Applied Mathematical Sciences, Vol. 1, 2007, no. 15, 711 - 722

Malmquist Productivity Index on Interval Data in Telecommunication Firms, Application of Data Envelopment Analysis

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Abstract

Performance evaluation of units under supervision in any organization is one of the most important topics of concern in management. Computing relative efficiency of related companies of an organization and presenting suitable remedies for inefficient units, eventuates to unit productivity increase and subsequently leads to the an increase in the overall organization productivity. In this article, efficiency and productivity of components in telecommunication companies and their improvement or deterioration is researched. To achieve this practical goal, use of DEA is made to calculate Malmquist productivity index and by applying Malmquist productivity index, productivity rate of growth for telecommunication companies is calculated.

Mathematics Subject Classification: Operations research, No. 90

Keywords: Data Envelopment Analysis, Malmquist productivity index, efficiency, Interval Data

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1 Introduction

Productivity development is one of the major resources for economic development. In recent years employers and productivity analysts, have had enormous scientific achievements regarding administration issues in factories and firms. These efforts mostly concentrated on information related to productivity and their experience. This issue has resulted to better understanding of efficiency and providing effective information for employers and private and public section designers. Previously, efficiency variations were just a piece of evidence to development or deterioration. But later on, it was observed that technical changes were also effective in productivity. In this regard, M.P.I was presented to deal with this matter. Fare (1992-1994) developed M.P.I which was first introduced by Malmquist (1953).In addition to this Robert M.Thrall(2000) has made use of one of the application of Malmquist criteria in DEA and Rikard Althin(2001) has addressed the application of this criteria in an article titled as "Measurement of productivity changes: two Malmquist index". He patched together Fare opinions based on efficiency measure with Kio opinion (1983) based on productivity measure and defined M.P.I for every unit with consumption inputs and production outputs.
In this project we need a technique, which could facilitate evaluation by combining different indices. In recent years, by showing partial views and considering weights for each of the indexes, a criterion named "total scare" has been developed by which practically with changes in individual opinions, different results have been obtained. M.P.I has been fully discussed in this article. Improvement or deterioration due to the benefit of the outputs is the basis of action.
Here, we first explain the fundamental definition of data envelopment analysis and later on we propose a definition for M.P.I, followed by one of its applications in telecommunication companies and we will study the result of this assessment.

2 Background

For every manager on the administrative level, data concerning the efficiency of the units is one of the most important managerial tools in calculating unit productivity. In any system, this productivity is dependent on the efficiency

and the effectiveness of the units under consideration.
Up to the eighty's, significant amount of research had been done to calculate the efficiency of different systems. By system we mean a set from which the decision making units under consideration were chosen and by the efficiency of a decision making unit we mean that the unit under consideration is making use of the available resources in the best way possible and has actually had an acceptable performance.
Let n be the number of DMU in the system under evaluation and with j=1,...,n , DMU_j would be the jth decision making unit. This DMU would include m input of x_{ij},..., x_{mj} and s output y_{ij},...,y_{sj}. The inputs and outputs of each DMU are nonnegative and at least one input and one output is positive.
Production possibility set T_c , which is obtained on the basis of observation coverage principle, efficiency in constant scale, possibility and convergence, is defined as:

$$T_c = \{(X,Y) \mid X \geq \sum_{j=1}^{n} \lambda_j X_j, Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \lambda_j \geq 0, \quad j = 1,...,n\} \quad \text{(2-1)}$$

This set is called the production possibility set or the CCR and has the following:

- Being piecewise linear, the boundary of T_c, is the boundary of efficiency.
- Every DMU located on this boundary, has a relative efficiency and is otherwise non-efficient.
- If DMU_0 is not located on the boundary, it could be directed to the border using different methods.

The input oriented CCR model is defined as follows:

$$\begin{array}{ll} Min\theta & (2-2) \\ S.t. & \\ \sum_{j=1}^{n} \lambda_j X_j \leq \theta X_0 & \\ \sum_{i=1}^{n} \lambda_j X_j \geq Y & \\ \lambda_j \geq 0 & j = 1,...,n. \end{array}$$

In which $\lambda_1, ..., \lambda_n, \theta$, are the unknown quantities.

3 Malmquist

3.1 Malmquist Productivity Index

Using linear programming techniques and data envelopment analysis, Fare (1957) defined a suitable method for evaluating experimental production function for multi-input and multi-output systems. In data envelopment analysis the best efficiency boundary is obtained by a set of $DMUs$, regardless of the inputs and outputs priorities. $DMUs$ on the efficiency boundary are units with maximum output area or with minimum input area. By identifying efficient units and efficient boundary, non-efficient units are analyzed. M.P.I is defined by patching together efficiency variations of each unit and technical variations, M.P.I could be calculated from the distance function below or from other similar functions.

$$D(X_0, Y_o) = inf\{\theta \mid (\theta X_0, Y_0) \in PPS\}$$

In addition to comparing the efficiency of each unit at present and in the past to calculate the corresponding advancement or regression, bases itself on the society transcendences movement in these two time intervals. Meaning that in the competition, the advancement or regression of the unit under consideration is defined on the advancement or regression of the society transcendences. The variations of the society transcendences are called technology variations. Owing to this in M.P.I, product of the following factors is an indication of advancement or regression.

M.P.I of each unit = unit efficiency variations * unit technology variations.

to calculate the quantity of the aforementioned index, by making use of data envelopment analysis fundamental model, let (X_j^t, Y_j^t) and (X_j^{t+1}, Y_j^{t+1}) be the coordinates of the jth decision making unit at moment t and $t+1$ respectively, four different efficiencies are calculated with respect to the different times.

- DMU_p Efficiency with coordinate of moment t, with respect to transcendent of moment $\theta^t(X_p^t, Y_p^t) = t$.
- DMU_p Efficiency with coordinate of moment t+1 , with respect to transcendent of moment $\theta^t(X_p^{t+1}, Y_p^{t+1}) = t$.

- DMU_p Efficiency with coordinate of moment t, with respect to transcendent of moment $\theta^{t+1}(X_p^t, Y_p^t) = t+1$.
- DMU_p Efficiency with coordinate of moment t+1 , with respect to transcendent of moment $\theta^{t+1}(X_p^{t+1}, Y_p^{t+1}) = t+1$.

Keep in mind that each of the above values is calculated using the optimization models of DEA. Hence we will have:

The variations of $DMU_p = [\frac{\theta^{t+1}(X_p^{t+1},Y_p^{t+1})}{\theta^t(X_p^t,Y_p^t)}]$ (3-1-1)

Technology variations $= [\frac{\theta^t(X_p^t,Y_p^t)}{\theta^{t+1}(X_p^t,Y_p^t)} \frac{\theta^t(X_p^{t+1},Y_p^{t+1})}{\theta^{t+1}(X_p^{t+1},Y_p^{t+1})}]^{1/2}$ (3-1-2)

Hence M.P.I is defined as follows:

$$MPI = \sqrt{\frac{(\theta^t+1(X_p^t+1,Y_p^t+1)).(\theta^t \ (X_p^t+1,Y_p^t+1))}{(\theta^t+1(X_p^t,Y_p^t)).(\theta^t(X_p^t,Y_p^t))}} \quad \text{(3-1-3)}$$

In which the value of M.P.I for the decision making unit is shown. Note that:
a) If $MPI > 1$, then DMU_p will have progress.
b) If $MPI < 1$, then DMU_p will have regression.
c) If $MPI = 1$, then DMU_p will have neither progress nor regression.

3.2 Interval Malmquist productivity index

As we mentioned previously, Malmquist productivity index measure the progress or regression in the period of (t, t+1)- that for calculating it we need to the simple and mixed efficiency that describe as following - and we notice that this model obtained from CCR model that the CCR model can be computed from DEA model. The CCR model with interval data consider as follows:

$$\begin{aligned}
&\theta^p(X_0^q, Y_0^q) = Min\theta \qquad\qquad (3-2-1)\\
&S.t.\\
&\textstyle\sum_{j=1}^n \lambda_j [X_j^{p,l}, X_j^{p,u}] \le \theta[X_0^{q,l}, X_0^{q,u}]\\
&\textstyle\sum_{i=1}^n \lambda_j [Y_j^{p,l}, Y_j^{p,u}] \ge [Y_0^{q,l}, Y_0^{q,u}]\\
&\lambda \ge 0 \ , \ j = 1, ..., n
\end{aligned}$$

For constructing dependent model with Malmquist productivity index, we consider these models as follows:

$$\theta^{p,m}(X_{p_0}^{p}, Y_{p_0}^{p}) = Min\theta \qquad (3-2-2)$$
$$S.t.$$
$$\sum_{j=1, j\neq p_0}^{n} \lambda_j X_j^{p,m} + \lambda_{p_0} X_{p_0}^{p,n} \leq \theta X_{p_0}^{p,n}$$
$$\sum_{j=1, j\neq p_0}^{n} \lambda_j Y_j^{p,n} + \lambda_{p_0} Y_{p_0}^{p,m} \geq Y_{p_0}^{p,n}$$
$$\lambda \geq 0\ ,\ j = 1, ..., n$$

$$\theta^{p,m}(X_{p_0}^{q}, Y_{p_0}^{q}) = Min\theta \quad (3-2-3)$$
$$S.t.$$
$$\sum_{j=1}^{n} \lambda_j X_j^{p,m} \leq \theta X_{p_0}^{p,n}$$
$$\sum_{j=1}^{n} \lambda_j Y_j^{p,n} \geq \theta Y_{p_0}^{p,n}$$
$$\lambda \geq 0\ ,\ j = 1, ..., n$$

For obtaining interval Malmquist productivity index we need to compute values that we can be obtained as following:

1. For obtaining $\theta^{t,l}(X_{p_0}^{t}, Y_{p_0}^{t})$, we need to put these values in the model of(3-2-2):
   p=t , m=l , n=u (3-2-4)

2. $\theta^{t,u}(X_{p_0}^{t}, Y_{p_0}^{t})$ Obtained when we put these values in (3-2-2) model,
   p=t , m=u , n=l (3-2-5)

3. $\theta^{t+1,l}(X_{p_0}^{t+1}, Y_{p_0}^{t+1})$ Obtained when we put these values in (3-2-2) model,
   p=t+1 , m=l , n=u (3-2-6)

4. $\theta^{t+1,u}(X_{p_0}^{t+1}, Y_{p_0}^{t+1})$ Obtained when we put these values in (3-2-2) model,
   p=t+1 , m=u , n=l (3-2-7)

5. $\theta^{t+1,u}(X_{p_0}^{t}, Y_{p_0}^{t})$ Obtained when we put these values in (3-2-3) model,
   p=t+1 , q=t , m=u , n=l (3-2-8)

6. $\theta^{t+1,l}(X_{p_0}^{t}, Y_{p_0}^{t})$ Obtained when we put these values in (3-2-3) model,
   p=t+1 , q=t , m=l , n=u (3-2-9)

7. $\theta^{t+1,l}(X_{p_0}^{t+1}, Y_{p_0}^{t+1})$ Obtained when we put these values in (3-2-3) model,
   p=t , q=t+1 , m=u , n=l , (3-2-10)

8. $\theta^{t,l}(X_{p_0}^{t+1}, Y_{p_0}^{t+1})$ Obtained when we put these values in (3-2-3) model,
   p=t , q=t+1 , m=u , n=l , (3-2-11)

Therefore, we can obtain interval MPI from above models to compute two efficiency indices (simple and mixed) of CCR model. MPI with regular inputs as shown in (3-1-3) model, but if the inputs and outputs of this model are interval, for both of efficiency indices in (3-1-3) model, we obtained interval models, Furthermore, the value if MIp for $DMUp$ will be related to one interval that the lowest and highest value of this interval (MI_pl, MI_pu) computed as below equations:

$$MP_p^l = \sqrt{\frac{(\theta^{t,l}(X_p^{t+1},Y_p^{t+1})).(\theta^{t+1,l}(X_p^{t+1},Y_p^{t+1}))}{(\theta^{t,u}(X_p^{t},Y_p^{t})).(\theta^{t+1,u}(X_p^{t},Y_p^{t}))}} \quad (3-2-12)$$

$$MP_p^u = \sqrt{\frac{(\theta^{t,u}(X_p^{t+1},Y_p^{t+1})).(\theta^{t+1,u}(X_p^{t+1},Y_p^{t+1}))}{(\theta^{t,l}(X_p^{t+1},Y_p^{t+1})).(\theta^{t+1,l}(X_p^{t},Y_p^{t}))}} \quad (3-2-13)$$

3.3 Describing interval Malmquist productivity index

For describing the interval MPI, below complex are consider as following:

$$M^{++} = DMU_j \mid MI_j^l > 1$$
$$M^{-} = DMU_j \mid MI_j^u < 1$$
$$M^{+} = DMU_j \mid MI_j^l \leq 1, MI_j^u \geq 1$$

M^{++} complex include DMUs that under each condition will have progress,
M^{-} complex include DMUs that for some point, will have progress or regression,
M^{+} Complex include DMUs that will have regression under each condition.

Hence,
1. If $MI_p^l = MI_p^u = 1$, then DMUp did not have progress and regression.
2. If $MI_p^u > 1, MI_p^l = 1$, then DMUp did have progress.
3. If $MI_p^u > 1, MI_p^l < 1$, then ρ is applied to determine the progress or regression for DMUp as shown in (3-3-1) equation.

$$\rho = \frac{MI_p^u - 1}{1 - MI_p^l}$$
$$0 < \rho < \infty$$

If $\rho > 1$, then the percent of progress is more than the regression,
And If $\rho < 1$, then the percent of regression is more than the progress.

4 Solving method and an example

In this section, we intend to apply the above proposed model to validate and verify our methodology in one illustration example. In this study, we consider five telecommunication unit of decision maker with respect to their performance in two periods (2003-2004). Also, we gather and analyze the efficient indices that can be influence on their efficiency that includes of age, personal skill, personal degree, costs and incomes in the first and last in two mentioned period, the number of delivered telephone line in this periods, number of the efficient personal for organization, technical equipments of units and increasing its and the number of specialist personal in this units in mentioned years.

	Inputs		*Outputs*
1	*Personalage*	1	*Income*
2	*Personalskill*	2	*Numberofdeliveredtelephoneline*
3	*Personaldegree*		
4	*Cost*		
5	*Numberofnodeliveredline*		
6	*Technicalequipment*		
7	*Specialistpersonal*		

Table 1: inputs and outputs indices

Input tables at T1 and T2,

	*I*1	*I*2	*I*3	*I*4	*I*5	*I*6	*I*7
1	20.643	11.128	4.051	4560300243	11966	1	0.49
2	8.214	7.760	5.674	977558726	27243	5	0.58
3	17.441	11.931	1.967	4300274680	21334	3	0.51
4	1.360	14.481	5.908	5853659315	25475	2	0.49
5	2.763	3.569	6.018	5939583362	27533	4	0.55

Table2: (I1L) Inputs of lower bound in 2003

	$I1$	$I2$	$I3$	$I4$	$I5$	$I6$	$I7$
1	63.902	25.199	4.051	4560300243	11966	1	0.49
2	78.285	29.502	5.675	6977558726	27243	5	0.58
3	64.979	23.772	1.967	4300274680	21334	3	0.51
4	81.222	23.592	5.908	5853659315	25475	2	0.49
5	82.264	36.904	6.018	5939583362	27533	4	0.55

Table3: (I1U) Inputs of upper bound in 2003

	$I1$	$I2$	$I3$	$I4$	$I5$	$I6$	$I7$
1	21.643	10.369	4.051	4560300243	12967	1	0.49
2	9.215	6.042	5.675	8968856066	30310	5	0.58
3	18.441	11.451	1.967	4850677630	21637	3	0.51
4	2.360	14.342	5.908	7317822797	28338	2	0.49
5	3.763	0.402	6.018	7202503310	33656	4	0.55

Table4: (I2L) Inputs of lower bound in 2004

	$I1$	$I2$	$I3$	$I4$	$I5$	$I6$	$I7$
1	64.902	27.958	19.731	4560300243	12967	1	0.49
2	79.285	33.220	18.825	6977558726	30310	5	0.58
3	65.979	26.253	20.305	4300274680	21637	3	0.51
4	82.223	25.730	19.838	5853659315	28338	2	0.49
5	83.264	42.071	18.260	5939583362	33656	4	0.55

Table5: (I2U) Inputs of upper bound in 2004

Output data at T1 and T2

	$O1$	$O2$
	$[Lowerbound, Upperbound]$	$[Lowerbound, Upperbound]$
1	$[9437099230, 9437099230]$	$[12967, 12967]$
2	$[14656051894, 14656051894]$	$[30310, 30310]$
3	$[9981365439, 9981365439]$	$[21637, 21637]$
4	$[11805660774, 11805660774]$	$[28338, 28338]$
5	$[17518577736, 17518577736]$	$[33656, 33656]$

Table6: (O1L) Outputs of lower and upper bound in 2003

	$O1$	$O2$
	$[Lowerbound, Upperbound]$	$[Lowerbound, Upperbound]$
1	$[9437099230, 9437099230]$	$[15307, 15307]$
2	$[17676716657, 17676716657]$	$[34284, 34284]$
3	$[11184972427, 11184972427]$	$[23049, 23049]$
4	$[13768348839, 13768348839]$	$[30198, 30198]$
5	$[21316294134, 21316294134]$	$[36146, 36146]$

Table7: (O2L) Outputs of lower and upper bound in 2004

Finally, these solutions are summarizing in table8 (column1, column2).

With applying models (3-2-12),(3-2-13) the productivity index of five decision maker units are computed. For each unit of decision makers, two indices of MPI under lower and upper bound are obtained. As we shown in table8, the DMU1 in the worst and best condition having 0.76 and 1.37 malmquist productivity index that it shows in the worst and the best condition we consider regression and progress respectively , ρ scale in(3-3-1) model is computed as follows:

$\rho = \frac{1.37-1}{1-o.76} = 1.54$

Also, ρ scale shows that the percent of progress DMU1 is more than the percent of regression.
Hence ρ scale for other DMUs shown in table8 (column3).

ρ	MI_p^u	MI_p^l	
1.54	1.372247	0.75869	$DMU1$
2.96	2.948734	0.340317	$DMU2$
1.2	1.701145	0.421779	$DMU3$
5	5.197737	0.161403	$DMU4$
9.7	8.654369	0.21186	$DMU5$

Table8: result of MPI and ρ scale

Hence, all of decision maker units have been the percent of progress more than the percent of regression and with refer to achieve scale the units which had more in compression with others are in following order.

$\rho_{DMU_5} > \rho_{DMU_4} > \rho_{DMU_2} > \rho_{DMU_1} > \rho_{DMU_3} >$

5 Conclusion

The purpose of this study was to develop the Malmquist productivity index to DMUs with interval data that since the level of inputs and outputs for DMUs are not know exactly. We try by using Malmquist productivity index to compute the progress and regression DMUs in telecommunication firms.

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Received: November 29, 2006