

# An Introduction of a Combined Space-Time Manifold

Gregory L. Light

Providence College  
Providence, Rhode Island 02918, USA  
glight@providence.edu

## Abstract

This paper introduces the term, "combined manifold," which essentially is the graph of a diffeomorphism from one manifold to another. Following a previously published article about "invisible energies" by the author, this present work models the known Universe as a combined space-time 4-manifold by considering two sets of Einstein Field Equations, and establishes a theorem that determines the gravitational motions therein.

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## 1 Introduction

This note introduces the term, "combined manifold," which essentially is the graph of a diffeomorphism from one manifold to another. We consider the possibility of a "combined particle" that is composed of a visible mass and an invisible mass - - invisible because of its neutron-like quality; visible masses interact only with other visible masses, and invisible masses interact only with other invisible masses (see [3]); as such, visible masses cause a distinct set of space-time curvatures, and likewise invisible masses cause yet another set of space-time curvatures (by [1]); accordingly, a combined particle resides in a combined space-time manifold (for a recent related work on 4-manifolds in general, see [5]). Our goal here is to combine the two gravitational motions from the visible manifold and the invisible manifold into one; this is done by a special weighted average of the two metrics of the two manifolds as determined by two sets of Einstein Field Equations. Section 2 below first defines "combined manifold" and "combined particle," then proceeds to prove a theorem that

establishes the gravitational motions of a combined particle, and ends with a corollary and some pertinent remarks. Section 3 concludes with a summary, emphasizing the experimental nature of the model.

## 2 The Dual Universe as a Combined 4-Manifold

**Definition 1** Let  $n \in \mathbb{N}$ ,  $\mathcal{M}^{[1]}$  and  $\mathcal{M}^{[2]}$  be two differentiable  $n$ -manifolds. Assume that there exists a diffeomorphism  $h : \mathcal{M}^{[1]} \rightarrow \mathcal{M}^{[2]}$ ; then the product manifold  $\mathcal{M}^{[3]} := \{(p^{[1]}, p^{[2]}) \in \mathcal{M}^{[1]} \times \mathcal{M}^{[2]} \mid h(p^{[1]}) = p^{[2]}\}$  is a combined  $n$ -manifold.

**Remark 1** By definition, the graph of any diffeomorphism  $h : \mathcal{M}^{[1]} \rightarrow \mathcal{M}^{[2]}$  is a combined manifold; also, if two manifolds,  $\mathcal{M}^{[1]}$  and  $\mathcal{M}^{[2]}$ , are parametrized by  $f^{[1]}$  and  $f^{[2]}$  on the same domain  $U \subset \mathbb{R}^n$ , then

$\mathcal{M}^{[3]} := \{p^{[3]} := (p^{[1]}, p^{[2]}) = (f^{[1]}(u), f^{[2]}(u)) \mid u \in U\}$  is a combined  $n$ -manifold; however, we note that

$$T_{p^{[3]}}\mathcal{M}^{[3]} \subset T_{(p^{[1]}, p^{[2]})}(\mathcal{M}^{[1]} \times \mathcal{M}^{[2]}) \simeq T_{p^{[1]}}\mathcal{M}^{[1]} \oplus T_{p^{[2]}}\mathcal{M}^{[2]} \quad (\text{cf. [2], 93}). \quad (1)$$

**Proposition 1** Let  $g_{p^{[1]}}^{[1]}$  and  $g_{p^{[2]}}^{[2]}$  be any bilinear forms on  $T_{p^{[1]}}\mathcal{M}^{[1]}$  and  $T_{p^{[2]}}\mathcal{M}^{[2]}$  respectively; then  $\forall w_1, w_2 \in [0, \infty)$

$$g_{p^{[3]}}^{[3]} := w_1 \cdot g_{p^{[1]}}^{[1]} + w_2 \cdot g_{p^{[2]}}^{[2]} \quad (2)$$

is a bilinear form on  $T_{p^{[3]}}\mathcal{M}^{[3]}$ .

**Proof.** Clearly the properties of symmetry and bilinearity are preserved in  $g_{p^{[3]}}^{[3]}$ . ■

**Notation 1** In the sequel,  $c \equiv$  the speed of light in the vacuum;

$$\eta := \text{diag}\left(1, -\frac{1}{c^2}, -\frac{1}{c^2}, -\frac{1}{c^2}\right)_{\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3, \mathbf{e}_4\}},$$

where  $\{\mathbf{e}_i^T := (\text{Kronecker } \delta_{i1}, \delta_{i2}, \delta_{i3}, \delta_{i4}) \mid i = 1, 2, 3, 4\}$

is the standard basis in  $\mathbb{R}^4$ ;

$\mathbb{R}^{1+3} := (\mathbb{R}^4, \eta)$ , and  $M$ , in a different font, denotes an inertial mass.

**Definition 2** *A combined particle  $P$  is a point-like object of inertial mass  $\|m^{[3]}\| \equiv \|(m^{[1]}, m^{[2]})\|_{l^1} = \|m^{[1]}\| + \|m^{[2]}\| = m^{[1]} + m^{[2]}$ , where - -*

(1)  $m^{[1]}$  engages in all the fundamental forces of nature, but  $m^{[2]}$  engages in only gravitational forces (see [3]);

(2)  $\forall i = 1, 2, m^{[i]}\mathbf{a}^{[i]} = \mathbf{F}^{[i]}$ , as determined by the Newton's equation of motion, with  $\mathbf{a}^{[i]}$  and  $\mathbf{F}^{[i]}$  being respectively accelerations and forces in  $\mathbb{R}^{1+3,[i]}$ ;

(3)  $\forall i = 1, 2, m^{[i]}\mathbf{a}^{[i]} = -\frac{G^{[i]}(M^{[i]} \cdot m^{[i]})}{r^2}$ ,  $r > 0$ , as determined by the Newton's law of gravitation, with  $M^{[i]}$  being another inertial mass and  $G^{[i]} > 0$  being a gravitational constant in  $\mathbb{R}^{1+3,[i]}$ ;

(4)

$$G^{[3]}M^{[3]}m^{[3]} \quad : \quad = G^{[3]}(M^{[3]} \cdot m^{[3]}) \tag{3}$$

$$= G^{[3]}(M^{[1]}m^{[1]} + M^{[2]}m^{[2]}) \quad (\text{see [3]}), \tag{4}$$

with  $G^{[3]} > 0$  being a gravitational constant,

$$G^{[3]} = G^{[1]} \text{ if } M^{[2]}m^{[2]} = 0, \text{ and} \tag{5}$$

$$G^{[3]} = G^{[2]} \text{ if } M^{[1]}m^{[1]} = 0. \tag{6}$$

**Remark 2** *The above definition decomposes any inertial mass  $m^{[3]}$  into two parts,  $m^{[1]}$  and  $m^{[2]}$ ;  $m^{[i]}$  engages in gravitational forces exclusively in  $\mathbb{R}^{1+3,[i]}$ ,  $i = 1, 2$ , so that  $m^{[3]}$  is located at  $p_{m^{[3]}}^{[3]} \in \mathcal{M}^{[3]}$ , where*

$$\mathcal{M}^{[3]} := \{p^{[3]} := (p^{[1]}, p^{[2]}) \mid p^{[1]} = f^{[1]}(\mathbf{u}) \in \mathcal{M}^{[1]}, \quad p^{[2]} = f^{[2]}(\mathbf{u}) \in \mathcal{M}^{[2]}\}, \tag{7}$$

$f^{[1]} : U \subset \mathbb{R}^{1+3} \longrightarrow \mathcal{M}^{[1]}$  is a local parametrization of  $\mathcal{M}^{[1]}$ , and  $f^{[2]} : U \subset \mathbb{R}^{1+3} \longrightarrow \mathcal{M}^{[2]}$  is a local parametrization of  $\mathcal{M}^{[2]}$ ; i.e.,  $\mathcal{M}^{[3]}$  is a combined space-time 4-manifold.

**Theorem 2** *Assume that  $M^{[1]}m^{[1]}M^{[2]}m^{[2]} \neq 0$  and*

$$G^{[3]} = \frac{G^{[1]}G^{[2]}}{G^{[1]} + G^{[2]}}; \tag{8}$$

*then the Newtonian approximation of gravitational motions in  $\mathcal{M}^{[3]}$  is,  $\forall r > 0$ ,*

$$\begin{aligned} & \|m^{[3]}\| \mathbf{a}^{[3]} \\ &= - \left[ \left( \frac{G^{[2]}}{G^{[1]} + G^{[2]}} \right) \left( \frac{G^{[1]}M^{[1]}m^{[1]}}{r^2} \right) + \left( \frac{G^{[1]}}{G^{[1]} + G^{[2]}} \right) \left( \frac{G^{[2]}M^{[2]}m^{[2]}}{r^2} \right) \right]. \end{aligned} \tag{9}$$

**Proof.**  $\forall i = 1, 2$ , the metric  $g^{[i]}$  for  $\mathcal{M}^{[i]}$  is determined by Einstein Field Equations (see [1])

$$R_{\mu\nu}^{[i]} - \frac{1}{2}R^{[i]} \cdot g_{\mu\nu}^{[i]} = \frac{-8\pi G^{[i]}}{c^2}T_{\mu\nu}^{[i]}, \tag{10}$$

where  $\forall \{\mu, \nu\} \subset \{1, 2, 3, 4\}$ ,  $R_{\mu\nu}^{[i]}$  and  $R^{[i]}$  are the Ricci curvature tensors of types (0, 2) and (0, 0), as dependent on the metric  $g^{[i]}$ ,  $T_{\mu\nu}^{[i]}$  is the given energy-momentum tensor. By Proposition 1, define for  $\mathcal{M}^{[3]}$  the metric

$$g^{[3]} := \frac{G^{[2]}}{G^{[1]} + G^{[2]}} \cdot g^{[1]} + \frac{G^{[1]}}{G^{[1]} + G^{[2]}} \cdot g^{[2]}. \tag{11}$$

Since  $\forall i = 1, 2$ ,

$$g_{11,r}^{[i]} \approx 1 - \frac{2G^{[i]}M^{[i]}}{rc^2} \equiv 1 - \frac{2G^{[i]} \|M^{[i]}\|}{rc^2}, \quad r > 0, \tag{12}$$

(see [1], 817; [4], 288), we thus have

$$\begin{aligned} & g_{11,r}^{[3]} \\ & \approx \left( \frac{G^{[2]}}{G^{[1]} + G^{[2]}} \right) \left( 1 - \frac{2G^{[1]}M^{[1]}}{rc^2} \right) + \left( \frac{G^{[1]}}{G^{[1]} + G^{[2]}} \right) \left( 1 - \frac{2G^{[2]}M^{[2]}}{rc^2} \right) \tag{13} \\ & = 1 - \left( \frac{2}{rc^2} \right) \left( \frac{G^{[1]}G^{[2]}}{G^{[1]} + G^{[2]}} \right) (M^{[1]} + M^{[2]}) \\ & = 1 - \frac{2G^{[3]} \|M^{[3]}\|}{rc^2}; \end{aligned} \tag{14}$$

i.e., equation (12) is also preserved in  $\mathcal{M}^{[3]}$ . Accordingly,  $\mathcal{M}^{[3]}$  has the same form of Newtonian approximation

$$\begin{aligned} & \|m^{[3]}\| \mathbf{a}^{[3]} \\ &= - \frac{G^{[3]} (M^{[3]} \cdot m^{[3]})}{r^2} \quad (r > 0; \text{ cf. equations (3), (4)}) \end{aligned} \tag{15}$$

$$= - \left( \frac{G^{[2]}}{G^{[1]} + G^{[2]}} \right) \left( \frac{G^{[1]}M^{[1]}m^{[1]}}{r^2} \right) - \left( \frac{G^{[1]}}{G^{[1]} + G^{[2]}} \right) \left( \frac{G^{[2]}M^{[2]}m^{[2]}}{r^2} \right) \tag{16}$$

■

**Corollary 3**  $\mathbf{a}^{[3]} = -\frac{G^{[3]}M^{[1]}}{r^2} \left( \frac{m^{[1]}}{m^{[1]}+m^{[2]}} \right) - \frac{G^{[3]}M^{[2]}}{r^2} \left( \frac{m^{[2]}}{m^{[1]}+m^{[2]}} \right)$ ,  $r > 0$ .

**Remark 3** *The above corollary shows that the proportionality of  $\left( \frac{m^{[1]}}{m^{[1]}+m^{[2]}} \right) \equiv \left( \frac{m^{[1]}}{\|m^{[3]}\|} \right) \equiv \kappa$  for any material may be experimentally ascertained by pairing it with another material sharing the same compositions (i.e., ratios among neutrons, protons, and electrons), by solving the following quadratic equation for  $\kappa$ :*

$$\mathbf{a}^{[3]} = -\frac{G^{[3]} \|M^{[3]}\|}{r^2} \left( \frac{M^{[1]} m^{[1]}}{\|M^{[3]}\| \|m^{[3]}\|} + \frac{M^{[2]} m^{[2]}}{\|M^{[3]}\| \|m^{[3]}\|} \right) \tag{17}$$

$$= -\frac{G^{[3]} \|M^{[3]}\|}{r^2} [\kappa^2 + (1 - \kappa)^2], \tag{18}$$

where  $G^{[3]}$  is the recognized gravitational constant,  $\mathbf{a}^{[3]}$ ,  $r$ , and  $M^{[3]}$  are all observables. As for the selection between the two roots for  $\kappa$ , we consider the energy  $E^{[3]}$  of  $M^{[3]}$ :

$$E^{[3]} = M^{[3]}c^2 = M^{[1]}c^2 + M^{[2]}c^2 = E^{[1]} + E^{[2]} \text{ (see [3])}; \tag{19}$$

since the observed quantities  $E^{[1]}$  and  $M^{[3]}$  in experiments did not reveal  $E^{[1]} < M^{[3]}c^2$ , we conclude that  $E^{[1]} \lesssim M^{[3]}c^2$ , i.e.,  $\kappa \lesssim 1$ , which determines the selection of the two roots for  $\kappa$ .

**Remark 4** *Assume that photons carry energies in the form of either  $(E, 0)$  or  $(0, E)$  (see [3]); then in  $\mathcal{M}^{[1]}$  we have  $G^{[1]}$  applying to light (cf. equation (5)). Since experiments of the deflection of light by gravity in  $\mathcal{M}^{[1]}$  have confirmed General Relativity, which went by  $G^{[3]}$ , we conclude that*

$$G^{[3]} = G^{[1]} \left( \frac{G^{[2]}}{G^{[1]} + G^{[2]}} \right) \lesssim G^{[1]}; \tag{20}$$

i.e.,  $\left( \frac{G^{[2]}}{G^{[1]}+G^{[2]}} \right) \equiv w^{[1]} \lesssim 1$ , with

$$G^{[1]} = \frac{G^{[3]}}{w^{[1]}}, \text{ and} \tag{21}$$

$$G^{[2]} = \frac{G^{[3]}}{1 - w^{[1]}}. \tag{22}$$

**Remark 5** *Dark matter*  $(0, M^{[2]})$  manifests itself in  $\mathcal{M}^{[3]}$  by

$$\mathbf{a}^{[3]} = -\frac{G^{[2]}M^{[2]}}{r^2} \left( \frac{m^{[2]}}{m^{[1]} + m^{[2]}} \right). \quad (23)$$

**Remark 6** *Clearly the general gravitational motions in  $\mathcal{M}^{[3]}$  are determined by the geodesics as measured by  $g^{[3]}$ .*

### 3 Summary Remark

This note has decomposed the known Universe  $\mathcal{M}^{[3]}$  into the visible  $\mathcal{M}^{[1]}$  and the invisible  $\mathcal{M}^{[2]}$ , whereby we have introduced the term "combined manifold." Since our construct of  $\mathcal{M}^{[3]}$  lends itself to experimental tests, we maintain that our idea is not an abstract speculation; to wit, the proportionalities of  $\left( \frac{m^{[1]}}{\|\mathcal{M}^{[3]}\|} \right) \equiv \kappa$  and  $\left( \frac{G^{[2]}}{G^{[1]}+G^{[2]}} \right) \equiv w^{[1]}$  are subject to experimental tests. Otherwise, the dual representations of matter as (particle,wave) provide a distinct support of our model.

### 4 References

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