_1* &= x_2*, \\ \dot{x}_2* &= u. \end{cases} \tag{17}$$

As for above example, a simple computation yield

$$N(\theta) = \begin{pmatrix} \dfrac{2\left(\dfrac{\alpha}{\beta}\theta^{\frac{1}{\alpha}}\right)^3}{(\alpha+1)(\alpha+2)(\alpha+3)} & \dfrac{-\left(\dfrac{\alpha}{\beta}\theta^{\frac{1}{\alpha}}\right)^2}{(\alpha+1)(\alpha+2)} \\ \dfrac{-\left(\dfrac{\alpha}{\beta}\theta^{\frac{1}{\alpha}}\right)^2}{(\alpha+1)(\alpha+2)} & \dfrac{\dfrac{\alpha}{\beta}\theta^{\frac{1}{\alpha}}}{(\alpha+1)} \end{pmatrix}$$

and

$$N^{-1}(\theta) = \begin{pmatrix} \dfrac{(\alpha+2)^2(\alpha+3)}{\left(\dfrac{\alpha}{\beta}\theta^{\frac{1}{\alpha}}\right)^3} & \dfrac{(\alpha+2)(\alpha+3)}{\left(\dfrac{\alpha}{\beta}\theta^{\frac{1}{\alpha}}\right)^2} \\ \dfrac{(\alpha+2)(\alpha+3)}{\left(\dfrac{\alpha}{\beta}\theta^{\frac{1}{\alpha}}\right)^2} & \dfrac{2(\alpha+2)}{\dfrac{\alpha}{\beta}\theta^{\frac{1}{\alpha}}} \end{pmatrix}$$

Hence, it follows that

$$\theta(x) = \frac{(\alpha+2)^2(\alpha+3)x_1^2*}{\left(\dfrac{\alpha}{\beta}\theta^{\frac{1}{\alpha}}\right)^3} + \frac{2(\alpha+2)(\alpha+3)x_1 * x_2*}{\left(\dfrac{\alpha}{\beta}\theta^{\frac{1}{\alpha}}\right)^2} + \frac{2(\alpha+2)x_2^2*}{\dfrac{\alpha}{\beta}\theta^{\frac{1}{\alpha}}}$$

So by application of Theorem 1, system (16) can be globally stabilized by the feedback law

$$u(x) = -\frac{(\alpha+2)(\alpha+3)x_1*}{2\left(\dfrac{\alpha}{\beta}\theta^{\frac{1}{\alpha}}\right)^2} - \frac{(\alpha+2)x_2*}{\dfrac{\alpha}{\beta}\theta^{\frac{1}{\alpha}}} \tag{18}$$

The simulation results are shown in Fig.1 and Fig.2, where it is clear that the control effect using our feedback controller is as good as the case of controlling the trajectory $x(t)$ to the origin.

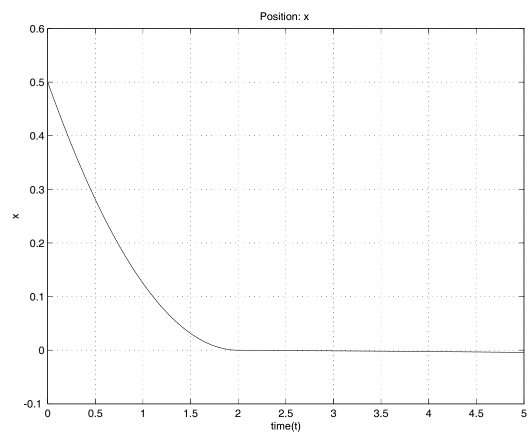

Figure 1: Example 1 under robust feedback (15) when $\alpha = \beta = 1$

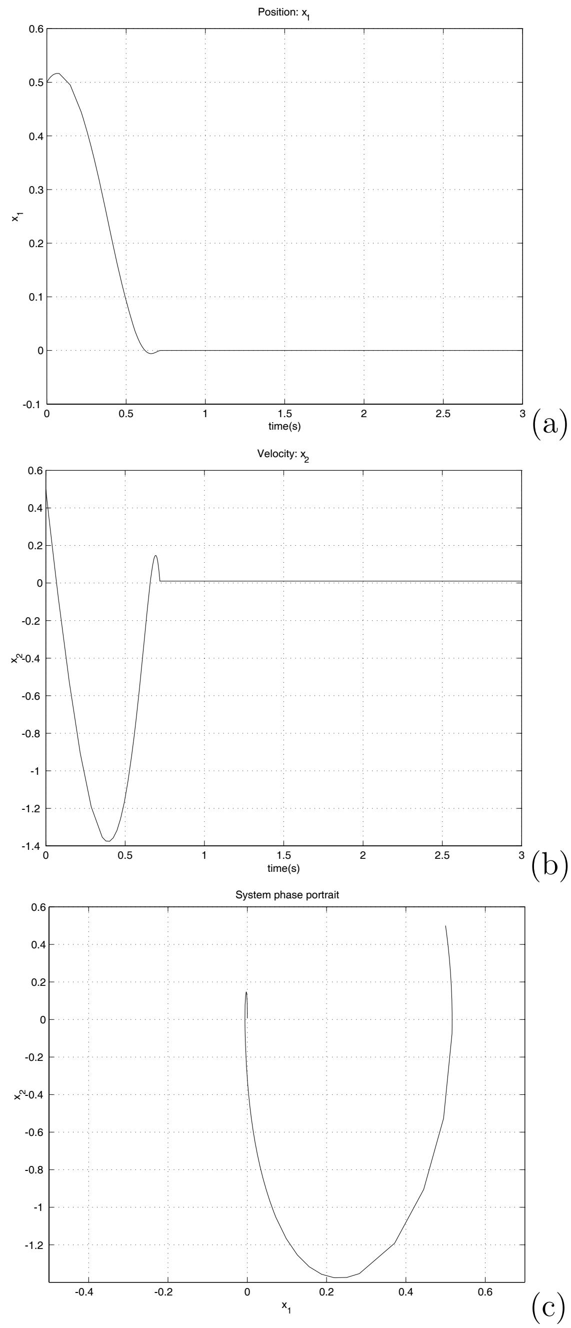

Figure 2: Example 2 under robust feedback (18) when $\alpha = 1$ and $\beta = 50$

5 Conclusion

In this paper, we have constructed a family of feedbacks (which contains arbitraily small bounded functions) which globally asymptotically stabilize linear systems. A recursive application of the design we have proposed is possible and provides us with a new technique for dealing with the stabilization problems for linear systems. The feedback design is inherently robust to some types of disturbances and can be robustified with respect to many other types.

In future works, we will investigate under slightly more restrictive assumption, the stabilizability problem of a class of nonlinear system using the controllability function technique and illustrate our design of control law on the magnetically levitated ball system.

A Proof of Lemma 1

It is clear that $N(\theta)$ is a symmetric matrix and a simple computation proves that it is the solution of equation (5).

Suppose that for $\theta > 0$, there exists a vector $X_0* \neq 0$ such that $\langle N(\theta)X_0, X_0* *\rangle = 0$. From (6) one gets

$$\int_0^\theta *\|\tau^{\frac{1}{2\alpha}} B^T e^{-\frac{\alpha}{\beta}A^T(\theta^{\frac{1}{\alpha}}-\tau^{\frac{1}{\alpha}})} X_0 * \|^2 d\tau = 0, \quad for\ \ all\ \ \tau \in [0, \theta]$$

which implies

$$B^T e^{-\frac{\alpha}{\beta}A^T(\theta^{\frac{1}{\alpha}}-\tau^{\frac{1}{\alpha}})} X_0 = 0*, \quad for\ \ all\ \ \tau \in [0, \theta]$$

If one computes the (n-1) derivative of the above equation with respect τ and by setting $\tau = \theta$ one has

$$B^T X_0*, \;\; B^T A^T X_0*, \ldots, B^T (A^T)^{n-1} X_0* = 0$$

Since the pair (A, B) is controllable, this lead to a contradiction and the matrix $N(\theta)$ is symmetric positive definite.

△

B Proof of Lemma 2

1. We first establish that $\theta(x)$ is the positive solution of equation (8). Consider the function

$$F(\theta, x) = \theta - \langle N^{-1}(\theta)x, x\rangle, \quad \theta \neq 0, \;\; and \;\; x \neq 0$$

Let us evaluate its derivative with respect θ for $x \neq 0$

$$\frac{\partial F}{\partial \theta}(\theta, x) = 1 + \left\langle N^{-1}\frac{dN(\theta)}{\theta}N^{-1}(\theta)x, x\right\rangle$$

Using (4) one has

$$\frac{dN(\theta)}{d\theta} = \frac{1}{\frac{\alpha}{\beta}\theta^{1+\ frac1\alpha}}\int_0^{\frac{\alpha}{\beta}\theta^{\frac{1}{\alpha}}} * \left(1 - \frac{s}{\frac{\alpha}{\beta}\theta^{\frac{1}{\alpha}}}\right)^{\alpha-1} se^{-As}BB^Te^{-A^Ts}ds > 0. \quad (19)$$

By application of Lemma 1, (19) is positive definite and it turns out that $\frac{\partial F}{\partial \theta}(\theta, x) > 1$. So, for θ large enough $F(\theta, x) > 0$ and $F(\theta, x)$ is a strict increasing function with respect to θ.
On the other hand, since $N(\theta)$ is symmetric positive definite matrix, $N^{-1}(\theta)$ is also symmetric positive definite and it follows that

$$\langle N^{-1}(\theta)x, x\rangle \geq \frac{\langle x, x*\rangle}{\|N(\theta)\|}.$$

Following (6), $\|N(\theta)\| \to 0$ when $\theta \to 0+$, and for θ sufficient small one can deduce that

$$F(\theta, x) \leq \theta - \frac{\langle x, x*\rangle}{\|N(\theta)\|} < 0$$

Thus since $\theta(x)$ is a continuous function, the equation $\theta = \langle N^{-1}(\theta)x, x\rangle$ admits a unique positive solution $\theta(x)$.
2. Now, We prove that the function $\theta(x)$ is of class C^1.
Since $\frac{\partial F}{\partial \theta}(\theta, x) > 1$ for each $x \neq 0$, by applying the implicit function theorem, the function $\theta(x)$ is the solution of $F(\theta, x) = 0$ in a neighbourhood of $x \neq 0$ and of class C^1 in a neighbourhood of this point.
For $x \neq 0$, the function $\langle N^{-1}(\theta)x, x\rangle$ is monotone decreasing with respect θ so the function $\|N^{-1}(\theta)\|$ is also a monotone decreasing function. Thus for all $\varepsilon > 0$ we have

$$\|N^{-1}(\theta)\| \leq \mathcal{R}(\varepsilon) < \infty, \ \ for\ all\ \theta \geq \varepsilon$$

Choising $\rho_0* > 0$ sufficient small such that

$$\varepsilon > \mathcal{R}(\varepsilon)\rho_0^2 * . \quad (20)$$

and consider the closed-loop

$$S_{\rho_0*}* = \{x \in \mathbb{R}^n,\ \ \|x\| \leq \rho_0*\}$$

we want to prove that $\theta(x) < \varepsilon$, for all $x \in S_{\rho_0*}*$.

Suppose that $\theta(x) \geq \varepsilon$ for certain $x \in S_{\rho_0 *}*$. In the one hand we have

$$\langle N^{-1}(\theta)x, x\rangle \leq \|N^{-1}(\theta)\| \, \|x\|^2 \leq \mathcal{R}(\varepsilon)\rho_0^2 * .$$

On the other hand, using (8) and (20)

$$\theta(x) \geq \varepsilon > \mathcal{R}(\varepsilon)\rho_0^2 * \geq \theta(x)$$

This leads to a contradiction which implies $\theta(x) \leq \varepsilon$ for $x \in S_{\rho_0 *}*$. The results follows.

△

References

[1] Korobov, V. I.: A general approach to synthesis problem, Dokl. Acad. Nauk SSSR T.248(1979), N0 5, pp. 1051-1055

[2] LaSalle, J. P.: Letschetz, S., Stability by Liapunov's direct method, with application, Accademic Press, New York, 1961

[3] Liapunov, A. M.: Problème général de la stabilité du mouvement (en russe) Soc. Math. KHARKOV, 1892

Traduction française : Ann. Fac. Sci. Toulouse, 1907, 9, 203-474, Reed. Princiton University Press, 1949

[4] Marinitch, A. P., Rodoumta, K.: Sur la structure d'une famille de fonction de controllabilité dans le probléme de synthese de commandes bornées,

La méthode de fonctions de Liapunov dans les mathématiques contemporains, Acte du colloque scientifique 27-29 Mai 1986, Kharkiv, p.65

[5] Rodoumta, K.: A method of construction of controllability function, PHD thesis, Kharkiv, 1987

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