Computation of Area and Number Frequency Dimensions of Mountains Extracted from Multiscale Digital Elevation Models

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Abstract

Frequency dimension is used to characterize fractals that come as distinct individual objects which are spread over a range of space. In this paper, the number and area frequency dimensions of mountains extracted from multiscale digital elevation models (DEMs) are computed. First, the lifting scheme is used to generate multiscale DEMs. Mountain extraction is performed on the generated multiscale DEMs. Opening by reconstruction is performed iteratively on the generated mountains using square kernels of increasing size. The total number and total area of mountains remaining after each iteration of opening by reconstruction are computed, and used to perform the computation of the number and area distribution functions. The number-frequency distribution is the slope of the log-log plot of the number-distribution functions against the kernel size, while the area-frequency distribution is the slope of the log-log plot of the area-distribution functions against the kernel size.
Keywords: multiscale digital elevation models (DEMs), mountains, lifting scheme, opening by reconstruction, number and area frequency dimensions.

1 Introduction

Frequency dimension [2] is used to characterize fractals that come as distinct individual objects which are spread over a range of space. This parameter determines an order for a group of objects; i.e. the number of dimensions required to model the objects. It does not have to be an integer, and its decimal portion expresses a measure of system complexity. Frequency dimension is computed using the following power law:

\[ C(r) \sim r^v \]  
\[ \log C(r) = v \cdot \log r + c_v \]

where \( c_v \) is a constant of proportionality, \( C(r) \) is the distribution function, \( r \) is the kernel size, and \( v \) is the frequency dimension.

In this paper, the number and area frequency dimensions are employed to compute. In Section 2, the lifting scheme is used to generate multiscale DEMs. In Section 3, concepts of mathematical morphology are employed to compute the number and area frequency dimensions of the mountains extracted from the generated multiscale DEMs. Concluding remarks regarding the scope of the study is provided in the final section.

2 Generation of Multiscale DEMs using the Lifting Scheme

Feature detection and characterization often need to be performed at different of scales of measurement. Wood [12, 13] shows that analysis of a region at multiple scales allows for a greater amount of information to be extracted from the DEM about the spatial characteristics of a feature. The term scale refers to combination of both spatial extent and spatial detail or resolution [10]. In this paper, the variation in the spatial
extent over which mountains are defined is used as the basis to perform the computation of number and area frequency dimensions.

In this paper, multiscaling is performed using the lifting scheme [8, 9]. The lifting scheme is a flexible technique that has been used in several different settings, for easy construction and implementation of traditional wavelets and of second generation wavelets, such as spherical wavelets. Lifting consists of the following three basic operations (Figure 1):

**Step 1: Split**

The original data set $x[n]$ is divided into two disjoint subsets, even indexed points $x_e[n] = x[2n]$, and odd indexed points $x_0[n] = x[2n+1]$.

**Step 2: Predict**

The wavelet coefficients $d[n]$ are generated as the error in predicting $x_0[n]$ from $x_e[n]$ using the prediction operator $P$:

$$d[n] = x_0[n] - P(x_e[n])$$

**Step 3: Update**

Scaling coefficients $c[n]$ that represent a coarse approximation to the signal $x[n]$ are obtained by combining $x_e[n]$ and $d[n]$. This is accomplished by applying an update operator $U$ to the wavelet coefficients and adding to $x_e[n]$:

$$c[n] = x_e[n] + U$$
These three steps form a lifting stage. Using a DEM as the input, an iteration of the lifting stage on the output $c[n]$ creates the complete set of multiscale DEMs $c_j[n]$ and the elevation loss caused by the change of scale $d_j[n]$.

The DEM in Figure 2 shows the area of Great Basin, Nevada, USA. The area is bounded by latitude $38^\circ 15'$ to $42^\circ$ N and longitude $118^\circ 30'$ to $115^\circ 30'$ W. The DEM was rectified and resampled to 925m in both x and y directions. The DEM is a Global Digital Elevation Model (GTOPO30 DEM) and was downloaded from the USGS GTOPO30 website (http://edcwww.cr.usgs.gov/landdaac/gtopo30/gtopo30.html). GTOPO30 DEMs are available at a global scale, providing a digital representation of the Earth’s surface at a 30 arc-seconds sampling interval. The land data used to derive GTOPO30 DEMs are obtained from digital terrain elevation data (DTED), the 1-degree DEM for USA and the digital chart of the world (DCW). The accuracy of GTOPO30 DEMs varies by location according to the source data. The DTED and the 1-degree dataset have a vertical accuracy of $\pm$ 30m while the absolute accuracy of the DCW vector dataset is $\pm$2000m horizontal error and $\pm$650 vertical error [5].
Multiscale digital elevation models

Figure 2: The GTOPO30 DEM of Great Basin. The elevation values of the terrain (minimum 1005 meters and maximum 3651 meters) are rescaled to the interval of 0 to 255 (the brightest pixel has the highest elevation). The scale is approximately 1:3,900,00.

Multiscale DEMs of the Great Basin region are generated by implementing the lifting scheme on the DEM of Great Basin using scales of 1 to 20. As shown in Figure 3, as the scale increases, the merge of small regions into the surrounding grey level regions increases, causing removal of fine detail in the DEM.
Figure 3: Multiscale DEMs generated using scales of: (a) 1 (b) 3 (c) 5 (d) 10 (e) 15 (f) 20.

The mountains of the generated multiscale DEMs (Figure 4) are extracted using the mathematical morphological based mountains extraction algorithm proposed in Dinesh (2006). The peaks of the DEM are extracted by implementing ultimate erosion on the DEM. Conditional dilation is performed on the extracted peaks obtain the mountain of the DEM. As shown in Figures 4, the merging of small regions into the surrounding grey level regions and removal of fine detail in the DEM cause a reduction in the area of the extracted mountains.
3 Application of Opening by Reconstruction to Compute the Number and Area Frequency Dimensions of Mountains Extracted from Multiscale DEMs.

The number and area frequency dimensions of the extracted mountains are computed using concepts of mathematical morphology [4, 6, 7]. Mathematical morphology is a branch of image processing that deals with the extraction of image components that are useful for representational and descriptive purposes. Morphological operators generally require two inputs; the input image $A$, which can be in binary or grayscale form, and the kernel $B$, which is used to determine the precise effect of the operator.

Dilation sets the pixel values within the kernel to the maximum value of the pixel neighbourhood. The dilation operation is expressed as:

$$A \oplus B = \{a + b : a \in A, b \in B\}$$  \hspace{1cm} (5)

Erosion sets the pixels values within the kernel to the minimum value of the kernel. Erosion is the dual operator of dilation:
An opening (Equation 7) is defined as an erosion followed by a dilation using the same kernel for both operation. Binary opening preserves foreground regions that have a similar shape to this kernel, or that can completely contain the kernel, while eliminating all other regions of foreground pixels.

\[ A \ominus B = (A \ominus B) \oplus B \]  

Morphological reconstruction allows for the isolation of certain features within an image based on the manipulation of a mask image \( X \) and a marker image \( Y \). It is founded on the concept of geodesic transformations, where dilations or erosion of a marker image are performed until stability is achieved (represented by a mask image) [11].

The geodesic dilation, \( \delta^G \) used in the reconstruction process is performed through iteration of elementary geodesic dilations, \( \delta_{(l)} \), until stability is achieved.

\[ \delta^G(Y) = \delta_{(l)}(Y) \circ \delta_{(l)}(Y) \circ \delta_{(l)}(Y) \circ \text{...until stability} \]  

The elementary dilation process is performed using a standard dilation of size one followed by an intersection.

\[ \delta_{(l)}(Y) = Y \oplus B \cap X \]  

The operation in equation 9 is used for elementary dilation in binary reconstruction [11].

Morphological reconstruction is a useful filtering tool. Figure 5(a) shows an image with circles of various sizes. In order to filter the smaller sized circles, first opening is performed using a square kernel of size 30. The circles that are unable to completely contain the kernel are removed, while the shape of remaining circles is altered (Figure 5(b)). Morphological reconstruction is implemented with Figure 5(a) being the
mask and Figure 5(b) being the marker. This restores the original shape of the remaining circles (Figure 5(c)). This process is known as opening by reconstruction.

Opening by reconstruction is implemented on the mountains extracted from the generated multiscale DEMs using square kernels if increasing size. At each iteration of opening by reconstruction, the total number $N_n$ and total area $S_n$ of mountains removed are computed.
Figure 5: The application of morphological reconstruction in filtering. (a) The original image. (b) The opened image. (c) The reconstructed image.

The number-distribution function $C_N(r)$ and the area-distribution function $C_S(r)$ are computed as follows:

$$C_N(r) = \frac{N_n}{(N_0)^2}$$  \hspace{1cm} (10)

$$C_S(r) = \frac{A_n}{(A_0)^2}$$  \hspace{1cm} (11)

where $N_0$ and $A_0$ are number and area, respectively, of mountains prior to iteration of opening by reconstruction.

The number-frequency dimension $v_N$ is the slope of the log-log plot of $C_N(r)$ against $r$ (Figure 6), while the area-frequency dimension $v_S$ is the slope of the log-log plot of $C_S(r)$ against $r$ (Figure 7). The number and area frequency dimensions of the extracted mountains are shown in Table 1.
Figure 5.4: Log-log plots of the number-distribution function $C_N(r)$ against the kernel size $r$ for the corresponding mountains in Figure 4.
Figure 5.5: Log-log plots of the area-distribution function $C_S(r)$ against the kernel size $r$ for the corresponding mountains in Figure 4.

Table 1: The computed number and area frequency dimensions of the mountains extracted from the generated multiscale DEMs.

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<th>Area-frequency dimension</th>
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4 Conclusion

In this paper, concepts of mathematical morphology were employed to compute the number and area frequency dimensions of mountains extracted from multiscale DEMs. The proposed methodology is also applicable to other geomorphological features that form distinct individual objects, such as catchments and water bodies. Our future workplan is to perform the quantification of the spatial heterogeneity of mountains extracted from multiscale DEMs using the number and area frequency dimensions computed in this paper.

References


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