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Chemical Reaction, Heat and Mass Transfer on MHD Flow over a Vertical Isothermal Cone Surface in Micropolar Fluids with Heat Generation/Absorption

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Abstract

An analysis has been carried out to obtain the nonlinear MHD flow with heat and mass transfer characteristics of an incompressible, viscous, electrically conducting and Boussinesq fluid on a vertical isothermal cone surface in micropolar fluids with chemical reaction and heat generation/absorption. A discussion is provided for the effects of the micropolar parameters λ, Δ, chemical reaction parameter γ,heat generation absorption parameter α, Schmidt number Sc, and buoyancy parameter A on the friction factor, heat transfer rate, mass transfer rate and wall couple stress.

1 Introduction

In many engineering applications such as nuclear reactor safety, combustion systems, solar collectors, metallurgy, and chemical engineering there are many transport processes that are governed by the joint action of the buoyancy forces from both thermal and mass diffusion in the presence of chemical reaction effects. Representative applications of interest include: solidification of binary alloy and crystal growth, dispersion of dissolved materials or particulate water in flows, drying and dehydration operations in chemical and food processing plants, and combustion of atomized liquid fuels. Furthermore, the presence of a foreign mass in air or water is causes some kind of chemical reaction. During a chemical reaction between two species heat is also generated. Diffusion and chemical reactions in an isothermal laminar flow along a soluble flat plate were studied by Fairbanks and Wike [1]. The effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction were studied by Das et al.[2]. Andersson et al. [3] studied the flow and mass diffusion of a chemical species with first-order and higher order reactions over a linearly stretching surface. Anjalidevi and Kandasamy [4] studied the steady laminar flow along a semi-infinite horizontal plate in the presence of a species concentration and chemical reaction. Fan et al. [5]studied the mixed convective heat and mass transfer over a horizontal moving plate with a chemical-reaction effect. A similarity solution is obtain by applying transformation group theory. Takhar et al. [6] investigated the flow and mass diffusion of a chemical species with first-order and higher order reactions over a continuously stretching sheet with an applied magnetic field. The study of heat generation or absorption effects in moving fluids is important in view of several physical problems, such as fluids undergoing exothermic or endothermic chemical reactions, Vajravelu and Hadjinicolaou [7] and Vajravelu and Nayfch [8]. In many chemical engineering processes, chemical reactions take place between a foreign mass and the working fluid which moves due to the stretching of a surface. The order of the chemical reactions depends on several factors. One of the simplest chemical reactions is the first-order reaction in which the rate of reaction is directly proportional to the species concentration. Muthucumaraswamy [9] studied the effects of a chemical reaction on a moving isothermal vertical infinitely long surface with suction. Anjali Devi and Kandasamy [10] studied effects of chemical reaction, heat and mass transfer on non-linear MHD laminar boundary layer flow over a wedge with suction and injection. Chamkha [11] presented an analytical solutions for heat and mass transfer by laminar flow of a Newtonian, viscous, electrically, conducting and heat generation absorption. The effects of radiation and chemical reactions, in the presence of a transverse magnetic field, on free convective flow and mass transfer of an optically dense viscous, incompressible, and electrically conducting fluid past a vertical isothermal cone surface are investigated by Afify [12]. Kandasamy et al. [13] studied the nonlinear MHD flow with heat and mass transfer characteristics of an incompressible, viscous, electrically conducting and Boussinesq fluid on a vertical stretching surface with chemical reaction and thermal stratification effects. In the present work we study the effect of nonlinear MHD flow with heat and mass transfer characteristics of an incompressible, viscous, electrically conducting and Boussinesq fluid on with chemical reaction and heat generation. The surface of the cone is maintained at uniform constant temperature $\mathrm{T}_w^{'}$ and uniform concentration $\mathrm{C}_w^{'}$ and higher of that the fluid $\mathrm{T}_\infty^{'}$ and $\mathrm{C}_\infty^{'}$ respectively. Numerical solutions are obtained for different values of Schmidt number Sc, micrpolar parameters Δ, λ chemical reaction parameter γ heat generation absorption parameter α and buoyancy parameter A.

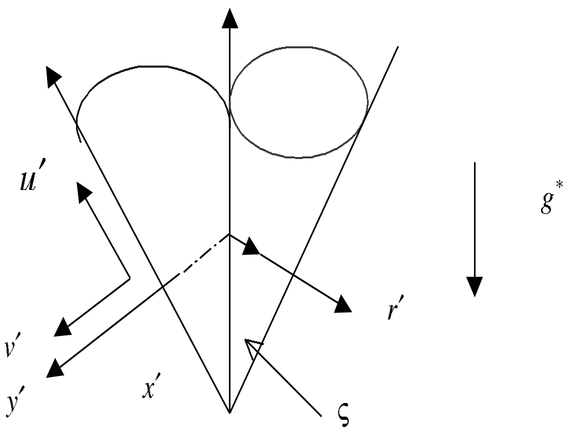

Figure 1: Geometry of the flow

2 Mathematical Analysis

Consider the laminar, steady, free convection and mass flow of an incompressible viscous fluid, over an isothermal vertical cone surface in a micropolar fluid. The fluid properties are not affected by the temperature differences except that the density in the body force term. Also the influence of the density variation in other terms of the momentum, angular momentum, energy and concentration equations and the variation of the expansion coefficients with temperature is negligible. We also consider that the boundary layer is thin relative to the local cone radius so that the local radius to a point inside the layer (see figure 1), can be replaced with the value at the cone surface. This condition is not satisfied in the neighborhood of the cone tip. On the other hand, the pressure gradient across the boundary layer is considered to be negligible, so that the equations governing the problem are strictly applicable to cones of small apex angle.Under the above assumption the laminar, steady, free convective and mass transfer flow, of an incompressible viscous fluid, over an isothermal vertical cone surface, is described by the following equations (1)-(4) and boundary conditions (5).

$$\frac{\partial(\mathrm{r}'\mathrm{u}')}{\partial \mathrm{x}'} + \frac{\partial(\mathrm{r}'\mathrm{v}')}{\partial \mathrm{y}'} = 0 \tag{1}$$

$$\mathrm{u}'\frac{\partial \mathrm{u}'}{\partial \mathrm{x}'} + \mathrm{v}'\frac{\partial \mathrm{u}'}{\partial \mathrm{y}'} = \frac{(K+\mu)}{\rho}\frac{\partial^2 \mathrm{u}'}{\partial \mathrm{y}'^2} + \frac{K}{\rho}\frac{\partial \mathrm{N}'}{\partial \mathrm{y}'} g^*[\beta(\mathrm{T}'-\mathrm{T}'_\infty)+\beta^*(\mathrm{C}'-\mathrm{C}'_\infty)]\,cos\varsigma - \frac{\sigma B_0^2}{\rho}\mathrm{u}', \tag{2}$$

$$\rho(\mathrm{u}'\frac{\partial \mathrm{N}'}{\partial \mathrm{x}'} + \mathrm{v}'\frac{\partial \mathrm{N}'}{\partial \mathrm{y}'}) = \frac{\varepsilon}{j}\frac{\partial^2 \mathrm{N}'}{\partial \mathrm{y}'^2} - \frac{K}{j}(\frac{\partial \mathrm{u}'}{\partial \mathrm{y}'} + 2\mathrm{N}'), \tag{3}$$

$$u'\frac{\partial T'}{\partial x'}+v'\frac{\partial T'}{\partial y'}=\frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2}-Q(T'-T'_\infty),\tag{4}$$

$$u'\frac{\partial C'}{\partial x'}+v'\frac{\partial C'}{\partial y'}=D\frac{\partial^2 C'}{\partial y'^2}-k_1(C'-C'_\infty).\tag{5}$$

With the boundary conditions

$$y'=0:u'=0,v'=0,N'=0,T'=T'_w,C'=C'_w,\tag{6}$$

$$y'\to\infty:u'=0,N'=0,T'=T'_\infty,C'=C'_\infty.\tag{7}$$

Introducing now the dimensionless variables:

$$x=x'/L,y=y'/L,r(x)=r'(x')/L,u=u'L/\nu,v=v'L/\nu,\tag{8}$$

$\theta=(T'-T'_\infty)/(T'_w-T'_\infty),\phi=(C'-C'_\infty)/(C'_w-C'_\infty),Gr_L=g^*\beta\cos\varsigma(T'_w-T'_\infty)/\nu^2$(Grashof number), $Pr=\rho\nu c_p/k$ (Prandtl number), $\Delta=K/\mu$ (Micropolar parameter),$Gc_L=g^*\beta^*\cos\varsigma(C'_w-C'_\infty)/\nu^2$ (modified Grashof number), $Sc=\nu/D$ (Schmidt number), $Mn=\sigma B_0^2/\rho,\gamma=k_1L^2/\nu$, and $\alpha=QL^2/\nu$, are magnetic field parameter,chemical reaction parameter and a heat generation parameter respectively, where L is the cone slant height.

The system of equations (1)-(5) and boundary conditions (6)and (7)become:

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial y}=0,\tag{9}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\Delta\frac{\partial N}{\partial y}+(1+\Delta)\frac{\partial^2 u}{\partial y^2}+Gr_L\theta+Gc_L\phi-Mnu,\tag{10}$$

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2}-\gamma\theta,\tag{11}$$

$$u\frac{\partial\phi}{\partial x}+v\frac{\partial\phi}{\partial y}=\frac{1}{Sc}\frac{\partial^2\phi}{\partial y^2}-\gamma\phi.\tag{12}$$

With the boundary conditions

$$y=0:u=0,v=0,N=0,\theta=1,\phi=1,\tag{13}$$

$$y\to\infty:u=0,N=0,\theta=0,\phi=0.\tag{14}$$

Next the following transformations are introduced to obtain the equations in terms of the generalized stream, temperature, concentration and microrotation functions.

$$\eta=Gr_L^{1/4}yx^{-1/4},\psi(x,y)=Gr_L^{1/4}x^{3/4}f(\eta),N(x,y)=Gr_L^{3/4}x^{1/4}g(\eta).\tag{15}$$

Defining now the velocity component as:.

$$u = \frac{1}{r}\frac{\partial \psi}{\partial y} = Gr_L^{1/2} x^{1/2} f(\eta), v = -\frac{1}{r}\frac{\partial \psi}{\partial x} = Gr_L^{1/4}\frac{x^{-1/4}}{4}(\eta f' - 3f). \quad (16)$$

Where dashes mean differentiation with respect to η, the continuity equation is automatically satisfied and the system of equations (7)-(10) becomes:

$$(1+\Delta)f''' + \frac{3}{4}ff'' - \frac{1}{2}f'^2 + \theta + A\phi + \Delta g' - Mnf' = 0, \quad (17)$$

$$\lambda g'' - \Delta B(f'' + 2g) + \frac{1}{4}(3fg' - f'g) = 0, \quad (18)$$

$$\frac{1}{Pr}\theta'' + \frac{3}{4}f\theta' - \alpha\theta = 0, \quad (19)$$

$$\frac{1}{Sc}\phi'' + \frac{3}{4}f\phi' - \gamma\phi = 0, \quad (20)$$

With the boundary conditions:

$$\eta = 0 : f = 0, f' = 0, g = 0, \theta = 1, \phi = 1, \quad (21)$$

$$\eta \to \infty : f' = 0, g = 0, \theta = 0, \phi = 0. \quad (22)$$

Where $B = Gr_L^{-1/2} x^{1/2}/j, \lambda = \frac{\varepsilon}{\mu L^2 j}$ are dimensionless material parameter and A is the ratio of modified Grashof number Gc_L to the Grashof number Gr_L $(A = Gc_L/Gr_L)$.

Results and Discussions

In order to get the physical insight, the system of ordinary differential equations (17)-(20) along with the boundary conditions (21)-(22) are integrated numerically by means of the fourth-order Runge-Kutta method with shooting technique. The step size $\eta = 0.05$ is used while obtaining the numerical solution with $\eta_{max} = 8.0$. Numerical computation were carried out for Prandtl number Pr, Schmidt number Sc, the micropolar parameters Δ, λ, chemical reaction parameter g, magnetic field parameter Mn, heat generation absorption α and the relative effect of mass and thermal diffusion A, are summarized with $Pr = 0.71, B = 0.1, Sc = 0.6, A = 1.0, \lambda = 0.5, \Delta = 0.0, 0.5, 1.5, 5.0, Mn = 0.0, 2.0$, and $\alpha = -0.1, 0.0, 0.1$.

In the absence of chemical reaction, magnetic field and heat generation / absorption parameters, the results have been compared with that of previous work [14] and it is found that they are in good agreement. Effect due to magnetic field and chemical reaction at the wall of the cone over the velocity, temperature concentration and angular velocity are shown through Figs (2)-(5). Figure (2) and (4) depicts the dimensionless velocity and angular velocity profiles for different values of magnetic field and chemical reaction parameters. It observes that the velocity and angular velocity component of the fluid along the wall of the cone increase with decrease of the strength of the magnetic field, on the contrary, the dimensionless temperature and concentration of the fluid increase with increase of the strength of the magnetic field. On the other hand the dimensionless velocity, angular velocity and temperature of the fluid reduce with an increase of chemical reaction parameter while the dimensionless concentration has the opposite behavior. The influence of heat generation/absorption over velocity, temperature, concentration and angular velocity along the wall of the cone are elucidated with the help of Figures (6)-(9). It is clear that the velocity, temperature, concentration and angular velocity of the fluid reduce with increase of heat generation parameter and increase with the increasing in the micropolar parameter.

3 Concluding Remarks

We conclude the following from the previous results and discussions:

1- In the presence of uniform magnetic field, increases of the strength of the applied magnetic field decelerates the fluid motion along the wall of the cone inside the boundary layer whereas the temperature and concentration of the fluid along the wall of the cone increase as the strength of applied magnetic field increases.

2-The influence of chemical reaction, the fluid flow along the wall of the cone accelerate with increase of chemical reaction parameter, on the other hand, temperature of the fluid increases with increase of chemical reaction parameter but concentration of the fluid reduces with it.

3-Due to heat generation and micropolar parameters, increases of heat generation parameter decelerates the fluid motion, temperature distribution and concentration of the fluid along the wall and micropolar parameter accelerates the fluid motion decelerates the fluid motion, temperature distribution and concentration of the fluid along the wall of the cone.

Nomenclature

A	Buoyancy ratio, $\frac{Gr_c}{Gr_T}$
B	Dimensionless parameter
B_0	Magnetic induction
C	Concentration
D	Mass diffusion coefficient
f	Dimensionless velocity
g	Dimensionless microrotation
g^*	Acceleration due to gravity
Gr_L	Grashof number
Gc_L	Modified Grashof number
j	Microinertia per unit mass
k	Thermal conductivity of fluid
k_1	Reaction rate constant
K	Vertex viscosity
N	Angular velocity
Pr	Prandtl number
Q	Heat generation/absorption coefficient
T	Temperature
u	Velocity component in x -direction
v	Velocity component in y-direction
x	Horizontal co-ordinate
y	Vertical co-ordinate
	Greek symbols
ς	Cone angle
α	Heat generation parameter
β	Thermal expansion coefficient
γ	Chemical reaction parameter
ε	Spin-gradient viscosity
η	Dimensionless co-ordinate
λ	Dimensionless material parameter
μ	Dynamic viscosity
υ	Kinematic viscosity
θ	Dimensionless temperature
ϕ	Dimensionless concentration
ψ	Stream function
ρ	Density of the fluid
σ	Electric conductivity of the fluid
	Subscript
w	Wall condition
∞	Ambient condition
	Superscript
$'$	Differentiation with respect to η

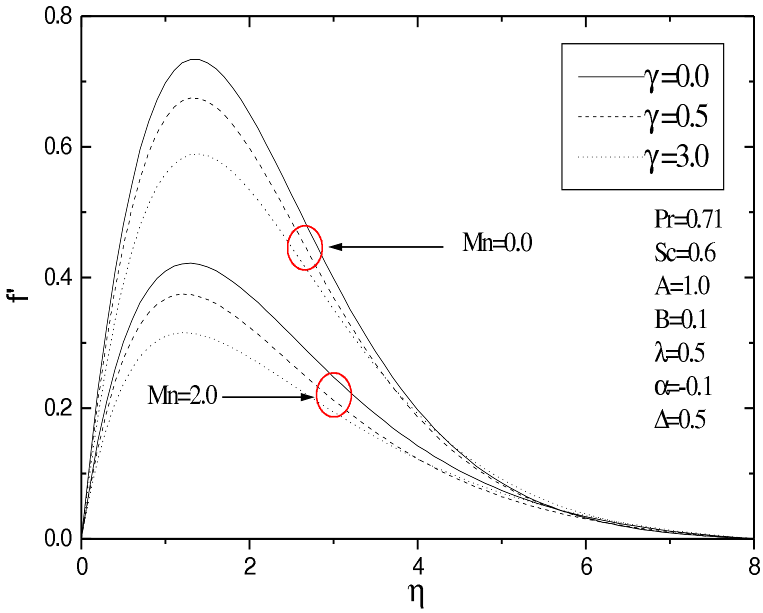

Figure 2: Velocity profiles for different values of chemical reaction γ and magnetic Mn parameters

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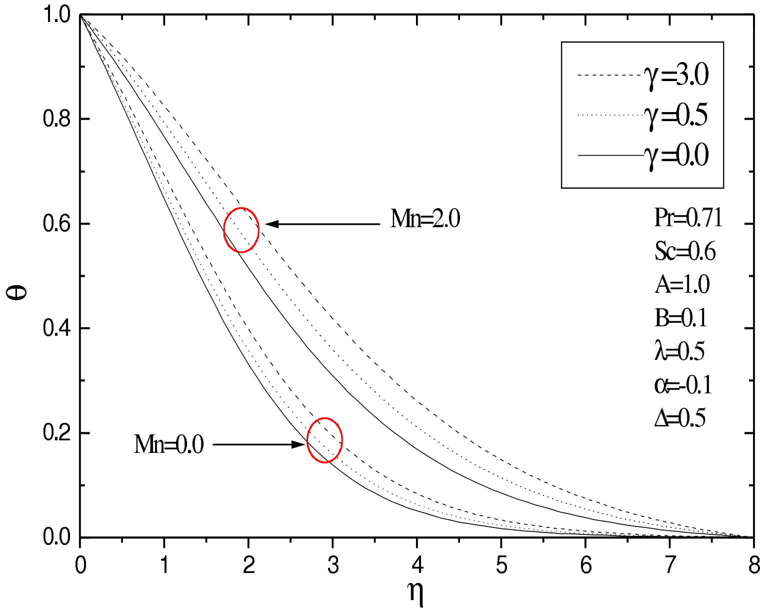

Figure 3: Temperature profiles for different values of chemical reaction γ and magnetic Mn parameters

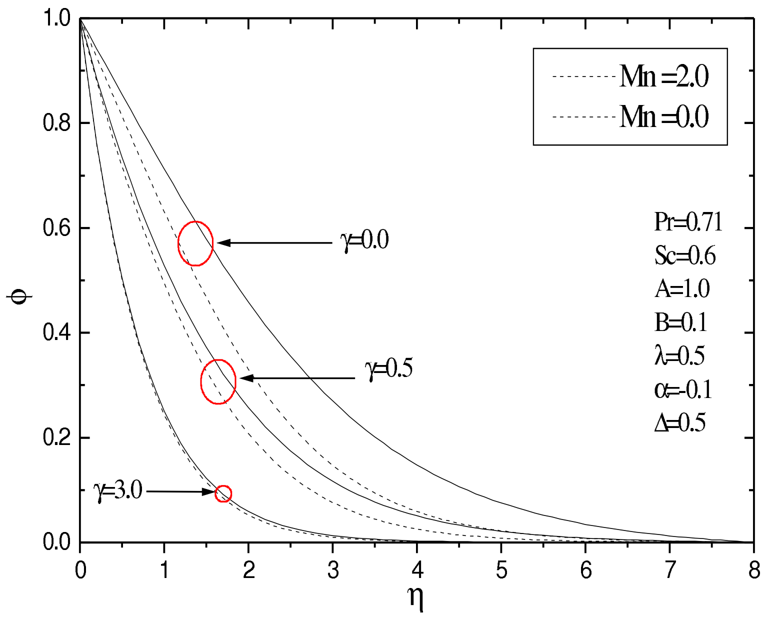

Figure 4: Concentration profiles for different values of chemical reaction γ and magnetic Mn parameters

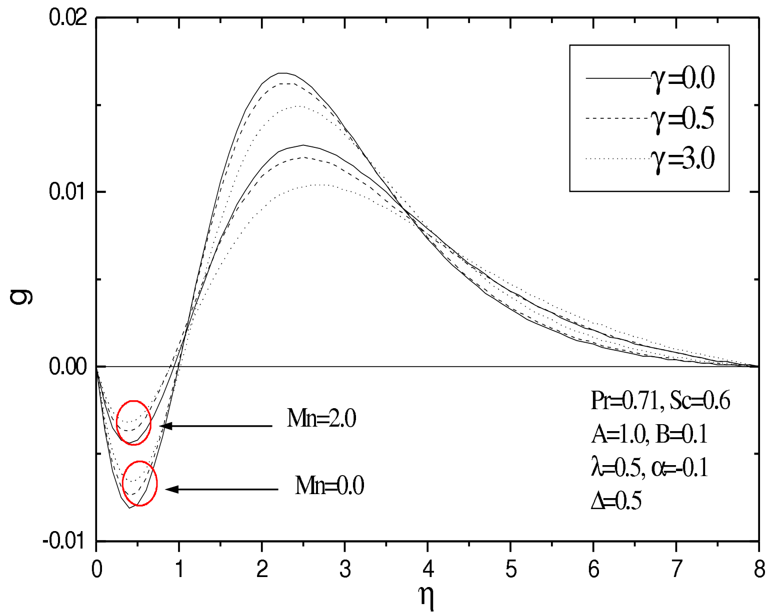

Figure 5: Angular velocity profiles for different values of chemical reaction γ and magnetic Mn parameters

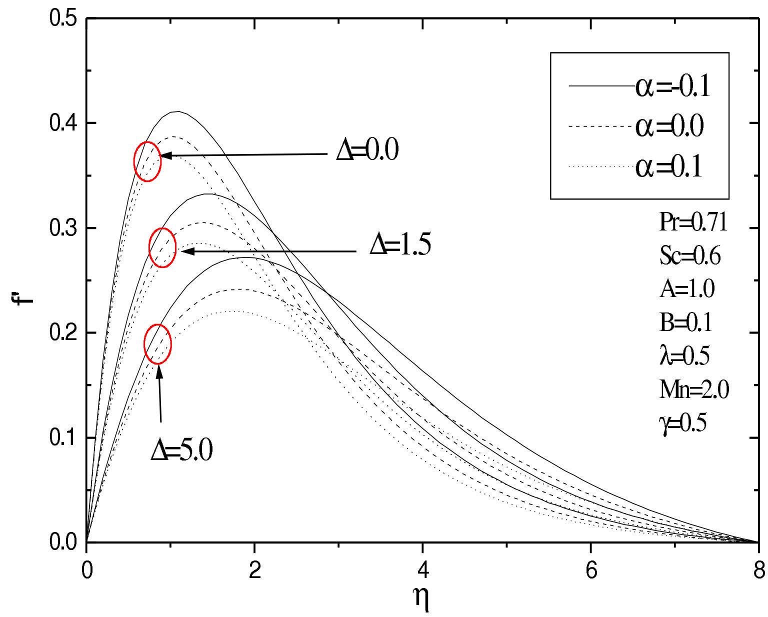

Figure 6: Velocity profiles for different values of heat generation α and micropolar Δ parameters

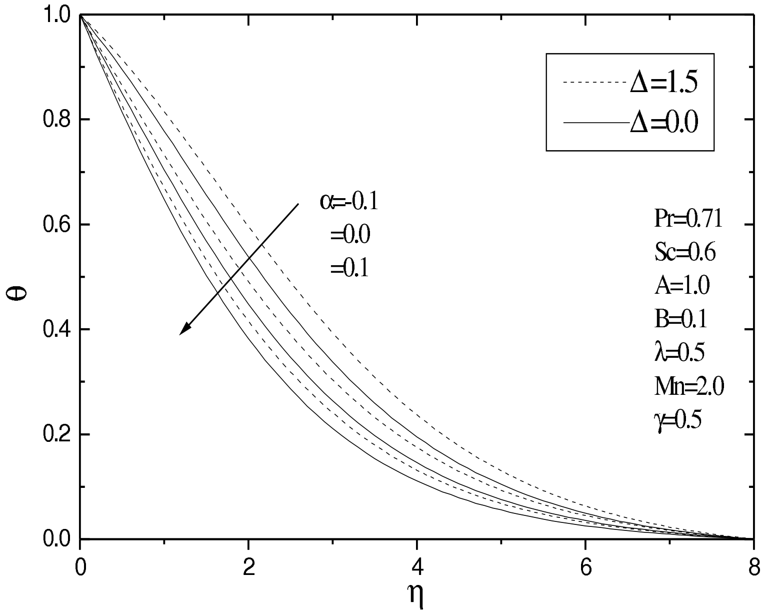

Figure 7: Temperature profiles for different values of heat generation α and micropolar Δ parameters

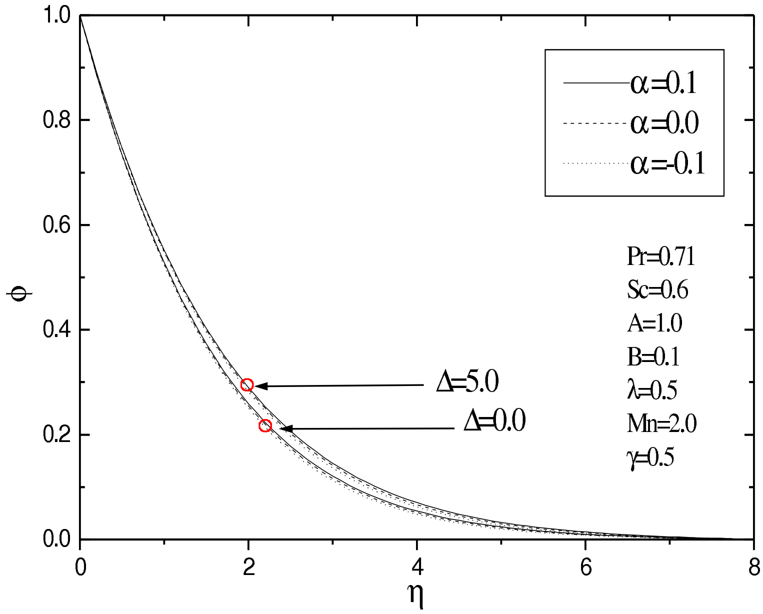

Figure 8: Concentration profiles for different values of heat generation α and micropolar Δ parameters

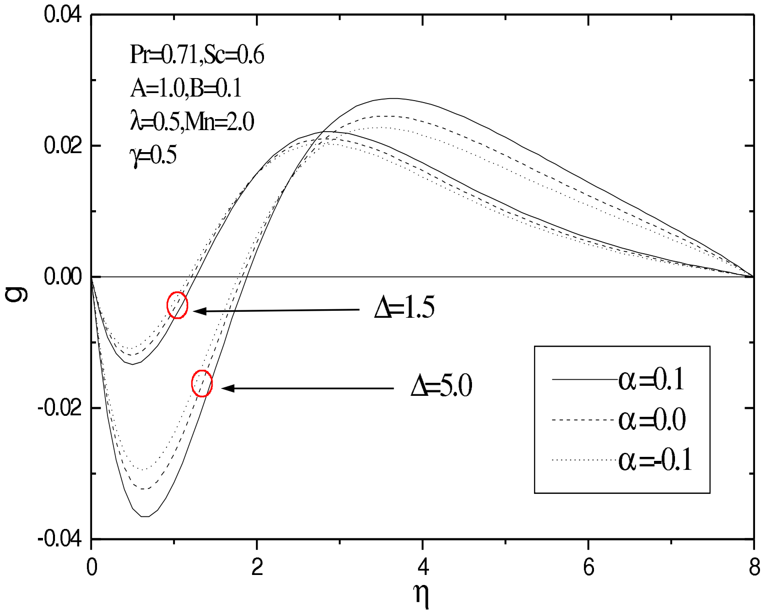

Figure 9: Agular velocity profiles for different values of heat generation α and micropolar Δ parameters

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