Abstract

A current problem in institutional finance is to devise a means of computing the maximum profit that can be made by a depository financial institution (DFI). Such institutions are characterized by their depository and loan issuing activities. Our main objective is to investigate the stochastic dynamics of DFI assets, liabilities and capital under the influence of macroeconomic factors. A feature of our approach is that we can incorporate inherent cyclical effects in credit prices, risk-weightings, provisioning, profitability and capital in the modeling of the aforementioned items. As a consequence, we are able to formulate a maximization problem that involves optimal choices of the depository consumption, value of the DFI’s investment in loans and provisions for
loan losses. In particular, we demonstrate that a DFI is able to maximize its expected utility of discounted depository consumption during a fixed time interval, \([t, T]\), and final profit at time \(T\). Here, the associated Hamilton-Jacobi-Bellman (HJB) equation has a smooth solution when the optimal controls are computed by means of power, logarithmic and exponential utility functions. This enables us to make a direct comparison between the economic properties of the solutions for different choices of utility function. By way of conclusion, we provide an analysis of the economic aspects of the DFI modeling and optimization discussed in the main body of the paper.

**Mathematics Subject Classification:** 60G44, 90A09, 93B15

**Keywords:** Depository Financial Institutions; Stochastic Modeling; Optimization

## 1 INTRODUCTION

A depository financial institution (DFI) is characterized by its main activities of deposit taking, holding and surrendering and loan issuing. Financial intermediaries that qualify as DFIs are commercial banks and, in recent times, many insurance companies and investment banks. A popular approach to the study of DFI dynamics and optimization involves a financial system that is assumed to be imperfectly competitive. As a consequence, profits are ensured by virtue of the fact that the net loan interest margin is greater than the marginal resource cost of outstanding debts and loans. Besides competition policy, the decisions related to capital structure play a significant role in DFI behaviour. Here, the relationship between DFI capital and lending and macroeconomic activity is of crucial importance. By way of addressing these issues, we present a continuous-time DFI model involving balance sheet items such as assets (loans, bonds and reserves), liabilities (outstanding debts) and DFI capital (shareholder equity and subordinate debt). In turn, the aforementioned models enable us to formulate an optimization problem that seeks to establish optimal DFI depository consumption on a finite time interval and terminal profit by choosing the appropriate depository consumption, value of the DFI’s investment in loans and provisions for loan losses. Here profits are only expressed as a function of assets and liabilities.

Another factor influencing the optimization procedure is regulation and supervision. For banks, this regulation takes the form of the Basel II Capital Accord (see [4] and [5]) that is to be implemented on a worldwide basis by 2007. In the case of insurance companies, Solvency 2 constitutes a fundamental review of the capital adequacy regime for insurers that aims to establish a
revised set of capital requirements in the European Union (see [38]). In both cases, the proposed regulation adopts a three pillared approach with the ratio of DFI capital to risk-weighted assets (RWAs), also called the capital adequacy ratio (CAR), playing a major role as an index of the adequacy of capital held by DFIs. The CAR forms the cornerstone of the minimum capital requirement (Pillar 1 for Basel II and Solvency 2) and has the form

\[
\text{Capital Adequacy Ratio} = \frac{\text{Indicator of Absolute Amount of Capital}}{\text{Indicator of Absolute Level of Risk}}.
\]

This ratio provides an indication of whether the absolute amount of DFI capital is adequate when compared to a measure of absolute risk. Our study expresses the CAR as

\[
\text{CAR} (\rho) = \frac{\text{DFI Capital} (C)}{\text{Total RWAs} (A^{\omega})},
\]

where the total RWAs, \(A^{\omega}\), are constituted by risk-weighted loans, \(\lambda\), and bonds, \(B\). In situations where the value of \(\rho\) is smaller than a certain closure or corrective action threshold, \(\rho^c = \rho^l\) or \(\rho^r = \rho^a\), respectively, regulators may pressurize DFIs to increase the value of their CARs. Basel II and Solvency 2 also introduces two other pillars that involve internal assessments of capital adequacy (subject to supervisory review: Pillar 2) and market discipline (through enhanced transparency: Pillar 3) as important components of prudential regulation (see, for instance, [18]). The impact of a risk-sensitive framework such as Basel II and Solvency 2 on macroeconomic stability of DFIs is an important issue. For instance, the question of the procyclical effects of the new capital adequacy regulation is of major interest. In this regard, it is likely that during a recession a decrease in CARs and an increase in regulatory requirements necessitated by the fall in the risk profile of assets may increase the possibility of a credit crunch and result in poor economic growth. Also, since RWAs are sensitive to risk changes, the CAR may increase while the actual levels of DFI capital may decrease. This means that a given CAR can only be sustained if DFIs hold more regulatory capital.

### 1.1 RELATION TO PREVIOUS LITERATURE

In this subsection, we consider the association between our contribution and previous literature. The issues that we highlight include the role of DFI capital, credit models for monetary policy, macroeconomic activity, cyclicality concerns and stochastic modeling and optimization.

The most important role of capital is to mitigate the moral hazard problem that results from asymmetric information between DFIs, depositors and borrowers. The Modigliani-Miller theorem forms the basis for modern thinking
on capital structure (see [29]). In an efficient market, their basic result states that, in the absence of taxes, insolvency costs and asymmetric information, the DFI value is unaffected by how it is financed. In this framework, it does not matter if DFI capital is raised by issuing equity or selling debt or what the dividend policy is. By contrast, in our contribution, in the presence of loan market frictions, the value of the DFI is dependent on its financial structure (see, for instance, [6], [19], [28] and [36] for banking). In this case, it is well-known that the DFI’s decisions about lending and other issues may be driven by the CAR (see, for instance, [16], [17], [30], [35] and [37]). Further evidence of the impact of capital requirements on the lending activities of DFIs are provided by [22] and [40].

A new line of research into credit models for monetary policy has considered the association between DFI capital and loan demand and supply (see, for instance, [1], [8], [11], [13], [39], [41] and [42]). This credit channel is commonly known as the DFI capital channel and propagates that a change in interest rates can affect lending via DFI capital. We also discuss the effect of macroeconomic activity on a DFI’s capital structure and lending activities (see, for instance, [21]). With regard to the latter, for instance, there is considerable evidence to suggest that macroeconomic conditions impact the probability of default and loss given default on loans (see, for instance, [21] and [25]). Throughout our contribution, gross domestic product (GDP) may be considered to be a proxy for macroeconomic activity. In particular, shocks to the macroeconomy may be classified as either a GDP demand shock (for example, a change in purchases by governments or consumer confidence) or a GDP supply shock (for example, a dramatic shift in the oil price).

It is a widely accepted fact that certain financial indicators (for instance, credit prices, asset prices, bond spreads, ratings from credit rating agencies, provisioning, profitability, capital, leverage and risk-weighted capital adequacy ratios, other ratios such as write-off/loan ratios and perceived risk) exhibit cyclical tendencies. In particular, "procyclicality" has become a buzzword in discussions around the new regulatory framework offered by Basel II and Solvency 2. In the sequel, the movement in a financial indicator is said to be "procyclical" if it tends to amplify business cycle fluctuations. A consequence of procyclicality is that banks tend to restrict their lending activity during economic downturns because of their concern about loan quality and the probability of loan defaults. This exacerbates the recession since credit constrained businesses and individuals cut back on their investment activity. On the other hand, banks expand their lending activity during boom periods, thereby contributing to a possible overextension of the economy that may transform an economic expansion into an inflationary spiral. Our interest in cyclical tendencies extends to its relationship with credit prices, risk-weightings, provisioning, profitability and capital (see, for instance, [2], [8], [9], [11], [12] and
As an example, we incorporate in our models, the fact that provisioning behaves procyclically by falling during economic booms and rising during recessions.

Several discussions related to DFI optimal control problems in discrete- and continuous-time settings have recently surfaced in the literature (see, for instance, [10], [21], [28], [31] and [35]). Also, some recent papers using dynamic optimization methods in analyzing bank regulatory capital policies include [32] for Basel II and [3], [15] and [27] for Basel market risk capital requirements. In [35], a discrete-time dynamic DFI model of imperfect competition is presented, where the DFIs can invest in a prudent or a gambling asset. For both these options, a maximization problem that involves the DFI value for shareholders is formulated. On the other hand, [31] examines a problem related to the optimal risk management of DFIs in a continuous-time stochastic dynamic setting. In particular, we minimize market and capital adequacy risk that involves the safety of the assets held and the stability of sources of capital, respectively. In this regard, we suggest an optimal portfolio choice and rate of DFI capital inflow that will keep the loan level as close as possible to an actuarially determined reference process. This set-up leads to a nonlinear stochastic optimal control problem whose solution may be determined by means of the dynamic programming algorithm.

1.2 OUTLINE OF THE PAPER

The main problems to emerge from the above discussion that are solved in this paper can be formulated as follows:

**Problem 1.1 (DFI Dynamic Modelling):** Can we construct mathematical models to describe the dynamics of DFI assets, liabilities and capital in an economically sound manner? (Section 2).

**Problem 1.2 (DFI Maximization Problem):** Which decisions about the depository consumption, value of the investment in loans and provisions for loan losses must be made in order to attain a maximal profit for DFIs? (Theorems 3.2 and 3.7 in Section 3).

Certain ramifications of the solutions to the modeling and maximization problems posed in Problems 1.1 and 1.2 are discussed in Section 4. Finally, Section 5 offers a few concluding remarks and topics for possible future research.
2 STOCHASTIC MODEL FOR DEPOSITORY FINANCIAL INSTITUTIONS

The Basel II capital accord and Solvency 2 (see, for instance, [4] and [5]) permits internal models, that satisfy certain criteria, that may be used by DFIs to determine the riskiness of their portfolios and the required capital cushion. In this spirit, we construct a continuous-time stochastic dynamic model that consists of assets (uses of funds) and liabilities (sources of funds) that are balanced by DFI capital (see, for instance [19]) according to the well-known relation

\[ \text{Total Assets (} \text{A} \text{)} = \text{Total Liabilities (} \text{\Gamma} \text{)} + \text{Total DFI Capital (} \text{C} \text{)}. \]

In our contribution, these items on the DFI balance sheet can specifically be identified as

\[ A_t = \Lambda_t + B_t + R_t; \quad \Gamma_t = \Delta_t; \quad C_t = n_t E_t - O_t, \]

where \( \Lambda, B, R, \Delta, n, E \) and \( O \) are loans, bonds, reserves, outstanding debts, number of shares, DFI equity and subordinate debt, respectively.

2.1 ASSETS

In this subsection, the DFI assets that we discuss are loans, bonds, reserves and risk-weighted assets. In the sequel, we suppose that \((\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbf{P})\) is a filtered probability space.

2.1.1 Loans

We suppose that, after providing liquidity, the DFI grants loans at the interest rate on loans or loan rate, \( r_t^L \). Due to the expenses related to monitoring and screening, we assume that these loans incur a constant marginal cost, \( c^L \). In addition, we introduce the generic variable, \( M_t \), that represents the level of macroeconomic activity in the DFI’s loan market. We suppose that macroeconomic process, \( M = \{M_t\}_{t \geq 0} \), follows the geometric Brownian motion process

\[ dM_t = M_t \left[ \mu_t^M dt + \sigma_t^M dZ_t^M \right], \]

where \( \sigma_t^M \) and \( Z_t^M \) denote volatility in macroeconomic activity and the Brownian motion driving the macroeconomic activity, respectively.

In this paragraph, we provide a brief discussion of loan demand and supply. Taking our lead from the equilibrium arguments in [39], we denote both these
credit price processes by $\Lambda = \{\Lambda_t\}_{t \geq 0}$. In this regard, the DFI faces a Hicksian demand for loans given by

$$\Lambda_t = l_o - l_1 \int_0^t r^\Lambda_s ds + \int_0^t \sigma^\Lambda_s dZ^d_s + l_2 M_t,$$

(2)

where $\sigma^\Lambda_t$ and $Z^d_t$ denote volatility in the loan demand and the Brownian motion driving the demand for loans (which may be correlated with the macroeconomic activity), respectively. We note that the loan demand in (2) is an increasing function of $M$ and a decreasing (increasing) function of $\int_0^t r^\Lambda_s ds > 0$ ($< 0$). Also, we assume that the loan supply process, $\Lambda$, follows the geometric Brownian motion process

$$d\Lambda_t = \Lambda_t \left\{ \left( r^\Lambda(t) - c^\Lambda \right) dt + \sigma_t dZ_t \right\},$$

(3)

where $\sigma_t > 0$ denotes the volatility in the loan supply and $Z_t$ is a standard Brownian motion with respect to a filtration, $(\mathcal{F}_t)_{t \geq 0}$, of the probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, P)$. The value of the DFI loan investment, $\lambda$, at $t$ is expressed as

$$\lambda_t = n_t^\Lambda \Lambda_t,$$

where $n_t^\Lambda$ is the number of loans at $t$.

2.1.2 Provisions for Loan Losses

In line with reality, the DFI is allowed to suffer loan losses for which provision can be made. The accompanying default risk is modeled as a compound Poisson process where $N$ is a Poisson process with a deterministic frequency parameter, $\phi(t)$. Here $N$ is stochastically independent of the Brownian motion, $Z$, given in (3). Furthermore, we introduce the value of loan losses as

$$L(M_t, t) = r^d(M_t) \lambda_t,$$

(4)

where $L$ is independent of $N$. The formula for, $L(M_t, s)$, presented in (4) can be expressed in terms of profit, $\Pi$, as $L(\Pi_t, s)$, by virtue of the evidence from empirical studies that suggest that a strong correlation between $M_t$ and $\Pi_t$ exists (see the discussion on the procyclicality of bank profitability in, for instance, [2] and [9]). Also, we assume that the default or loan loss rate, $r^d \in [0, 1]$, increases when macroeconomic conditions deteriorate according to

$$0 \leq r^d(M_t) \leq 1, \quad \frac{\partial r^d(M_t)}{\partial M_t} < 0.$$
As was the case with the relationship between profit and macroeconomic activity, the above description of the loan loss rate is consistent with empirical evidence that suggests that bank losses on loan portfolios are correlated with the business cycle under any capital adequacy regime (see, for instance, [2], [9] and [12]). Furthermore, we assume that the contribution made by the DFI to the provision for loan losses takes the form of a continuous expense that can be expressed as

\[
\left[1 + \theta(s)\right] \phi(s) \mathbb{E}[P_s(L)],
\]

where \( \theta \) is a loading term dependent on the level of credit risk, \( \theta(t) \geq 0 \) and \( P_t \) is the actual provision for loan losses. This means that if the DFI suffers a loan loss of \( \lambda = l \) at time \( t \), the provisions, \( P_t(l) \), covers these losses. The actual manner in which DFIs make provision for loan losses can differ greatly. However, there is invariably some cost incurred by the DFI in administering the process. In this regard, we denote the costs associated with DFI provisioning for loan losses by \( c^P \) (see, for instance, [2], [12] and [26]).

### 2.1.3 Bonds

Depending on the DFI in question, there are roughly two types of bonds: treasury bonds and savings bonds. All bonds besides savings bonds are very liquid. They are heavily traded on the secondary market. We denote the interest rate on bonds or bond rate by \( r_t^B \) and assume that \( r_t^A - c^A > r_t^B \) for all \( t \).

### 2.1.4 Reserves

*DFI reserves* are the deposits held in accounts with a national agency (for instance, the Federal Reserve for banks) plus money that is physically held by DFIs (vault cash). Such reserves are constituted by money that is not lent out but is earmarked to cater for withdrawals by depositors. Since it is uncommon for depositors to withdraw all of their funds simultaneously, only a portion of total outstanding debts will be needed as reserves. As a result of this description, we may introduce a reserve-deposit ratio, \( \gamma \), for which

\[
R_t = \gamma \Delta_t.
\]

The DFI uses the remaining outstanding debts to earn profit, either by issuing loans or by investing in assets such as bonds and stocks.
2.1.5 Risk-Weighted Assets

We consider risk-weighted assets (RWAs) that are defined by placing each on- and off-balance sheet item into a risk category. The more risky assets are assigned a larger weight. Figure 1 below provides a few illustrative risk categories, their risk-weights and representative items.

<table>
<thead>
<tr>
<th>Risk Category</th>
<th>Risk-Weight</th>
<th>DFI Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 %</td>
<td>Cash, Reserves, Bonds</td>
</tr>
<tr>
<td>2</td>
<td>20 %</td>
<td>Shares</td>
</tr>
<tr>
<td>3</td>
<td>50 %</td>
<td>Home Loans</td>
</tr>
<tr>
<td>4</td>
<td>100 %</td>
<td>Loans to Private Agents</td>
</tr>
</tbody>
</table>

Figure 1: Risk Categories, Risk-Weights and Representative Items

As a result, RWAs are a weighted average of the various assets of the DFIs. In the sequel, we denote the risk-weight on bonds and loans by $\omega^B$ and $\omega^\lambda$, respectively. With regard to the latter, we can identify a special risk-weight on loans $\omega^\lambda = \omega(M_t)$ that is a decreasing function of current macroeconomic conditions, i.e.,

$$\frac{\partial \omega(M_t)}{\partial M_t} < 0.$$

This is in line with the procyclical notion that during booms, when macroeconomic activity increases, the risk-weights will decrease. On the other hand, during recessions, risk-weights may increase because of an elevated probability of default and/or loss given default on loans (see, for instance, [2], [9] and [12]).

2.2 CAPITAL

In this subsection, we discuss total DFI capital, binding capital constraints and retained earnings for a DFI.

2.2.1 Total DFI Capital

The DFI’s total capital, $C$, has the form

$$C_t = C_t^{T1} + C_t^{T2},$$

where $C_t^{T1}$ and $C_t^{T2}$ are Tier 1 and Tier 2 capital, respectively. Tier 1 (T1) capital is the book value of DFI capital defined as the difference between the accounting value of the assets and liabilities. In our contribution, Tier 1 capital
is represented at $t^-$’s market value of the DFI equity, $n_t E_t^-$, where $n_t$ is the number of shares and $E_t$ is the market price of the DFI’s common equity at $t$. Tier 2 (T2) capital consists of preferred stock and subordinate debt. Subordinate debt is subordinate to outstanding debts and hence faces greater default risk. Tier 2 capital, $O_t$, issued at $t^-$ are represented by bonds that pay an interest rate, $r^O$ (see, for instance, [1]).

2.2.2 Binding Capital Constraints

To reflect the book value property of regulatory capital and its market valuation sensitivity, we assume that at $t^-$, the market value of equity and bonds determines the capital constraint to which the DFI is subjected at $t$. While there are several capital constraints associated with Basel II and Solvency 2, it is easy to show that the binding one is the total capital constraint. This constraint requires

$$\rho_t = \frac{C_t}{a_t} \geq 0.08.$$  

For the regulatory ratio of total capital to risk-weighted loans plus bonds, $\rho^r$, a capital constraint may be represented by

$$\rho^r \left[ \omega^\lambda \lambda_t + \omega^B B_t \right] \leq n_t E_t^- + O_t.$$  

(5)

As a result of (5), it is not necessary to differentiate between the relative cost of raising debt versus equity. Moreover, when maximizing profits, we consider the regulatory ratio of total capital to risk-weighted loans, $\rho^r$, as an appropriate capital constraint. This means that we may set $\omega^\lambda = \omega(M_t)$ and $\omega^B = 0$ in (5) and express the binding capital constraint as

$$\rho^r \omega(M_t) \lambda_t \leq n_t E_t^- + O_t.$$  

(6)

The exact value of the regulatory ratio, $\rho^r$, may vary quite considerably from institution to institution (see, for instance, [36] and [37]). In fact, subject to an appropriate choice for $\rho^r$, some DFIs may consider that equality in (6) implies an optimal choice of the investment in loans, $\lambda$, so that

$$\lambda_t^* = \frac{n_t E_t^- + O_t}{\rho^r \omega(M_t)}.$$  

(7)

2.2.3 Retained Earnings

To establish the relationship between DFI profitability and Basel II and/or Solvency 2 a model of DFI financing is introduced that is based on [1]. We
know that *DFI profits*, \( \Pi_t \), are used to meet the DFI’s commitments that include *dividend payments on equity*, \( n_t d_t \), and *interest and principal payments on bonds*, \( (1 + r^O_t)O_t \). The *retained earnings* subsequent to these payments may be computed by using

\[
\Pi_t = E_t^r + n_t d_t + (1 + r^O_t)O_t.
\]

In standard usage, retained earnings refer to earnings that are not paid out in dividends, interest or taxes. They represent wealth accumulating in the DFI and should be capitalized in the value of the DFI’s equity. Retained earnings also are defined to include DFI charter value income. Normally, charter value refers to the present value of anticipated profits from future lending.

DFIs constantly invest in fixed assets (including buildings and equipment) which we denote by \( F_t \). The DFI is assumed to maintain these assets throughout its existence so that the DFI must only cover the costs related to the *depreciation of fixed assets*, \( F_t \). These activities are financed through retaining earnings and the eliciting of additional debt and equity, so that

\[
F_t = E_t^r + (n_t^+ - n_t)E_t + O_t^+.
\]

### 2.3 LIABILITIES

In our study, the only liability that we consider is outstanding debts.

#### 2.3.1 Deposits

The DFI takes deposits, \( \Delta_t \), at a constant *marginal cost*, \( c^\Delta \), that may be associated with cheque clearing and bookkeeping. It is assumed that deposit taking is not interrupted even in times when the *interest rate on deposits* or *deposit rate*, \( r^\Delta_t \), is less than the bond rate, \( r^B_t \). We suppose that the dynamics of the deposit rate process, \( r^\Delta = \{r^\Delta_t\}_{t \geq 0} \), is determined by the geometric Brownian motion process

\[
dt{r^\Delta_t} = r^\Delta_t \left\{ \mu^\Delta dt + \sigma^\Delta_t dZ^r_{t} \right\},
\]

where \( \mu^\Delta \) and \( \sigma^\Delta_t \) are the drift coefficient and volatility in the deposit rate, respectively. In the sequel, we express the sum of the marginal cost and deposit rate components, \( k \), as

\[
k_t = \left[ r^\Delta_t + c^\Delta \right] \Delta_t.
\]

The term \( k \) in (9) represents the consumption of the DFI’s wealth/profit by the holding and taking of deposits and is sometimes referred to as the *depository consumption*. 
We have to consider the possibility that \textit{unanticipated deposit withdrawals}, $u$, will occur. By way of making provision for these withdrawals, the DFI is inclined to hold bonds that are very liquid. In our contribution, we assume that $u$ is related to the probability density function, $f(u)$, that is independent of time. For sake of argument, we suppose that the unanticipated deposit withdrawals have a uniform distribution with support $[\Delta, \Delta]$ so that the \textit{cost of liquidation}, $c_l$, or additional external funding is a quadratic function of bonds. In addition, for any $t$, if we have that

$$u > B_t,$$

then DFI assets are liquidated at some \textit{penalty rate}, $r^p_t$. In this case, the \textit{cost of deposit withdrawals} is

$$c_w(B_t) = r^p_t \int_{B_t}^{\Delta} (u - B_t) f(u) du = \frac{r^p_t}{2\Delta} [\Delta - B_t]^2. \quad (10)$$

\subsection*{2.4 \textbf{PROFIT}}

Suppose that the value of the DFI loan supply, $\lambda$, loan losses, $L(\Pi, t)$, and cost of withdrawals, $c_w(B_t)$, are given by (3), (4) and (10), respectively. The expression for the dynamics of profits that may be deduced from the above is of the form

$$d\Pi_s = \left[ r^B(s) \Pi_s + \left( r^\lambda(s) - c^\lambda - r^B(s) \right) \lambda_s + \mu^a(s) - k_s - [1 + \theta(s)] \phi(s) \bar{E}[P_s(L)] \right] ds$$

$$- c_w(B_s) + \sigma(s) \lambda_s dZ_s$$

$$- \left\{ L(\Pi_s, s) - P_s(L(\Pi_s, s)) \right\} dN_s, \quad s \geq t, \quad \Pi_t = \pi, \quad (11)$$

where $\mu^a(s)$ is the rate term for auxiliary profits that may be generated from activities such as special screening, monitoring, liquidity provision and access to the payment system. Also, this additional profit may be generated from imperfect competition, barriers to entry, exclusive access to cheap deposits or tax benefits. The stochastic model for profit in (11) can be considered to be the natural analogue of the corresponding discrete-time model presented in [1].

\section*{3 \textbf{OPTIMIZATION FOR A DEPOSITORY FINANCIAL INSTITUTION}}

In this section, we make use of the modeling of the preceding discussion to solve a DFI optimization problem.
3.1 STATEMENT OF THE OPTIMIZATION PROBLEM

For sake of argument, we consider a special case of \(11\) with \(c^w(B_t) = 0, r^B(s) = r^B, r^A(s) = r^A\) and \(\sigma(s) = \sigma\). In this regard, we have that

\[
d\Pi_s = \left[ r^B \Pi_s + (r^A - r^B)\lambda_s + \mu^a(s) - k_s - [1 + \theta(s)]\phi(s)E[P_s(L)] \right] ds \\
+ \sigma \lambda_s dZ_s - \left\{ L(\Pi_s, s) - P_s(L(\Pi_s, s)) \right\} dN_s, \quad s \geq t, \quad \Pi_t = \pi. \tag{12}
\]

We identify the control variates as the depository consumption, value of the investment in loans and provisions for loan losses that are all endogenous processes. We assume that the DFI seeks to optimize (over allowable \(\{k_t, \lambda_t, P_t\}\)) its expected utility of the discounted depository consumption during a fixed time interval \([t, T]\) and final profit at time \(T\). The set of admissible controls, \(A\), has the form

\[
A = \left\{ (k_t, \lambda_t, P_t) : \text{measurable w.r.t. } F_t, \ (12) \text{ having a unique solution} \right\}. \tag{13}
\]

The associated objective function of this problem is given by

\[
V(\pi, t) = \sup_{(k_t, \lambda_t, P_t)} E \left[ \int_t^T \exp\{-\delta(s-t)\} U^{(1)}(k_s) ds + \exp\{-\delta(T-t)\} U^{(2)}(\Pi_T) | \Pi_t = \pi \right] \tag{14}
\]

where \(U^{(1)}\) and \(U^{(2)}\) are twice-differentiable, increasing, concave utility functions and \(\delta > 0\) is the rate at which the depository consumption and terminal profit are discounted. We considered several options for the mathematical form of the utility functions. Of course, in principle, one can formulate any utility function. The question then is whether the resulting Hamilton-Jacobi-Bellman (HJB) equation can be solved (smoothly) analytically? In the sequel, we obtain an analytic solution for the choices of power, logarithmic and quadratic utility functions. In this regard, we align our optimization procedure with the methodology suggested in such contributions as [7].

We are now in a position to state the stochastic optimization problem for the DFI depository consumption and terminal profit on a fixed time horizon.

**Problem 3.1 (Optimal Depository Consumption and Profit):** Suppose that \(A \neq \emptyset\), where the admissible class of control laws, \(A\) is given by (13). Also, consider the SDE for the \(\Pi\)-dynamics from (12) and the objective function, \(V : A \to \mathbb{R}_+\), given by (14). In this case, solve

\[
\sup_{k_t, \lambda_t, P_t} V(\Pi; k_t, \lambda_t, P_t),
\]

and the optimal control law \((k^*_t, \lambda^*_t, P^*_t)\), if it exists,

\[
(k^*_t, \lambda^*_t, P^*_t) = \arg \sup_{k_t, \lambda_t, P_t} V(\Pi; k_t, \lambda_t, P_t) \in A.
\]
3.2 SOLUTION TO THE OPTIMIZATION PROBLEM

In this subsection, we determine a solution to Problem 3.1 in the case where the time horizon \([t, T]\) is fixed. In Theorem 3.2 below, \(D_t V(\pi, t)\), \(D_\pi V(\pi, t)\) and \(D_{\pi\pi} V(\pi, t)\) denote first and second order partial derivatives of \(V\) with respect to the variables \(t\) and \(\pi\) where appropriate. For example, \(D_{\pi\pi} V(\pi, t)\) is the second partial derivative of \(V\) with respect to \(\pi\).

**Theorem 3.2 (Optimal Depository Consumption and Profit):** Suppose that the objective function, \(V(\pi, t)\), is described by (14). In this case, a solution to the optimization problem is of the form

\[
\lambda_t^* = -\left(\frac{r^\Lambda_c - r^B}{\sigma^2}\right) \frac{D_\pi V(\Pi_t^*, t)}{D_{\pi\pi} V(\Pi_t^*, t)},
\]

where \(\Pi_t^*\) is the optimally controlled profit. Also, the optimal depository consumption, \(\{k_t^*\}\), solves the equation

\[
D_k U^{(1)} (k_t^*) = D_\pi V(\Pi_t^*, t).
\]

**Proof.** In our proof, via the dynamic programming approach, \(V\) solves the Hamilton-Jacobi-Bellman (HJB) equation

\[
\begin{align*}
\delta V(\pi, t) &= D_t V(\pi, t) + \max_k \left[U^{(1)}(k) - k D_\pi V(\pi, t)\right] + \left(r^B \pi + \mu^o(t)\right) D_\pi V(\pi, t) \\
&\quad + \max_\lambda \left[(r^\Lambda_c - r^B) \lambda D_\pi V(\pi, t) + \frac{1}{2}\sigma^2 \lambda^2 D_{\pi\pi} V(\pi, t)\right] \\
&\quad + \max_P \left[\phi(t) \left\{E D_t V\left(\pi - (\lambda - P(\lambda)), t\right) - D_t V(\pi, t)\right\}\right] \\
&\quad - \left(1 + \theta(t)\right) \phi(t) E[P(\lambda)] D_\pi V(\pi, t) \\
V(\pi, T) &= U^{(2)}(\pi).
\end{align*}
\]

The objective function, \(V\), is increasing and concave with respect to profit, \(\pi\), because the utility functions \(U^{(1)}\) and \(U^{(2)}\) are increasing and concave and because the differential equation for profit is linear with respect to the controls. Thus the optimal investment strategy in (15) holds. □

An alternative method of proof of Theorem 3.2 is by using the martingale approach that is expounded on in [14] and [24]. In order to determine an exact solution for our optimization problem in Theorem 3.2, we are required to make a specific choice for the utilities \(U^{(1)}\) and \(U^{(2)}\). Essentially these utilities can be almost any function involving \(k\) and \(\pi\), respectively. However, in order to obtain smooth analytic solutions to the maximization problem, in the ensuing discussion, we choose power, logarithmic and exponential utility functions and analyze the effect of the different choices.
3.2.1 Optimal Provisioning Process

With regard to the optimal provisioning process, $P^* = \{P^*_t\}_{0 \leq t \leq T}$, we have the following proposition via standard arguments.

**Theorem 3.3 (Optimal Provisioning Process):** The optimal provisioning process, $P^*$, is either no provisioning or per-loss provisioning, with the provisioning costs varying with respect to time. Specifically, at a given time, the optimal provisioning costs $c_{P^*} = \{c_{P^*_t}\}_{0 \leq t \leq T}$ solves

$$
\left(1 - \theta(t)\right) D_{\pi} V(P_{\Pi^*_t}, t) = D_{\pi} V(P_{\Pi^*_t} - c_{P_t}, t).
$$

(16)

No provisioning is optimal at time $t$ if and only if

$$
\left[1 - \theta(t)\right] D_{\pi} V(P_{\Pi^*_t}, t) \geq D_{\pi} V(P_{\Pi^*_t} - \operatorname{ess sup} \lambda(P_{\Pi^*_t}, t), t).
$$

**Corollary 3.4 (Optimal Provisioning Process):** An increase in the instantaneous price of provisioning reduces the instantaneous inclination towards provisioning.

**Proof.** Suppose $\theta(t)$ increases at time $t$ for an infinitesimal length of time, then this change has no effect on the properties of the objective function $V$. Because $D_{\pi} V(\pi, t) < 0$, we have that $c_{P^*_t}$ also increases at time $t$ for an infinitesimal length of time. □

**Corollary 3.5 (Optimal Provisioning Process):** Suppose that an optimal provisioning cost, $c_{P^*_t}$, exists. In this case, if $V$ in (14) exhibits decreasing absolute risk aversion with respect to profit, then the inclination towards provisioning decreases with increasing profit.

**Proof.** Here we follow [33] with the absolute risk aversion of $V$ with respect to $\pi$ being

$$
-\frac{\pi D_{\pi} V(\pi, t)}{D_{\pi} V(\pi, t)}.
$$

□

3.2.2 Optimization with Exponential Utility

Suppose

$$
U^{(1)}(k) = 0 \text{ and } U^{(2)}(\pi) = -\frac{1}{\gamma} \exp \left\{ -\gamma \pi \right\}, \quad \gamma > 0.
$$

(17)

From $U^{(1)}(k) = 0$, it follows that the optimal consumption is identically 0. In this regard, we can verify the following result.
Theorem 3.6 (Optimization with Exponential Utility): Suppose the exponential utilities are given as in (17) and assume that the loan loss, $L$, is independent of profit, although we allow the probability distribution of $L$ to vary deterministically with respect to time. In this case, we have that

$$V(\pi, t) = -\frac{1}{\gamma} \exp \left\{ -\gamma \pi \exp \left[ r(T - t) - \frac{(r^\Lambda - c^\Lambda - r^B)^2}{2\sigma^2} (T - t) \right] \right\} \Omega(t),$$

where $\Omega$ solves

$$\left\{ \begin{array}{l}
D_t \Omega(t) + \Omega(t) \left[ \phi(t) \left\{ M_{L(t) \wedge c^p_t} \left( \gamma \exp \left( r^B (T - t) \right) \right) - 1 \right\} 
+ \phi(t) \left( 1 + \theta(t) \right) E \left( L(t) - c^p_t \right) + \gamma \exp \left( r^B (T - t) \right) 
- \mu^a(t) \gamma \exp \left( r^B (T - t) \right) - \delta \right] = 0,
\end{array} \right.$$ $\Omega(T) = 1.$

and $M_{L(t) \wedge c^p_t}$ is the moment generating function of

$$L(t) \wedge c^p_t = \min \left[ L(t), c^p_t \right].$$

It follows that the optimal provisioning cost is given by

$$c^p_t^\ast = \min \left[ \frac{1}{\gamma} \exp \left\{ -r^B (T - t) \right\} \ln \left( 1 + \theta(t) \right), \text{ess sup} L(t) \right].$$

and the optimal DFI investment in loans is

$$\lambda^*_t = \frac{r^\Lambda - c^\Lambda - r^B}{\sigma^2 \gamma} \exp \left\{ -r^B (T - t) \right\}.$$

3.2.3 Optimization with Power Utility

In this regard, suppose that

$$U^{(1)}(k) = \frac{k^\gamma}{\gamma} \quad \text{and} \quad U^{(2)}(\pi) = b \frac{\pi^\gamma}{\gamma}$$

for some $\gamma < 1$, $\gamma \neq 0$ and $b \geq 0$. In other words, in this context, our choice of utility for the depository consumption and profits is the power utility. The parameter $b$ represents the weight that the DFI assigns to terminal profit versus depository consumption and can be viewed as a measure of the DFI’s propensity towards deposit taking. This leads to the following important result
Theorem 3.7 (Optimization with Power Utility): Suppose the additional profit rate is zero, the power utilities are given as in (18) and that the loan loss is proportional to profit via

\[ L(\Pi_t, t) = \beta(t)\Pi_t, \]

for some deterministic loan loss severity function, \( \beta \), where \( 0 \leq \beta(t) \leq 1 \). In this case, we have that

\[ V(\pi, t) = \pi^{\frac{\gamma}{\gamma}} \xi(t), \]

where \( \xi \) is represented by

\[ \xi(t) = \left[ b^{\frac{1}{1-\gamma}} \exp \left( -\int_t^T H(s) \, ds \right) + \int_t^T \exp \left( -\int_s^T H(u) \, du \right) \, ds \right]^{1-\gamma}, \]

and \( H \) is given by

\[
H(t) = \delta + \phi(t) - \kappa\gamma + (1 + \theta(t))\phi(t)\gamma \max \left( 0, (1 + \theta(t))^{\frac{1}{1-\gamma}} - (1 - \beta(t)) \right) \\
-\phi(t) \max \left( (1 + \theta(t))^{\frac{1}{1-\gamma}}, 1 - \beta(t) \right)
\]

and

\[ \kappa = r^B + \frac{(r^A - c^A - r^B)^2}{2\sigma^2(1-\gamma)}. \]

Thus, the optimal provisioning cost is

\[ c_t^P = \min \left( 1 - (1 + \theta(t))^{\frac{1}{1-\gamma}}, \beta(t) \right) \Pi_t^*, \]

the optimal depository consumption is

\[ k_t^* = D_\pi \Pi^{\frac{1}{1-\gamma}} = \xi(t)^{-\frac{1}{1-\gamma}} \Pi_t^*, \]

and the optimal DFI investment in loans is

\[ \lambda_t^* = \frac{r^A - c^A - r^B}{\sigma^2(1-\gamma)} \Pi_t^*. \tag{19} \]

We note that with the choice of power utility the optimal provisioning cost, depository consumption and investment in loans given by \( c_t^P, k_t^* \) and \( \lambda_t^* \), respectively, can all be expressed as a linear function of the optimal profit, \( \Pi_t^* \). The following corollary to Theorem 3.7 comments on the relationship between formulas for the optimal DFI investment in loans, \( \lambda_t^* \), obtained in (7) and (19).
Corollary 3.8 (Optimization with Power Utility): Suppose that the optimal DFI investment in loans, \( \lambda^* \), given by (7) and (19) are equal. Then a formula for the optimal profit is given by

\[
\Pi_t^* = \frac{\sigma^2(1 - \gamma)(n_tE_t + O_t)}{\rho^\omega(M_t)(r^A - c^A - r^B)}.
\]

3.2.4 Optimization with Logarithmic Utility

Suppose we let \( \gamma \to 0 \) in Theorem 3.7, so that

\[
U^{(1)}(c) = \ln c \quad \text{and} \quad U^{(2)}(\pi) = b \ln \pi.
\]

In this case, it is clear that the following corollary to Theorem 3.7 holds.

Corollary 3.9 (Optimization with Logarithmic Utility): Suppose the logarithmic utilities are given as in (20). In this case, we have that the objective function has the form

\[
V(\pi, t) = \xi(t) \ln \pi + \Omega(t),
\]

where we have

\[
\xi(t) = \left( b - \frac{1}{\delta} \right) \exp\{-\delta(T - t)\} + \frac{1}{\delta}.
\]

The optimal controls have a similar form as in Theorem 3.7 where \( \gamma \neq 0 \).

4 ANALYSIS OF THE MAIN ECONOMIC ISSUES

In accordance with the dictates of Basel II and Solvency 2, the models of DFI items constructed in this paper are related to the methods currently being used to assess the riskiness of DFI portfolios and their minimum capital requirement (see [4] and [5]).

4.1 STOCHASTIC DEPOSITORY FINANCIAL INSTITUTION MODEL

In this subsection, we analyse aspects of the DFI model presented in Section 2.
4.1.1 Assets

Subsubsection 2.1.1 of Subsection 2.1 provides us with a description of the main contributors to a DFI’s lending activities. DFIs respond differently to shocks that affect the value of the loan demand, $\lambda$, when the minimum capital requirements for Basel II and Solvency 2 are calculated by using risk-weighted assets. In the Hicksian case typified by (2), these responses are usually sensitive to macroeconomic conditions that are related to the term $l_2M_t$. We note that, the elasticity of demand, $d^e$, may be expressed as

$$d^e = \frac{l_1r^A_t}{\lambda_t}. $$

It is well-known that if the DFI is perfectly competitive, $d^e$ will tend to $\infty$. This, of course, is not the case in our imperfectly competitive paradigm. Loan defaults are independent of the capital adequacy paradigm that is chosen. In this regard, empirical evidence supports the opinion that better macroeconomic conditions reduce the loan default rate and thus the loan marginal cost (see the discussion on the procyclicality of loan losses in, for instance, [2], [9] and [12]).

4.1.2 Capital

Despite the analysis in Subsection 2.2, DFI capital is notoriously difficult to define, monitor and measure. With regard to the latter, the measurement of equity depends on how all of a DFI’s financial instruments and other assets are valued. In our case, the modeling of the shareholder equity component of DFI capital, $E$, is largely motivated by the following two observations. In the first place, it is meant to reflect the nature of the book value of equity and, secondly, to recognize that the book and market value of equity is highly correlated.

Under Basel II and Solvency 2, DFI capital requirements have replaced reserve requirements (see Subsubsection 2.1.4) as the main constraint on the behaviour of DFIs. A first motivation for this is that DFI capital has a major role to play in overcoming the moral hazard problem arising from asymmetric information between DFIs, creditors and debtors. Also, DFI regulators require capital to be held to protect themselves against the costs of financial distress, agency problems and the reduction of market discipline caused by the safety net.

Subsection 2.2.2 suggests that a close relationship exists between DFI capital holding and macroeconomic activity in the loan market. As was mentioned before, Basel II and Solvency 2 dictate that a macroeconomic shock will affect the loan risk-weights in the CAR. In general, a negative (positive) shock
results in the tightening (loosening) of the capital constraint from (6). As a consequence, in terms of a possible binding capital constraint, DFIs are free to increase (decrease) the loan supply when macroeconomic conditions improve (deteriorate). On the other hand, if the risk-weights are constant, a shock does not affect the loan supply but rather results in a change in the loan rate when the capital constraint binds. It is not always true that Basel II and Solvency 2 risk-sensitive weights lead to an increase (decrease) in DFI capital when macroeconomic activity in the loan market increases (decreases). A simple explanation for this is that macroeconomic conditions do not necessarily only affect loan demand but also influences the total capital constraint from (6). Furthermore, DFIs do not necessarily need to raise new capital to expand their loan supply, since a positive macroeconomic shock may result in a decrease in the RWAs with a commensurate increase in CARs (compare (1)). Similarly, DFIs are not compelled to decrease their capital when the loan demand decreases since the capital constraint usually tightens in response to a negative macroeconomic shock. A further complication is that an improvement in the latter conditions may result in an increase in the loan demand and, as a consequence, an increase in the probability that the capital constraint will be binding. DFIs may react to this situation by increasing capital to maximize profits (compare the definition of the return on equity (ROE) measure of profitability). Our main conclusion is that DFI capital is procyclical because it is dependent on fluctuations in loan demand which, in turn, is reliant on macroeconomic activity.

As far as DFI profit is concerned, an interesting scenario from Subsubsection 2.2.3 to consider, is when \( F_t = 0 \) in (8). This provides another expression for profit of the form

\[
\Pi_t = n_t d_t + (1 + r^O_t)O_t - (n_{t+} - n_t)E_t - O_{t+}.
\]

If, in addition, \((1 + r^O_t)O_t = O_{t+}\), then we may conclude that

\[
\Pi_t = n_t d_t - (n_{t+} - n_t)E_t.
\]

In turn, this results in the inequalities

\[
\Pi_t > n_t d_t \Rightarrow n_{t+}E_t < n_t E_t \text{ and } \Pi_t < n_t d_t \Rightarrow n_{t+}E_t > n_t E_t.
\]

Essentially, under the assumption that \( F_t = 0 \), the first statement implies that if the profit exceeds the dividends at \( t \), then there will be a decline in the \( t^+ \) shareholder equity when compared with equity at \( t \). The opposite is true for the second statement.

4.1.3 Liabilities

In some quarters, the deposit rate \( r^D \), described in Subsection 2.3 is considered to be a strong approximation of DFI monetary policy. Since such policy is
usually affected by macroeconomic activity, $M$, we expect the aforementioned items to share an intimate connection. However, in our analysis, we assume that the shocks $\sigma_{D_t}$ and $\sigma_{M_t}$ to $r^D$ and $M$, respectively, are uncorrelated. Essentially, this means that a precise monetary policy is lacking in our DFI model. This interesting relationship is the subject of further investigation.

4.2 AN OPTIMIZATION PROBLEM FOR DFIs

In this subsection, we discuss some of the issues related to the optimal DFI optimization problem presented in Section 3.

4.2.1 Statement of the Optimization Problem

Problem 3.1 in Subsection 3.1 (see, also, Problem 1.2) addresses issues in DFI operations that is related to the optimal implementation of financial economic principles. The parameter $\delta$ is an idiosyncratic discount rate that is not a market parameter, but rather part of the utility functional. The function $U^{(1)}$ measures the utility of the depository consumption, while $U^{(2)}$ measures the utility of terminal profit. Note also that we assume that $U^{(1)}$ and $U^{(2)}$ are stochastically monotone and therefore additively separable. Such a separability property does not necessarily hold for all DFIs.

4.2.2 Optimal Provisioning Process

The identity in (16) has an economic interpretation. The left-hand-side is the marginal cost of decreasing the cost of provisioning (or increasing provisioning). On the other hand, the right-hand-side is the marginal benefit of increasing provisioning. A nice consequence of this is that optimality in the provisioning process occurs when the marginal cost equals the marginal benefit.

4.2.3 Optimization with Exponential Utility

The results in Subsubsection 3.2.2 suggest that $c^p(t)$ and $\pi^r(t)$ are deterministic and independent of profit. Also, the optimal DFI loan investment strategy is not reliant on profit and, thus, independent of whether the DFI makes provisions for loan losses. Such independence from profit is generally observed in calculations with exponential utility because the absolute risk aversion (see [33]) is constant (equal to $\gamma$). Moreover, note that $c^p(t)$ is not reliant on the parameters of the loan process since we are dealing with exponential utility. Note that as DFI risk aversion increases (measured by $\gamma$), the allocation to the loan decreases. Moreover, as the provisioning cost, $c^p$, decreases, the DFI’s inclination towards provisioning increases. Also, as the load, $\theta(t)$, increases, the inclination towards provisioning decreases because it becomes more expensive.
Thus, Corollary 3.4 holds for permanent increases in \( \theta(t) \) because the profit effect is zero. Note also that as the horizon, \( T \), increases, the provisioning costs and the allocation to loans decrease. As a result, it seems that a DFI with a longer horizon behaves more conservatively.

4.2.4 Optimization with Power Utility

We note from Theorem 3.2 of Subsubsection 3.2.3, that [7] can be used to derive the associated HJB equation. In addition, certain verification theorems claim that if the objective function, \( V \), is smooth and the related HJB equation has a smooth solution, \( \tilde{V} \), then under certain regularity conditions, \( V = \tilde{V} \).

We are able to appeal to the theory of viscosity solutions in those cases for which a smooth solution does not exist.

As was mentioned before, the optimal provisioning cost, depository consumption and investment in loans, obtained in Theorem 3.7 of Subsubsection 3.2.3, can each be represented as a linear function of optimal profit. A possible explanation for this is that the choice of a power utility function naturally leads to constant relative risk aversion (see, for instance, [33]). Symbolically, this can be expressed as

\[
-\frac{\pi D_{\pi\pi}U(\pi)}{D_{\pi}U(\pi)} = 1 - \gamma.
\]

In this regard, we note that an increase in relative risk aversion results in the proportion of profit from loans decreases. Accompanying this is a increase in the demand for provisioning with a commensurate decrease in the cost of provisioning, \( c^P_\pi \). Another observation is that \( \xi \), and hence \( V \) and \( k^* \), are affected explicitly by the horizon, \( T \), and that \( \pi^* \) and \( c^P_\pi \) are affected by \( T \) only through \( k^*/\mu \)'s impact on profit. Note also that \( \xi \), and hence \( k^* \) and \( V \), depend on the frequency and severity parameters for loan losses, \( \phi \) and \( \beta \), respectively. Moreover, we see that the optimal provisioning costs, \( c^P_\pi \), are not reliant on the loan price parameters and the optimal allocation \( \lambda^* \) to loans is independent of loan losses. Next, consider the effect of provisioning on the optimal depository consumption. If we solve Problem 3.1 with \( P_t \) being identically zero, then we find that an objective function, \( \tilde{V} \), is similar to the objective, \( V \), when \( H \) is substituted by

\[
\tilde{H}(t) = \delta + \phi(t) - \mu^\alpha \gamma - \phi(t)(1 - \beta(t)).
\]

When \( \gamma < 0 \), we can use the fact that \( g(x) = x^\gamma \) is decreasing and convex to show that \( H(t) \geq \tilde{H}(t) \), for all \( t \leq T \). From this, it follows that the relative depository consumption (RDC) with provisioning is greater than the RDC without provisioning. Here, we measure RDC with respect to the optimally controlled profit. Similarly, when \( \gamma \in (0, 1) \), we can use the fact that \( g(x) = x^\gamma \)
is increasing and concave to show that $H(t) \leq \hat{H}(t)$, for all $t \leq T$, from which it follows that the RDC with provisioning is less than the RDC without provisioning. Thus, when $\gamma \in (0, 1)$, a DFI with no provisioning spends more on the depository consumption. To explain this counterintuitive behavior, we note that when $\gamma < 0$, the utility approaches $-\infty$ as $\pi \to 0^+$; however, when $\gamma \in (0, 1)$, the utility approaches 0 as $w \to 0^+$. In other words, when $\gamma < 0$, there is an infinite penalty for losing everything. When $\gamma \in (0, 1)$, one can increase utility in the face of a larger potential loan loss by charging more for deposits; the penalty for losing everything is finite. These results appear to be counterintuitive when $0 < \gamma < 1$.

Corollary 3.7 of Subsubsection 3.2.3 has interesting ramifications for procyclicality in DFI profitability. An immediate consequence of the formulation is that as the risk-weight (dependent on macroeconomic activity) increases, DFI profitability decreases. In other words, there will be a commensurate decrease in profits as macroeconomic conditions deteriorate. Another observation is that optimal profits will depend on aspects of on-balance sheet (assets, liabilities and capital) and off-balance sheet (auxiliary profits) activities.

4.2.5 Optimization with Logarithmic Utility

From Subsubsection 3.2.4, we observe that the optimal consumption, $k^*$, does not depend on the frequency and severity parameters, $\phi$ and $\beta$, of the loan loss process. This does not appear to be the case when $\gamma \neq 0$. As in the case of power utility ($\gamma \neq 0$), we see that $\xi$, and hence $V$ and $c^*$, are affected explicitly by the horizon $T$ and that $\pi^*$ and $c^P*$ are affected by $T$ only through $k^*$'s impact on profit. More specifically, if $\delta b > 1$, $\xi$ is a decreasing function of $T$ and, therefore, a DFI with a longer horizon consumes a larger proportion of profit. Similarly, if $\delta b < 1$, a DFI with a longer horizon consumes a smaller proportion of profit. The intuition behind this result is not clear. On the one hand, $b$ measures the DFI's propensity toward saving. On the other hand, $\delta$ measures the DFIs preference for the immediate versus the deferred depository consumption.

5 CONCLUDING REMARKS

In this paper, we analyzed the effect of macroeconomic shocks on models for DFI assets, liabilities and capital. Furthermore, we solved an optimal valuation problem that, amongst other things, maximized profit under constraint. A comprehensive illustration of some of the concepts discussed in the main body of the paper was provided. Finally, the most important outcomes of the paper were discussed from an economic perspective that mainly involved contemporary DFI capital adequacy regulation.
The main thrust of future research will involve models of DFI items driven by Lévy processes (see, for instance, Protter in [34, Chapter I, Section 4]). Such processes have an advantage over the more traditional modeling tools such as Brownian motion in that they describe the non-continuous evolution of the value of economic and financial items more accurately. For instance, because the behaviour of DFI loans, profit, capital and CARs are characterized by jumps, the representation of the dynamics of these items by means of Lévy processes is more realistic. As a result of this, recent research has strived to replace the existing Brownian motion-based DFI models (see, for instance, [17], [20], [28] and [37]) by systems driven by more general processes. Also, a study of the optimal capital structure should ideally involve the consideration of taxes and costs of financial distress, transformation costs, asymmetric DFI information and the regulatory safety net. Another research area that is of ongoing interest is the (credit, market, operational, liquidity) risk minimization of DFI operations within a regulatory framework (see, for instance, [23] and [31]).

References


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