A Mizuno-Todd-Ye Predictor-Corrector
Infeasible-Interior-Point Method with
the $l_1$-Norm Wide Neighborhood for
Linear Programming

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Abstract

A new Mizuno-Todd-Ye predictor-corrector infeasible-interior-point algorithm for linear programming problem is proposed, which is based on one-norm wide neighborhood. We represent the complexity analysis of the algorithm and conclude that its iteration bound is $O(n \log \varepsilon^{-1})$, where $n$ is the larger dimension of a standard linear programming problem and $\varepsilon$ is the required precision. To our knowledge, this is the best complexity result obtain so far for infeasible interior-point methods with wide neighborhood.

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1 Introduction

Since Karmarkar’s ground-breaking paper [1] for linear programming (LP), interior-point methods (IPMs) has attracted the attention of many researchers, and has become the frontier research direction in the optimization field. Many scholars have made outstanding work in the direction of interior point algorithm. The IPMs can be classified in a variety of ways: the primal-dual path-following, affine scaling methods, potential reduction methods and etc. Among all variations of IPMs, the primal-dual IPM is the most efficient [2]. The Mehrotra(M)-type predictor-corrector algorithm [3] and the Mizuno-Todd-Ye(MTY) predictor-corrector algorithm [4] are two typical representative of primal-dual IPMs. Different from the M-type predictor-corrector interior algorithm, the predictor step and the corrector step of the MTY predictor-corrector interior point algorithm are iterated in different neighborhoods, and the predictor step iterates in the larger center path neighborhood, which promotes the iterative point reaches the optimal solution as quickly as possible, and the corrector step reduces the center neighborhood to control the iterative point not to be too far from the center path. Zhang and Zhang [5] proposed the complexity and complexity bounds of two M-type second-order algorithms. Later, Zhang [6] proposed the asymptotic convergence rate for the Mehrotra-type predictor-corrector interior-point algorithms under the condition of preserving Q-quadratic convergence. Since the 1990s, researchers began to pay attention to the MTY predictor-corrector algorithm [7]-[11], because it had the property of the best iteration complexity in IPMs.

All of the above researches require the initial point is strictly feasible for LP, but it is difficult to find these initial points in practice. In that case, Lustig [12] and Tanabe [13] proposed the infeasible-IPMs. Kojima et al.[14] was the first give theoretical results for infeasible-IPM. Mizuno [15] proposed a kind of primal-dual infeasible-IPM for LO. Later, Rangarajan [16] proposed infeasible-IPM for symmetric optimization (SO) and obtained that the complexity bound is $O(r^2 \log \varepsilon^{-1})$ for the Nesterov and Todd(NT) search direction and $O(r^{5/2} \log \varepsilon^{-1})$ for $x_s$ and $s_x$ search direction, where $r$ is the larger dimension of a standard LP.

As we all known, the small neighborhood algorithm work well good in theory, but not in practice. Later, Ai and Zhang [17] introduced a new one-norm neighborhood interior-point algorithm for linear complementarity problem (LCP), which decomposes the classical Newton direction into two orthogonal directions according to the positive and negative parts of the right-hand side of the centering equation. They proved that the algorithm stops after at most $O(\sqrt{n}L)$ iterations. This was the first wide neighborhood algorithm with the same theoretical complexity as a small neighborhood algorithm for LCP. Yang et al.[18] proposed an arc-search infeasible-interior-point algorithm for
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solving LP based on the Ai-Zhang wide neighborhood, and the algorithm uses the ellipses that approximate the entire central path. Based on proposed one-norm neighborhood by Liu et al.[19], it is shown that the interior complexity bound of the algorithm is $O(r^{\frac{5}{4}}\log^{-1})$ for the NT-directions.

Motivated by these, in this paper we present an infeasible-IPM with $l_1$-norm wide neighborhood for LP and established polynomial complexity of the proposed algorithm. Driven by the work of predecessors, we proposes a MTY predictor-corrector interior point algorithm in $l_1$-norm neighborhood of which advantage is that the decrease of the dual gap is proportional to the algorithm complexity. Finally, numerical experiments shows that our proposed algorithm is efficient and reliable.

Throughout the paper, we use the following notations: the symbol $e$ represents the vector of all ones, with dimension given by the context. $\| \cdot \|$ and $\| \cdot \|_1$ are used to represent the Euclidean norm and the $l_1$-norm of a vector, respectively. The $i$-th component of vector $x \in \mathbb{R}^n$ is denoted by $x_i$. We denote component-wise operations on vectors by the usual notations for real numbers. Thus, given two vectors $x, s$ of the same dimension, $xs, xs_1, (xs)_1, (xs)_2$ will denote the vectors with components $x_i s_i, x_i s_i^{-1}, (x_i s_i)^{-1}, (x_i s_i)^{-\frac{1}{2}}$.

The positive and negative parts of a vector $u \in \mathbb{R}^n$ are defined by $u^+ = \max\{u, 0\}, u^- = \min\{u, 0\}$, so that $u^+ \geq 0, u^- \leq 0$, and $u = u^+ + u^-$.  

2 Preliminary Discussions and Algorithm

2.1 Preliminary discussions

In this paper, we considers LP in the standard form of primal and dual problems as follows:

$$(P) \min \{c^T x : Ax = b, x \geq 0\}, \quad (D) \max \{b^T y : A^T y + s = c, s \geq 0\}.$$  

where $A \in \mathbb{R}^{m \times n}$, $c, x, s \in \mathbb{R}^n$, $b, y \in \mathbb{R}^m$.

Moreover, we denote the primal-dual feasibility set and strictly feasibility set of (P) and (D) by

$$F = \{(x, y, s) \in \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^n : Ax = b, A^T y + s = c, (x, s) \geq 0\};$$

$$F^0 = \{(x, y, s) \in \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^n : Ax = b, A^T y + s = c, (x, s) > 0\}.$$  

Under the assumptions that $F^0$ is nonempty and the matrix $A$ has full rank, i.e. rank($A$) = $m$, then $x_*$ and $(y_*, s_*)$ are optimal solution if and only if they satisfy the optimality conditions,

$$Ax = b \quad x \geq 0, \quad A^T y + s = c \quad s \geq 0, \quad xs = 0.$$  


The basic idea of primal-dual IPMs is to represent $x s = 0$ by the parameterized equation $x s = \mu e$ with $\mu > 0$. The system (3) can be written as

$$Ax = b, \quad x \geq 0, \quad A^Ty + s = c \quad s \geq 0, \quad x s = \mu e.$$  

(4)

2.2 Predictor search direction and step size

In the predictor step, we compute the predictor directions by solving the following systems.

$$A\Delta x_a = r_p, \quad A^T\Delta y_a + \Delta s_a = r_d, \quad s\Delta x_a + x\Delta s_a = r_a,$$

(5)

where $r_p = b - Ax, r_d = c - s - A^Ty, r_a = (\tau \mu e - xs)^+ + \sqrt{n}(\tau \mu e - xs)^+$. Let $\alpha_a$ be the step size taken along $(\Delta x_a, \Delta y_a, \Delta s_a).$ Then the next iterate and the duality gap are given by

$$\langle x(\alpha_a), y(\alpha_a), s(\alpha_a) \rangle = \langle x, y, s \rangle + \alpha_a(\Delta x_a, \Delta y_a, \Delta s_a),$$

(6)

$$\langle x(\alpha_a), s(\alpha_a) \rangle = x^T s + \alpha_a e^T(x\Delta s_a + s\Delta x_a) + \alpha_a^2(\Delta x_a)^T \Delta s_a$$

$$= x^T s + \alpha_a e^T r_a + \alpha_a^2(\Delta x_a)^T \Delta s_a,$$

(7)

$$\mu(\alpha_a) = \frac{1}{n} \langle x(\alpha_a), s(\alpha_a) \rangle$$

$$= [1 - (1 - \tau)\alpha_a]\mu + \alpha_a \sqrt{n - 1} e^T (\tau \mu e - xs)^+ + \frac{\alpha_a^2}{n} (\Delta x_a)^T \Delta s_a.$$  

(8)

In this paper, we define a new wide neighborhood as follows:

$$\mathcal{N}_1(\tau, \beta) := \{(x, y, s) \in F^0 : ||(\tau \mu e - xs)^+||_1 \leq \beta \tau \mu \},$$

(9)

where $\beta \leq \frac{1}{2}, \tau \leq \frac{1}{4}$. The new neighborhood is similar to the proposed neighborhood by AI and Zhang [17].

For choice of $\alpha_a$, we are based on several considerations as follows:

A1. The step size $\alpha_a$ guarantees sufficient reduction of the duality gap $\mu(\alpha_a)$. Let

$$\alpha_1 = \arg\min \{ \mu(\alpha_a) : \alpha_a \in [0, 1] \}.  \quad (10)$$

A2. The step size $\alpha_a$ ensures that the reduction of infeasibility is at least as fast as the duality gap. Let

$$\alpha_2 = \max \{ \alpha_a : \langle x(\alpha_a), s(\alpha_a) \rangle \geq (1 - \alpha_a) v(\alpha_a) s, \alpha_a \in [0, 1] \}.  \quad (11)$$

where $v = v^k \in [0, 1]$ is constant (See Step 4 in Algorithm 1)

A3. The step size $\alpha_a$ ensures $((x(\alpha_a), y(\alpha_a), s(\alpha_a)) \in \mathcal{N}_1(\tau, \beta)$. Let

$$\alpha_3 = \max \{ \alpha_a : ((x(\alpha_a), y(\alpha_a), s(\alpha_a)) \in \mathcal{N}_1(\tau, \beta) \}.  \quad (12)$$

In the predictor step, the largest iterate step size is given by

$$\bar{\alpha} = \max \{ \alpha_a : (x(\alpha_a), y(\alpha_a), s(\alpha_a)) \in \mathcal{N}_1(\tau, \beta), \alpha_a \in [0, \min \{ \alpha_1, \alpha_2 \}] \}.  \quad (13)$$
2.3 Corrector search direction and step size

First, we define \((\bar{x}, \bar{y}, \bar{s}) = (x(\bar{\alpha}), y(\bar{\alpha}), s(\bar{\alpha}))\), \(\bar{\mu} = \mu(\bar{\alpha})\). If \((\bar{x}, \bar{y}, \bar{s}) \in N_1(\tau, \frac{\beta}{2})\), we skip immediately to the next iterate. Otherwise, we carry out the corrector step. We compute the corrector direction by solving following system:

\[
A \Delta x_c = 0, \quad A^T \Delta y_c + \Delta s_c = 0, \quad \bar{s} \Delta x_c + \bar{x} \Delta s_c = r_c, \tag{14}
\]

where \(r_c = (\tau \bar{\mu} e - \bar{x} \bar{s})^- + \rho (\tau \bar{\mu} e - \bar{x} \bar{s})^+\), \(\rho = \frac{\| (\tau \bar{\mu} e - \bar{x} \bar{s})^- \|_1}{\| (\tau \bar{\mu} e - \bar{x} \bar{s})^+ \|_1}\), \(\rho\) is a finite constant. Moreover, the next iterate is given by

\[
(x(\alpha_c), y(\alpha_c), s(\alpha_c)) = (\bar{x}, \bar{y}, \bar{s}) + \alpha_c(\Delta x_c, \Delta y_c, \Delta s_c), \tag{15}
\]

where \(\alpha_c\) is the step size taken along \((\Delta x_c, \Delta y_c, \Delta s_c)\). For the duality gap, we have

\[
\mu(\alpha_c) = \frac{1}{n} \langle x(\alpha_c), s(\alpha_c) \rangle = \frac{1}{n} \langle \bar{x} + \alpha_c \Delta x_c, \bar{s} + \alpha_c \Delta s_c \rangle
\]

\[
= \frac{1}{n} \left[ \bar{x}^T \bar{s} + \alpha_c (\bar{x} \Delta s_c + \bar{s} \Delta x_c) + \alpha_c^2 (\Delta x_c)^T \Delta s_c \right]
\]

\[
= \frac{1}{n} \bar{x}^T \bar{s} + \frac{1}{n} \alpha_c \left[ -\frac{\| (\tau \bar{\mu} e - \bar{x} \bar{s})^- \|_1}{1} + \rho \frac{\| (\tau \bar{\mu} e - \bar{x} \bar{s})^+ \|_1}{1} \right] = \bar{\mu}. \tag{16}
\]

For choice of \(\alpha_c\), we only require it to satisfy \((x(\alpha_c), y(\alpha_c), s(\alpha_c)) \in N_1(\tau, \frac{\beta}{2})\).

2.4 Algorithmic framework

In what follows we describe the MTY predictor-corrector infeasible-IPM

\[\text{Algorithm 1:}\] Let \(\epsilon > 0, \tau \leq \frac{1}{4}, \beta \leq \frac{1}{2}\), and \((x_0, y_0, s_0) \in N_1(\tau, \frac{\beta}{2})\). Calculate \(\mu_0 = \langle x_0, s_0 \rangle / n, k := 0\).

\begin{itemize}
  \item Step 1: If \(\mu_k \leq \epsilon \mu_0\), then stop.
  \item Step 2: \text{(Predictor step)} Solve the predictor direction \((\Delta x_a, \Delta y_a, \Delta s_a)\) by (4),
  \begin{itemize}
    \item Compute the step size \(\alpha_k\) by (12).
    \item Set \((x_{k+1}, y_{k+1}, s_{k+1}) = (x(\alpha_k), y(\alpha_k), s(\alpha_k))\),
    \item If \((x_{k+1}, y_{k+1}, s_{k+1}) \in N_1(\tau, \frac{\beta}{2})\), go to Step 4.
    \item If \((x_{k+1}, y_{k+1}, s_{k+1}) \in N_1(\tau, \beta) \setminus N_1(\tau, \frac{\beta}{2})\), go to Step 3.
  \end{itemize}
  \item Step 3: \text{(Corrector step)} Solve the corrector direction \((\Delta x_c, \Delta y_c, \Delta s_c)\) by (13),
  \begin{itemize}
    \item Compute the largest step size \(\alpha_c\) such that \((x(\alpha_c), y(\alpha_c), s(\alpha_c)) \in N_1(\tau, \frac{\beta}{2})\),
    \item Set \((x_{k+1}, y_{k+1}, s_{k+1}) = (x(\alpha_c), y(\alpha_c), s(\alpha_c))\), go to Step 4.
  \end{itemize}
  \item Step 4: Compute \(\mu_{k+1} = \langle x_{k+1}, s_{k+1} \rangle / n\), and \(v_{k+1} = (1 - \alpha_k) v_k\).
  \item Set \(k := k + 1\), and go to Step 1.
\end{itemize}
For Algorithm 1, the following proposition is readily verified.

**Remark 1** Let \((x_k, y_k, s_k)\) be generated by Algorithm 1. Then for \(k \geq 0\), \(Ax_{k+1} - b = v_{k+1}(Ax_0 - b)\), and \(AT_{y_{k+1}} + s_{k+1} - c = v_{k+1}(AT_{y_0} + s_0 - c)\), where \(v_0 = 1\) and \(v_{k+1} = (1 - \bar{\alpha}_{k+1})v_k = \prod_{i=0}^{k} (1 - \bar{\alpha}_{i+1}) \in (0, 1]\).

Form Remark 1, we have

\[
v_k = \frac{\|Ax_k - b\|}{\|Ax_0 - b\|} = \frac{\|AT_{y_k} + s_k - c\|}{\|AT_{y_0} + s_0 - c\|},
\]

which implies \(v_k\) is represent of the relative infeasibility at \((x_k, y_k, s_k)\), By (10), one has \(\langle x_{k+1}, s_{k+1} \rangle \geq (1 - \bar{\alpha}_{k+1})v_k \langle x_0, s_0 \rangle = v_{k+1} \langle x_0, s_0 \rangle\), which ensures that the infeasibility approaches to zero as the complementarity \(\langle x, s \rangle\) approaches to zero.

Let \(\chi^* = \|x^*\| + \|s^*\|\), where \(x^*\) and \(y^*, s^*\) are optimal solution. We choose \((x_0, y_0, s_0)\) such that

\[
x_0 = s_0 = \chi^* e. \tag{17}
\]

holds, which implies that \(x_0 > 0, s_0 > 0\) and \(x_0 - x^* > 0, s_0 - s^* > 0\). Moreover, we directly compute and obtain \(\chi^* = \frac{n\|x^*\| + \|s^*\|}{n} \geq \frac{e^T x^* + e^T s^*}{n}\).

### 3 Analysis of the Algorithm

In this section, we first recall some useful results that will be used frequently in the analysis. Then, in order to prove the convergence of the proposed algorithm, some technical lemmas are provided.

#### 3.1 Technical lemmas

We first give the definition of optimal solution and introduce the partition of interval.

**Lemma 3.1.** [17] For any \(u, v \in \mathbb{R}^n\) and \(q \geq 1\), we have

\[
\|u + v\|^q_1 \leq \|u\|^q_1 + \|v\|^q_1, \quad \|(u + v)^+\|^q_1 \leq \|u^+\|^q_1 + \|v^+\|^q_1, \quad \|(u + v)^-\|^q_1 \leq \|u^-\|^q_1 + \|v^-\|^q_1.
\]

**Lemma 3.2.** [5] Let \(u, v \in \mathbb{R}^n\) and \(p = (x^{-1}s)^{\frac{1}{2}}\), then

\[
\|uv\|_1 \leq \|pu\| \|p^{-1}v\| \leq \frac{1}{2} (\|pu\|^2 + \|p^{-1}v\|^2).
\]

**Lemma 3.3.** [17] Let \(u, v \in \mathbb{R}^n\), \(u^Tv \geq 0\), and \(z = u + v\), then

\[
\|(uv)^-\|_1 \leq \|(uv)^+\|_1 \leq \frac{1}{4} \|z\|^2.
\]
Lemma 3.4. Let \( u \in \mathbb{R}_{++}^n, v \in \mathbb{R}_+^n \), then

\[
\begin{align*}
(1) \quad & \|u^{-\frac{1}{2}} v\|^2 \leq \frac{\|v\|^2}{u_{\min}}; \quad (2) \quad \|u^{-\frac{1}{2}} v\|_1 \leq \frac{\|v\|_1}{\sqrt{u_{\min}}},
\end{align*}
\]

Proof. Using the definition of norm and scaling, we have

\[
\begin{align*}
(1) \quad & \|u^{-\frac{1}{2}} v\|^2 = \sum_{i=1}^n (u_i^{-\frac{1}{2}} v_i)^2 = \sum_{i=1}^n \frac{v_i^2}{u_i} \leq \sum_{i=1}^n \frac{v_i^2}{u_{\min}} = \frac{\|v\|^2}{u_{\min}}, \\
(2) \quad & \|u^{-\frac{1}{2}} v\|_1 = \sum_{i=1}^n |u_i^{-\frac{1}{2}} v_i| = \sum_{i=1}^n \frac{|v_i|}{\sqrt{u_i}} \leq \sum_{i=1}^n \frac{|v_i|}{\sqrt{u_{\min}}} = \frac{\|v\|_1}{\sqrt{u_{\min}}}.
\end{align*}
\]

Lemma 3.5. Let \( (x, y, s) \in \mathcal{N}_1(\tau, \beta) \), we have \( (1-\tau)\mu n \leq \|\mu e - x s\|_1 \leq (1-\tau)\mu n + \sqrt{n}\beta \tau \mu \).

Proof. From the reference [17], we have

\[
\begin{align*}
e^T (\mu e - x s)^- = -(1-\tau)\mu n - e^T (\mu e - x s)^+ \leq -(1-\tau)\mu n,
\end{align*}
\]

\[
\begin{align*}
e^T (\mu e - x s)^+ \leq \sqrt{n} \| (\mu e - x s)^+ \| \leq \sqrt{n}\beta \tau \mu.
\end{align*}
\]

Combine the above two inequality, we have

\[
(1-\tau)\mu n \leq \|\mu e - x s\|_1 \leq (1-\tau)\mu n + \sqrt{n}\beta \tau \mu.
\]

Lemma 3.6. [17] Let \( (x, y, s) \in \mathcal{N}_1(\tau, \frac{\beta}{2}) \), we have,

\[
e^T (\mu e - x s)^+ \leq \frac{1}{2} \beta \tau \mu.
\]

Proof. From the defined of the neighborhood of \( \mathcal{N}_1(\tau, \frac{\beta}{2}) \), we have

\[
e^T (\mu e - x s)^+ = \| (\mu e - x s)^+ \|_1 \leq \frac{1}{2} \beta \tau \mu.
\]

3.2 Predictor step

Firstly, given the following notation:

\[
\begin{align*}
& \| (\mu (\alpha_a) e - x (\alpha_a) s (\alpha_a))^+ \|_1 = \| (\mu (\alpha_a) e - x s - \alpha_a r_a - \alpha_a^2 \Delta x^a \Delta s^a)^+ \|_1 \\
& \leq \| (\mu (\alpha_a) e - x s - \alpha_a r_a)^+ \|_1 + \alpha_a^2 \| (\Delta x^a \Delta s^a)^- \|_1,
\end{align*}
\]

where the last inequality follows from Lemma 3.1. Then, we have the next lemma.
Lemma 3.7. Let $\alpha_a$ such that $0 < \mu(\alpha_a) \leq \mu$, and $(x, y, s) \in \mathcal{N}_1(\tau, \frac{\beta}{2})$, 
\[ \text{If } \alpha_a \geq \frac{1}{\sqrt{n}}, \text{ then } \| (\tau \mu(\alpha_a)e - xs - \alpha_ar_a)^+ \|_1 = 0; \]
\[ \text{If } \alpha_a < \frac{1}{\sqrt{n}}, \text{ then } \| (\tau \mu(\alpha_a)e - xs - \alpha_ar_a)^+ \|_1 \leq \frac{1}{2}(1 - \alpha_a\sqrt{n})\beta \tau \mu(\alpha_a). \]

Proof. By the fact that $0 < \mu(\alpha_a) \leq \mu$, we have,
\[
\begin{align*}
&\| (\tau \mu(\alpha_a)e - xs - \alpha_ar_a)^+ \|_1 \\
&= \mu(\alpha_a) \| (\tau \mu e - \mu(\alpha_a)(xs + \alpha_ar_a))^+ \|_1 \\
&\leq \mu(\alpha_a) \| ((1 - \alpha_a)(\tau \mu e - xs)^- + (1 - \alpha_a\sqrt{n})(\tau \mu e - xs)^+)^+ \|_1 \\
&\leq \mu(\alpha_a) \| ((1 - \alpha_a)(\tau \mu e - xs)^-)^+ \|_1 + \mu(\alpha_a) \| (1 - \alpha_a\sqrt{n})(\tau \mu e - xs)^+ \|_1 \\
&= \mu(\alpha_a) \frac{1}{\mu} \| ((1 - \alpha_a\sqrt{n})(\tau \mu e - xs)^+)^+ \|_1.
\end{align*}
\]
where the first inequality follows from $\mu(\alpha_a) < \mu$, the second inequality follows from Lemma 3.1, the last equation follows from $\| ((1 - \alpha_a)(\tau \mu e - xs)^-)^+ \|_1 = 0$.

**case 1**, $\alpha_a \geq \frac{1}{\sqrt{n}}$, since $1 - \alpha_a\sqrt{n} \leq 0$, we have $\| (\tau \mu(\alpha_a)e - xs - \alpha_ar_a)^+ \|_1 = 0$;

**case 2**, $\alpha_a < \frac{1}{\sqrt{n}}$, since $1 - \alpha_a\sqrt{n} > 0$, we have,
\[
\begin{align*}
&\| (\tau \mu(\alpha_a)e - xs - \alpha_ar_a)^+ \|_1 \\
&\leq \mu(\alpha_a) \frac{1}{\mu} (1 - \alpha_a\sqrt{n}) \| (\tau \mu e - xs)^+ \|_1 \\
&\leq \frac{1}{2}(1 - \alpha_a\sqrt{n})\beta \tau \mu(\alpha_a),
\end{align*}
\]
where the last inequality follows from Lemma 3.6.

Taking two cases into account, we complete the proof.

Due to $\| (\tau \mu(\alpha_a)e - x(\alpha_a)s(\alpha_a))^+ \|_1 \leq \beta \tau \mu(\alpha_a)$ holds if
\[
\| (\tau \mu(\alpha_a)e - xs - \alpha_ar_a)^+ \|_1 + \alpha_a^2 \| (\Delta x_a \Delta s_a)^- \|_1 \leq \beta \tau \mu(\alpha_a).
\]

Using Lemma 3.7, we define
\[
g(\alpha_a) = \begin{cases} 
\alpha_a^2 \| (\Delta x_a \Delta s_a)^- \|_1 - \beta \tau \mu(\alpha_a), & \text{if } \alpha_a \geq \frac{1}{\sqrt{n}} \\
\frac{1}{2}(1 - \alpha_a\sqrt{n})\beta \tau \mu + \alpha_a^2 \| (\Delta x_a \Delta s_a)^- \|_1 - \beta \tau \mu(\alpha_a), & \text{if } \alpha_a < \frac{1}{\sqrt{n}} 
\end{cases}
\]

Therefore, we define $\bar{\alpha}$ as following:
\[
\bar{\alpha} = \max \{ \alpha : g(\alpha_a) \leq 0, \alpha \in [0, \min \{\alpha_1, \alpha_2\}] \}.
\]

Lemma 3.8. Let $(x, y, s) \in \mathcal{N}_1(\tau, \frac{\beta}{2})$, we have, $(xs)_{\min} \geq (1 - \frac{1}{2}\beta)\tau \mu.$
Proof. For \((x, y, s) \in \mathcal{N}_1(\tau, \frac{\beta}{2})\), we have
\[
\frac{1}{2} \beta \tau \mu \geq \| (\tau \mu e - xs)^+ \|_1 = \sum_{i=1}^{n} \max \{ \tau \mu - (xs)_i, 0 \} \geq \tau \mu - (xs)_{\min}.
\]
where the last inequality follow from the define of the \(l_1\) norm.

Lemma 3.9. Let \((x, y, s) \in \mathcal{N}_1(\tau, \frac{\beta}{2}), p = (x^{-1}s)^{\frac{1}{2}}, \beta \leq \frac{1}{2}, \tau \leq \frac{1}{4}\) we have
\[
\| (xs)^{-\frac{1}{2}} r_a \| \leq \frac{5}{4} \mu.
\]
Proof. we have \([ (\tau \mu e - xs)^- ]^T (\tau \mu e - xs)^+ = 0\), Take the norm of the definition of \((xs)^{-\frac{1}{2}} r_a\),
\[
\| (xs)^{-\frac{1}{2}} r_a \|^2 = \| (xs)^{-\frac{1}{2}} (\tau \mu e - xs)^- \|^2 + n \| (xs)^{-\frac{1}{2}} (\tau \mu e - xs)^+ \|^2.
\]
For the first part , as evidenced by reference [20] \( \| (xs)^{-\frac{1}{2}} (\tau \mu e - xs)^- \|^2 \leq n \mu\);
For the second part,we have
\[
\| (xs)^{-\frac{1}{2}} (\tau \mu e - xs)^+ \|^2 \leq \| (\tau \mu e - xs)^+ \|^2 \leq \frac{(1/2) \beta \mu}{(1 - 1/2 \beta) \tau \mu} \leq \frac{\beta \mu}{4}.
\]
where the first inequality follows in Lemma 3.4 and \(\|x\|_2 \leq \|x\|_1, x \in \mathbb{R}^n\), the second inequality in Lemma 3.8, when \(\beta \leq \frac{1}{2}\) the last inequality is follows.

we have, \(\| (xs)^{-\frac{1}{2}} r_a \|^2 \leq n \mu + \frac{1}{2} n \beta \tau \mu = (1 + \frac{1}{4} \beta \tau ) n \mu \leq \frac{5}{4} n \mu\).

Lemma 3.10. [18] Let \((x, y, s) \in \mathcal{N}_1(\tau, \frac{\beta}{2}), p = (x^{-1}s)^{\frac{1}{2}}\) and \((x_0, s_0)\) be defined as in (17) and \((x_*, y_*, s_*) \in F^0\). Then, we have
\[
(\Delta x_a)^T \Delta s_a \geq -\xi t - n \mu,
\]
where \(\xi = v(\|p(x_0 - x_*)\| + \|p^{-1}(s_0 - s_*)\|)\) and \(t = (\|p \Delta x_a\|^2 + \|p^{-1} \Delta s_a\|^2)^{\frac{1}{2}}\).

Lemma 3.11. Suppose Lemma 3.10 holds, \(\beta \leq \frac{1}{2}\) and \(\tau \leq \frac{1}{4}\), we have
\[
\|p \Delta x_a\|^2 + \|p^{-1} \Delta s_a\|^2 \leq \omega^2 n^2 \mu.
\]
Proof. Multiplying the third equation of system by \((xs)^{\frac{1}{2}}\) and taking the norm-squared on both sides of the resulting equation, we obtain
\[
\|p \Delta x_a\|^2 + \|p^{-1} \Delta s_a\|^2 + 2(\Delta x_a)^T \Delta s_a = \| (xs)^{-\frac{1}{2}} r_a \| \leq \frac{5}{4} n \mu.
\]
where the last inequality is follows in Lemma 3.9. Now, using Lemma 3.10, we obtain
\[ t^2 - 2\xi t \leq 2n\mu + \frac{5}{4}n\mu = \frac{13}{4}n\mu. \]
The quadratic \( t^2 - 2\xi t - \frac{13}{4}n\mu = 0 \) has a unique positive root at \( t_+ = \left( \xi + \sqrt{\xi^2 + \frac{13}{4}n\mu} \right) \) and it is positive for \( t > t_+ \), thus we must have \( t \leq t_+ \), which is equivalent to
\[ t^2 \leq \left( \xi + \sqrt{\xi^2 + \frac{13}{4}n\mu} \right)^2. \]

And then we calculate the bound of \( \xi \), similar to reference [18] Lemma 6, we have several important results.
\[
\|p(x_0 - x_\ast)\|_2 = \|s(x_0 - x_\ast) / \sqrt{x_0s}\|_2 \leq \frac{1}{(1 - \frac{1}{2}\beta)\tau\mu}[s^T(x_0 - x_\ast)],
\]
\[
\|p^{-1}(s_0 - s_\ast)\|_2 \leq \frac{1}{(1 - \frac{1}{2}\beta)\tau\mu}[s^T(s_0 - s_\ast)],
\]
\[
v [s^T(x_0 - x_\ast) + x^T(s_0 - s_\ast)] \leq x^Ts + vx_0^Ts_0 + v[x_0^Ts_\ast + s_0^Tx_\ast] \leq 3n\mu.
\]
we have, \( \xi = \frac{1}{\sqrt{(1 - \frac{1}{2}\beta)\tau\mu}}[x^T(s_0 - s_\ast) + s^T(x_0 - x_\ast)] \leq \frac{3n\sqrt{n}}{\sqrt{(1 - \frac{1}{2}\beta)\tau}}. \)

Therefore, \( t^2 \leq \left[ \frac{3n\sqrt{n}}{\sqrt{(1 - \frac{1}{2}\beta)\tau}} + \sqrt{\frac{9n^2\mu}{(1 - \frac{1}{2}\beta)\tau} + \frac{13}{4}n\mu} \right]^2 \leq \omega^2n^2\mu, \)
where \( \omega = \frac{3}{\sqrt{(1 - \frac{1}{2}\beta)\tau}} + \sqrt{\frac{9n^2\mu}{(1 - \frac{1}{2}\beta)\tau} + \frac{13}{4}n} \geq 12 \) we obtain the result.

**Lemma 3.12.** [18] Let \((x, y, s) \in \mathcal{N}(\tau, \beta \frac{2}{3}), p = (x^{-1}s)^\frac{1}{2}, \) then
\[
\|\Delta x_a\Delta s_a\|_1 \leq \frac{1}{2} [\|p\Delta x_a\|^2 + \|p^{-1}\Delta s_a\|^2] \leq \frac{1}{2}\omega^2n^2\mu.
\]

The following three lemmas provided lower bounds on \( \alpha_1, \alpha_2 \) and \( \alpha_3 \).

**Lemma 3.13.** Let \((x, y, s) \in \mathcal{N}(\tau, \beta \frac{2}{3}), \beta \leq \frac{1}{2}, \tau \leq \frac{1}{4}, \) and \( \alpha_1 \) be defined in (10), then \( \alpha_1 \geq \frac{11}{16\omega^2n} \).

**Proof.** Taking the derivative of the function \( \mu(\alpha_a) \) with respect to \( \alpha_a \) in (8), we have, \( \mu(\alpha_a)^\prime = -(1 - \tau)\mu + \frac{\sqrt{n-1}}{n}e^T(\tau\mu e - xs)^+ + \frac{2}{n}\alpha_a(\Delta x_a)^T\Delta s_a. \)

Moreover, equation \( \mu(\alpha_a)^\prime = 0 \) has a unique root \( \alpha_0 \in (0, 1) \). By computing \( \alpha_0 \) directly, we have
\[
\alpha_0 = \frac{(1 - \tau)n\mu - (\sqrt{n} - 1)(\tau\mu e - xs)^+1_1}{2(\Delta x_a)^T\Delta s_a} \\
\geq \frac{(1 - \tau)n\mu - \frac{1}{2}(\sqrt{n} - 1)\beta\tau}{\omega^2n^2\mu} \geq \frac{1 - \tau - \frac{1}{2}\beta\tau}{\omega^2n} \geq \frac{11}{16\omega^2n}.
\]
where the first inequality from Lemma 3.6 and Lemma 3.12, the last inequality is due to $\tau \leq \frac{1}{3}, \beta \leq \frac{1}{2}$.

From the definition of $\alpha_1$, we have $\alpha_1 \geq \frac{11}{16}\omega n$.

**Lemma 3.14.** Let $(x, y, s) \in \mathcal{N}_1(\tau, \frac{\beta}{2})$, $\beta \leq \frac{1}{2}, \tau \leq \frac{1}{4}$, and $\alpha_2$ be defined in (11), then $\alpha_2 \geq \frac{2\tau}{\omega n}$.

**Proof.** Using (8), we have

\[
\langle x(\alpha_a), s(\alpha_a) \rangle - (1 - \alpha_a)\langle x, s \rangle = [1 - (1 - \tau)\alpha_a]\mu n + \alpha_a(\sqrt{n} - 1)e^T(\tau \mu e - xs)^+ + \alpha_a^2(\Delta x^a)^T\Delta s_a - (1 - \alpha_a)\mu n \\
\geq \alpha_a\tau \mu n + \alpha_a^2(\Delta x_a)^T\Delta s_a \geq \alpha_a\tau \mu n - \alpha_a^2\|\Delta x_a^a\|_1 \geq \alpha_a\tau \mu n - \frac{1}{2}\alpha_a^2\omega^2 n^2 \mu,
\]

where the first inequality is due to $\alpha_a(\sqrt{n} - 1)e^T(\tau \mu e - xs)^+ \geq 0$, the third inequality follows from Lemma 3.12.

It is clear that if $\alpha_a \leq \frac{2\tau}{\omega n}$, one has $\alpha_a(\tau \mu n - \frac{1}{2}\alpha_a^2\omega^2 n^2 \mu) \geq 0$, which implies $(x(\alpha_a), s(\alpha_a)) \geq (1 - \alpha_a)\langle x, s \rangle$.

From the definition of $\alpha_2$ in (11), we have, $\alpha_2 \geq \frac{2\tau}{\omega n}$.

**Lemma 3.15.** Let $(x, y, s) \in \mathcal{N}_1(\tau, \frac{\beta}{2})$, $\beta \leq \frac{1}{2}, \tau \leq \frac{1}{4}$, and $\alpha_3$ be defined in (12), then $\alpha_3 \geq \frac{2\sqrt{3}\tau}{3\omega n}$.

**Proof.** From (18), we have $\alpha_a \geq \frac{1}{\sqrt{n}}$ or $\alpha_a < \frac{1}{\sqrt{n}}$. If $\alpha_a \geq \frac{1}{\sqrt{n}}$, we immediately obtain the lower bound on $\alpha_a$. Thus, we only mainly consider $\alpha_a < \frac{1}{\sqrt{n}}$.

For $\alpha_a < \frac{1}{\sqrt{n}}$, $g(\alpha_a) \leq 0$ is equivalent to

\[
L(\alpha_a) := \frac{1}{2}(1 - \alpha_a\sqrt{n})\beta \tau \mu + \alpha_a^2\|\Delta x^a\Delta s^a\|_1 \leq \beta \tau \mu(\alpha_a).
\]

Defined by $\mu(\alpha_a)$ in (8)

\[
\mu(\alpha_a) = [1 - (1 - \tau)\alpha_a]\mu + \alpha_a\sqrt{n} - \frac{1}{n}e^T(\tau \mu e - xs)^+ + \alpha_a^2(\Delta x^a)^T\Delta s_a.
\]

we have, $\mu(\alpha_a) \geq [1 - (1 - \tau)\alpha_a]\mu + \alpha_a^2(\Delta x^a)^T\Delta s_a$.

Using Lemma 3.12, we have

\[
L(\alpha_a) \leq \frac{1}{2}(1 - \alpha_a\sqrt{n})\beta \tau \mu + \frac{1}{2}\omega^2 n^2 \mu \alpha_a^2 := M, \\
\beta \tau \mu(\alpha_a) \geq \beta \tau [1 - (1 - \tau)\alpha_a]\mu - \beta \tau \alpha_a^2(\Delta x^a)^T\Delta s_a \\
\geq \beta \tau \mu - (1 - \tau)\alpha_a \beta \tau \mu - \frac{1}{2}\omega^2 n^2 \beta \tau \mu \alpha_a^2 := N.
\]
Let $H(\alpha) := M - N$, we have

$$
H(\alpha) = \frac{1}{2}(1 - \alpha \sqrt{n}) \beta \tau \mu + \frac{1}{2} \omega^2 n^2 \mu \alpha^2 - \beta \tau \mu + (1 - \tau) \alpha \beta \tau \mu + \frac{1}{2} \omega^2 n^2 \mu \beta \tau \alpha^2
$$

$$
= \frac{1}{2} (1 + \beta \tau) \omega^2 n^2 \mu \alpha^2 + \left(1 - \tau\right) \beta \tau \mu - \frac{1}{2} \sqrt{n} \beta \tau \mu \right) \alpha - \frac{1}{2} \beta \tau \mu.
$$

The quadratic $\frac{1}{2} (1 + \beta \tau) \omega^2 n^2 \mu \alpha^2 + \left(1 - \tau\right) \beta \tau \mu - \frac{1}{2} \sqrt{n} \beta \tau \mu \right) \alpha - \frac{1}{2} \beta \tau \mu = 0$

has a unique positive root at

$$
\alpha_a = \left(\frac{\tau - 1}{n} + \frac{1}{2\sqrt{n}}\right) \beta \tau + \sqrt{\left(\frac{1 - \tau}{n} - \frac{1}{2\sqrt{n}}\right)^2 \beta^2 \tau^2 + \beta \tau \omega^2 (1 + \beta \tau) \omega^2 (1 + \beta \tau)}
$$

$H(\alpha_a) \leq 0$ holds if $\alpha_a \leq \frac{2\sqrt{\beta \tau}}{3\omega \sqrt{n}}$, which implies $H(\alpha_a) \leq 0, \forall \alpha_a \in \left[0, \frac{2\sqrt{\beta \tau}}{3\omega \sqrt{n}}\right]$.

From the definition of $\alpha_3$ in (12), we have $\alpha_3 \geq \frac{2\sqrt{\beta \tau}}{3\omega \sqrt{n}}$.

### 3.3 Corrector step

The convergence of Algorithm 1 will be discussed theoretically in this subsection.

**Lemma 3.16.** Let $(\bar{x}, \bar{y}, \bar{s}) \in \mathcal{N}_1(\tau, \beta) \setminus \mathcal{N}_1(\tau, \frac{\beta}{2}), \rho = ||(\tau \bar{\mu} e - \bar{x} s)^-||_1 ||(\tau \bar{\mu} e - \bar{x} s)^+||_1$, we have $\left(1 - \frac{\tau n}{\beta \tau}\right) \leq \rho \leq 2\sqrt{n} + \frac{2(1 - \tau n)}{\beta \tau}$.

**Proof.** Applying the definition of the neighborhood and $(\bar{x}, \bar{y}, \bar{s}) \in \mathcal{N}_1(\tau, \beta) \setminus \mathcal{N}_1(\tau, \frac{\beta}{2})$, we have $\frac{1}{2} \beta \tau \bar{\mu} \leq ||(\tau \bar{\mu} e - \bar{x} s)^+||_1 \leq \beta \tau \bar{\mu}$.

From Lemma 3.5, we have, $(1 - \tau) \bar{\mu} n \leq ||(\tau \bar{\mu} e - \bar{x} s)^-||_1 \leq (1 - \tau) \bar{\mu} n + \sqrt{n} \beta \tau \bar{\mu}$.

Combine the above two equations, we have

$$
\rho = \frac{||(\tau \bar{\mu} e - \bar{x} s)^-||_1}{||(\tau \bar{\mu} e - \bar{x} s)^+||_1} \geq \frac{(1 - \tau) \bar{\mu} n}{\beta \tau \bar{\mu}} = \frac{(1 - \tau) n}{\beta \tau}.
$$

$$
\rho = \frac{||(\tau \bar{\mu} e - \bar{x} s)^-||_1}{||(\tau \bar{\mu} e - \bar{x} s)^+||_1} \leq \frac{(1 - \tau) \bar{\mu} n + \beta \tau \bar{\mu} \sqrt{n}}{\frac{1}{2} \beta \tau \bar{\mu}} \leq 2\sqrt{n} + \frac{2(1 - \tau) n}{\beta \tau}.
$$

**Lemma 3.17.** If $\alpha_c \geq 1/\rho$, we have $||\tau \mu (\alpha_c) e - \bar{x} s - \alpha_c r_c||_1 = 0$.

**Proof.** Using $\mu(\alpha_c) = \bar{\mu}$, we have,

$$
||\tau \mu (\alpha_c) e - \bar{x} s - \alpha_c r_c||_1
\leq ||(1 - \alpha_c) \left(\tau \bar{\mu} e - \bar{x} s\right)^-||_1 + ||(1 - \alpha_c \rho) \left(\tau \bar{\mu} e - \bar{x} s\right)^+||_1
= ||(1 - \alpha_c \rho) \left(\tau \bar{\mu} e - \bar{x} s\right)^+||_1 = 0.
$$
where the first inequality follows from Lemma 3.1, the second equation is due to \( [(\tau \bar{\mu} e - \bar{x}s)]^+ = 0 \), the last equation is due to \( \alpha_e \geq 1/\rho \).

**Lemma 3.18.** Let \((x, \bar{y}, \bar{s}) \in \mathcal{N}_1(\tau, \beta) \setminus \mathcal{N}_1(\tau, \frac{\beta}{2}), \bar{p} = (\bar{x}^{-1} \bar{s})^{\frac{1}{2}}, \bar{r} = (\bar{x}s)^{-\frac{1}{2}} r_c\), we have, \( \|\Delta x_c \Delta s_c\|_1 \leq \frac{1}{4} \|\bar{r}\| \).

**Proof.** Multiplying the third equation of (14) by \( (\bar{x}s)^{-\frac{1}{2}} \), we obtain

\[
\bar{x}^{-\frac{1}{2}} \bar{s}^{\frac{1}{2}} \Delta x_c + \bar{s}^{-\frac{1}{2}} \bar{x}^{\frac{1}{2}} \Delta s_c = (\bar{x}s)^{-\frac{1}{2}} r_c, \quad \bar{p} \Delta x_c + \bar{p}^{-1} \Delta s_c = \bar{r}.
\]

we have \( (\bar{x}^{-\frac{1}{2}} \bar{s}^{\frac{1}{2}} \Delta x_c)^T (\bar{s}^{-\frac{1}{2}} \bar{x}^{\frac{1}{2}} \Delta s_c) = 0 \).

Combine Lemma 3.3, we have,

\[
\|\Delta x_c \Delta s_c\|_1 = \|\bar{p} \Delta x_c, \bar{p}^{-1} \Delta s_c\| \leq \frac{1}{4} \|\bar{r}\|.
\]

**Lemma 3.19.** Let \((x, y, s) \in \mathcal{N}_1(\tau, \beta) \setminus \mathcal{N}_1(\tau, \frac{\beta}{2})\), then

\[
\|\bar{x}s\| \leq 2 \beta \tau \bar{\mu}.
\]

**Proof.** Analogical Lemma 3.8, we have, \( \beta \tau \bar{\mu} \geq \|\tau \bar{\mu} e - \bar{x}s\|_1 \geq \tau \bar{\mu} - (\bar{x}s)_{min} \). Then \( (\bar{x}s)_{min} \geq (1 - \beta) \tau \bar{\mu} \).

As evidenced of Lemma 2.1 in reference [21]

\[
\|\bar{x}s\|_2^2 = \|\bar{x}s\|_2^2 (\tau \bar{\mu} e - \bar{x}s) - \|\bar{x}s\|_2^2 \rho (\tau \bar{\mu} e - \bar{x}s) + \|\bar{x}s\|_2^2 \rho (\tau \bar{\mu} e - \bar{x}s) + \|\bar{x}s\|_2^2 \rho (\tau \bar{\mu} e - \bar{x}s) \leq \|\tau \bar{\mu} e - \bar{x}s\|_2^2 (1 - \beta) \tau \bar{\mu} + \|\bar{x}s\|_2^2 (1 - \beta) \tau \bar{\mu} \leq 2 \beta \tau \bar{\mu}.
\]

where the first inequality follows from Lemma 3.4, the second inequality follows \( (\bar{x}s)_{min} \geq (1 - \beta) \tau \bar{\mu} \), the last inequality is due to \((\bar{x}, \bar{y}, \bar{s}) \in \mathcal{N}_1(\beta, \tau) \setminus \mathcal{N}_1(\tau, \frac{\beta}{2})\), the last inequality is due to \( \frac{\beta}{1 - \beta} \leq 1 \).

From Lemma 3.16, and \( \beta \leq \frac{1}{2}, \tau \leq \frac{1}{4} \), we have,

\[
\frac{(1 - \tau)n}{\beta \tau} \leq \rho \leq 2 \sqrt{n} + \frac{2 (1 - \tau)n}{\beta \tau}, \quad \frac{\beta \tau}{2 \sqrt{n} \beta \mu + 2 (1 - \tau)n} \leq \frac{1}{\rho} \leq \frac{\beta \tau}{(1 - \tau)n}.
\]

Then,

\[
1 - 1/\rho \leq 1 - \frac{\beta \tau}{2 \sqrt{n} \beta \mu + 2 (1 - \tau)n} = \frac{(2 \sqrt{n} - 1) \beta \tau + 2n (1 - \tau)}{2 \sqrt{n} \beta \mu + 2 (1 - \tau)n} > 0.
\]

That means that \( \alpha_e \in [1/\rho, 1] \) is not empty.
Lemma 3.20. Let $\beta \leq \frac{1}{2}, \tau \leq \frac{1}{4}, \alpha_c \in [1/\rho, 1]$, we have $(x(\alpha_c), y(\alpha_c), s(\alpha_c)) \in \mathcal{N}_1(\tau, \frac{\beta}{2})$.

Proof. By the lemma above, we have

$$
\| [\tau \mu(\alpha_c) e - x(\alpha_c) s(\alpha_c)]^+ \|_1 = \| (\tau \bar{\mu} e - \bar{x}^T s - \alpha_c r_c - \alpha^2 (\Delta x_c \Delta s_c) )^+ \|_1
\leq \| (\tau \bar{\mu} e - \bar{x}^T s - \alpha_c r_c )^+ \|_1 + \alpha^2 (\Delta x_c \Delta s_c) ^{-} \|_1 = \alpha^2 \| (\Delta x_c \Delta s_c)^{-} \|_1
\leq \frac{1}{4} \alpha^2 \| (\bar{x}^{-})^{-\frac{1}{2}} r_c \|_2^2 \leq \frac{1}{4} \alpha^2 \cdot 2\beta \bar{\mu} \leq \frac{1}{2} \beta \bar{\mu}.
$$

where the first inequality follows from Lemma 3.1, the second equation follows from Lemma 3.17, the second inequality follows from Lemma 3.18, the third inequality follows from Lemma 3.19.

4 Polynomial Complexity

Form (16), we obtain that $\mu(\alpha_c)$ of corrector step is the same as predictor step. Therefore, the following theorem gives an upper bound for the number of iterations in which Algorithm 1 stop with an $\varepsilon$-approximate solution.

Theorem 4.1. The Algorithm 1 will terminate in $\varepsilon$-approximate solution $(x^k, y^k, s^k)$ such that $\| Ax^k - b \| \leq \varepsilon \| Ax^0 - b \|$ and $\| A^T y^k + s^k - c \| \leq \varepsilon \| A^T y^0 + s^0 - c \|$ and $\langle x^k, s^k \rangle \leq \varepsilon \langle x^0, s^0 \rangle$ in at most $O(n \log \varepsilon^{-1})$ iterations.

proof Let $\bar{\alpha} = min\{\alpha_1, \alpha_2, \alpha_3\} = \frac{2\tau}{\omega n}$. By the definition of $\bar{\alpha}$ in (19), we have,

$$
\mu(\bar{\alpha}) \leq \mu(\bar{\alpha}) = [1 - (1 - \tau) \bar{\alpha}] \mu + \bar{\alpha} \sqrt{\frac{n}{n}} - \frac{1}{n} e^T (\tau \mu e - xs)^+ + \frac{\bar{\alpha}^2}{n} (\Delta x^o)^T \Delta s^o
\leq [1 - (1 - \tau) \bar{\alpha}] \mu + \frac{1}{2} \bar{\alpha} \beta \tau \mu + \frac{1}{2n} \bar{\alpha}^2 \omega^2 n^2 \mu
= \left[1 - (1 + \frac{1}{2} \beta \tau + \frac{1}{2n} \bar{\alpha} \omega^2 n^2) \right] \mu
= \left[1 - (1 + \frac{1}{2} \beta \tau) \right] \mu = (1 - \zeta \bar{\alpha}) \mu,
$$

where $\zeta = 1 + \frac{1}{2} \beta \tau$, Because we need to have $\mu(\bar{\alpha}) \leq \varepsilon \mu_0$, it suffices to have

$$
\left(1 - \frac{2\tau \zeta}{\omega^2 n} \right)^k \mu_0 \leq \varepsilon \mu_0.
$$
Substitution gives $k \geq \frac{\omega^2 n}{27\zeta} log \varepsilon^{-1}$. Thus, Algorithm 1 terminates after at most $rac{\omega^2 n}{27\zeta} log \varepsilon^{-1}$ steps. Using Remark 1, we have $v^k \leq \varepsilon$, which implies

$v^k = \frac{\|Ax^k - b\|}{\|Ax^0 - b\|} = \frac{\|A^T y^k + s^k - c\|}{\|A^T y^0 + s^0 - c\|} \leq \varepsilon$

This completes the proof.

5 Computational performance

In this section, we report some computational performance results for the test problems given in Table 1 that are taken from the standard NETLIB test repository [22]. We compare the proposed Algorithm 1 in this paper with the given algorithm in [23], which it was recorded as a symbol Algorithm 2. Moreover, we select the optimization parameter $\tau = 0.00001$, $\beta = 0.00005$ for the Algorithm 1 and $\beta = 0.5$ for the Algorithm 2.

Table 1: Linear programming

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<th>Dimension</th>
<th>Algorithm 1</th>
<th>Algorithm 2</th>
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6 Conclusion

In this paper, we proposes a Mizuno-Todd-Ye predictor-corrector infeasible-interior-point method for linear programming. The algorithm is based on using the $l_1$-norm wide neighborhood of the central path and used infeasible algorithm. With an elegant analysis, we provide the convergence of the proved that the algorithm has the iteration bound $O(n log \varepsilon^{-1})$, which is the currently best known iteration bounds.
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