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Budget Deficit in a Growing Monetary Economy

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Abstract

Using a traditional neoclassical two-period overlapping generations model that takes into account consumers' money holdings, we examine the existence of budget deficit in an economy which grows at the constant positive rate. The following results will be shown. 1) Budget deficit is necessary to achieve full employment under constant prices of goods. 2) If the actual budget deficit is larger than the value which is necessary and sufficient for full employment under constant prices, an inflation is triggered. 3) If the actual budget deficit is smaller than the value which is necessary and sufficient for full employment under constant prices, a recession occurs. Therefore, full employment at constant prices cannot be achieved with a balanced budget.

Mathematics Subject Classification: 91B64

Keywords: budget deficit, economic growth, overlapping generations model

1 Introduction

Using a traditional neoclassical two-period overlapping generations model by [1] that takes into account consumers' money holdings, we examine the existence of fiscal deficits in an economy growing at a constant positive rate. The significance of fiscal deficits and government debt in the economy and the intergenerational burden have been analyzed in various ways by J. Tanaka. References include [7], [8], [9], [10][1]. J. Tanaka focuses on the intergenerational

[1]In a previous study about the existence of involuntary unemployment, [11], we used a similar but different overlapping generations model according to [4], [5], [6].

economic welfare gap due to the presence or absence of government debt, but his main model does not include economic growth, and he assumes that all government debt is redeemed by taxes. The interest of this paper lies elsewhere. We are interested in proving that budget deficits are necessary and inevitable in a growing economy where consumers hold money. In our model, people save primarily through capital, but in contrast to previous traditional models, we consider that consumers are willing to hold money other than capital for reasons such as liquidity.

In the next section, we will present our model and prove that a budget deficit is necessary to achieve full employment under constant prices of goods. Section 3 briefly discusses recessions and inflation. If the actual budget deficit is larger than is necessary and sufficient for full employment at constant prices, inflation occurs; if the actual budget deficit is smaller than is necessary and sufficient for full employment at constant prices, recession occurs. Thus, full employment at constant prices cannot be achieved with a balanced budget[2].

In our model, money is supplied by wage payments, which consumers use to pay taxes and to pay for consumption and investment in capital in their youth. What remains after that is money holding.

In the appendix we will very briefly analyze the monetary policy by issuance of government bonds. It raises the rate of interest.

This paper is an example of an analysis using a very simple model of the following statement by J. M. Keynes.

"Unemployment develops, that is to say, because people want the moon; — men cannot be employed when the object of desire (i.e. money) is something which cannot be produced and the demand for which cannot be readily choked off. There is no remedy but to persuade the public that green cheese is practically the same thing and to have a green cheese factory (i.e. a central bank) under public control." ([2], Chap. 17)

2 Money demand and budget deficit

We introduce money demand or money holding of consumers into the traditional overlapping generations model over two periods according to [1]. We also refer to [7] about formulation of the overlapping generations model.

Consumers live over two periods, younger period and older period. Population of consumers increases at the rate of n from a period to the next period.

The utility of a younger consumer in Period 1 is represented by

$$u(c_1^y, c_2^o, \frac{m_1}{p_1}) = (c_1^y)^\beta (c_2^o)^\gamma \left(\frac{m_1}{p_1}\right)^{1-\beta-\gamma}, \quad 0 < \beta < 1, \; 0 < \gamma < 1, \; 0 < \beta + \gamma < 1.$$

[2]In another paper, [12], we will analyze a similar problem in an endogenous growth model according to [3].

c_1^y is his consumption in Period 1, c_2^o is his consumption in Period 2. In Period 2 he belongs to the older generation. m_1 is his money holding or demand for money in Period 1. Consumers derive utility from holding money as well as from consumption in youth and old age. The budget constraint is

$$p_2c_2^o = (1+r_1)[(w-t)l_1 - p_1c_1^y - m_1] + m_1. \tag{1}$$

w is the wage, t is the tax, and r_1 is the interest rate or the rate of return on capital in Period 1. The tax is payed by employed consumers. p_1 and p_2 are the price levels in Periods 1 and 2. l_1 is an indicator of whether the consumer is employed or not in Period 1, and takes the value of 1 if the consumer is employed and 0 if he is not employed. w also equals the supply of money for an employed consumer. Let denote it by $\overline{m}$. The (nominal) investment in capital is

$$(w-t)l_1 - p_1c_1^y - m_1 = (\overline{m}-t)l_1 - p_1c_1^y - m_1. \tag{2}$$

It generates income at the rate of r. Then, the consumption in Period 2 of an employed consumer equals (1). (1) is rewritten as

$$p_1c_1^y + \frac{1}{1+r_1}p_2c_2^o + \frac{r_1}{1+r_1}m_1 = (w-t)l_1.$$

The Lagrange function is

$$\mathcal{L}=(c_1^y)^\beta(c_2^o)^\gamma\left(\frac{m_1}{p_1}\right)^{1-\beta-\gamma} - \lambda\left[p_1c_1^y + \frac{1}{1+r_1}p_2c_2^o + \frac{r_1}{1+r_1}m_1 - (w-t)l_1\right].$$

The conditions for utility maximization are

$$\beta(c_1^y)^{\beta-1}(c_2^o)^\gamma\left(\frac{m_1}{p_1}\right)^{1-\beta-\gamma}=\lambda p_1,\ \gamma(c_1^y)^\beta(c_2^o)^{\gamma-1}\left(\frac{m_1}{p_1}\right)^{1-\beta-\gamma}=\frac{1}{1+r}\lambda p_2,$$

$$(1-\beta-\gamma)(c_1^y)^\beta(c_2^o)^\gamma\left(\frac{m_1}{p_1}\right)^{-\beta-\gamma}=\frac{r_1}{1+r_1}\lambda p_1.$$

From them, we obtain

$$\beta(c_1^y)^\beta(c_2^o)^\gamma\left(\frac{m_1}{p_1}\right)^{1-\beta-\gamma}=\lambda p_1c_1^y,\ \gamma(c_1^y)^\beta(c_2^o)^\gamma\left(\frac{m_1}{p_1}\right)^{1-\beta-\gamma}=\frac{1}{1+r}\lambda p_2c_2^o,$$

$$(1-\beta-\gamma)(c_1^y)^\beta(c_2^o)^\gamma\left(\frac{m_1}{p_1}\right)^{1-\beta-\gamma}=\frac{r_1}{1+r_1}\lambda m_1.$$

Then, for employed consumers

$$c_1^y=\frac{\beta}{p_1}(w-t),\ c_2^o= (1+r_1)\frac{\gamma}{p_2}(w-t),\ m_1=\left(\frac{1+r_1}{r_1}\right)(1-\beta-\gamma)(w-t). \tag{3}$$

m_1 is decreasing in r_1. For unemployed consumers $c_1^y = c_2^o = m_1 = 0$. However, unemployment insurance could also be incorporated into the model without changing the results. For simplicity, we exclude them. Let L_1 be the employment in Period 1, and denote the population or the employment in the full employment state by L_1^f. Also L_0 and L_0^f denote the employment and the population in Period 0 which is the previous period of Period 1. Since the population grows at the rate of n, we have

$$L_1^f = (1+n)L_0^f.$$

Let K_1 and K_2 be the capital in Period 1 and that in Period 2. The real value of capital in Period 2 is derived from (2) as follows.

$$\begin{aligned} K_2 &= \left(\frac{w-t-m_1}{p_1} - c_1^y\right)L_1 = (1-\beta)\frac{w-t}{p_1}L_1 - \frac{m_1}{p_1}L_1 \qquad (4) \\ &= \left[1-\beta-\frac{1+r_1}{r_1}(1-\beta-\gamma)\right]\frac{w-t}{p_1}L_1. \end{aligned}$$

This means

$$p_1K_2 = \left[1-\beta-\frac{1+r_1}{r_1}(1-\beta-\gamma)\right](w-t)L_1.$$

From this we verify the following relation.

$$\begin{aligned} &p_2c_2^oL_1 = (1+r_1)p_1K_2 + m_1L_1 \\ &= (1+r_1)\left[1-\beta-\frac{1+r_1}{r_1}(1-\beta-\gamma)\right](w-t)L_1 + \frac{1+r_1}{r_1}(1-\beta-\gamma)(w-t)L_1 \\ &= (1+r_1)[1-\beta-(1-\beta-\gamma)](w-t)L_1 = (1+r_1)\gamma(w-t)L_1. \end{aligned}$$

From (4)

$$\frac{m_1}{p_1}L_1 = \frac{w-t}{p_1}L_1 - c_1^yL_1 - K_2.$$

This equation implies that money holding is equal to wages paid minus taxes, investment in capital, and consumption, as described in the introduction.

The capital in Period 1 is

$$K_1 = \frac{1}{p_0}(w-t-p_0c_0^y - m_0)L_0.$$

p_0 is the price level in Period 0. From this

$$p_1c_1^oL_0 = (1+r_0)p_0K_1 + m_0L_0 = (1+r_0)\gamma(w-t)L_0. \qquad (5)$$

r_0 is the rate of return on capital in Period 0. The production function of firms is

$$Y_1 = K_1^{\alpha} L_1^{1-\alpha}.$$

Y_1 is the real GDP in Period 1. The profit of firms is

$$\pi_1 = p_1 Y_1 - p_1 r_1 K_1 - w L_1 = p_1 K_1^{\alpha} L_1^{1-\alpha} - p_0 r_1 K_1 - w L_1.$$

$p_0 K_1$ is the nominal amount of the capital at the time the investment was made, that is, Period 0. The conditions for profit maximization are

$$p_1 \alpha K_1^{\alpha-1} L_1^{1-\alpha} = p_0 r_1, \ p_1(1-\alpha) K_1^{\alpha} L_1^{-\alpha} = w. \tag{6}$$

From them we get

$$p_0 r_1 K_1 = p_1 \alpha K_1^{\alpha} L_1^{1-\alpha} = p_1 \alpha Y_1, \ w L_1 = p_1(1-\alpha) K_1^{\alpha} L_1^{1-\alpha} = p_1(1-\alpha) Y_1.$$

Then,

$$p_1 Y_1 = p_0 r_1 K_1 + w L_1. \tag{7}$$

Let G_1 be the fiscal spending in Period 1. The condition for equilibrium of the good market is

$$p_1 c_1^y L_1 + p_1 c_1^o L_0 + G_1 + p_1(K_2 - K_1) = p_1 Y_1. \tag{8}$$

$p_1(K_2 - K_1)$ represents the cost required so as to increase the real value of the capital from K_1 to K_2. The left-hand side is the total demand, and the right-hand side is the total supply. From (3),

$$p_1 c_1^y L_1 = \beta(w - t) L_1. \tag{9}$$

Substituting (5), (7) and (9) into (8),

$$\beta(w-t)L_1 + (1+r_0)p_0 K_1 + m_0 L_0 + G_1 + p_1(K_2 - K_1) = p_0 r_1 K_1 + w L_1.$$

This is rewritten as

$$p_0 K_1 + m_0 L_0 + G_1 - t L_1 + p_1(K_2 - K_1) = (1-\beta)(w-t) L_1 + p_0(r_1 - r_0) K_1.$$

From (4),

$$(1-\beta)(w-t)L_1 = p_1 K_2 + m_1 L_1.$$

Therefore,

$$p_0 K_1 + m_0 L_0 + G_1 - t L_1 + p_1(K_2 - K_1) = p_1 K_2 + m_1 L_1 + p_0(r_1 - r_0) K_1,$$

and so

$$G_1 - t L_1 = m_1 L_1 - m_0 L_0 + (p_1 - p_0) K_1 + p_0(r_1 - r_0) K_1. \tag{10}$$

In the steady state with full employment we have $r_1 = r_0$, $L_1 = (1+n)L_0$, and $m_1 = m_0$. Further, if the prices are constant, that is, $p_1 = p_0$, we obtain

$$\begin{aligned} G_1 - tL_1 &= \frac{n}{1+n} m_1 L_1 = \frac{n}{1+n} \frac{1+r_1}{r_1} (1-\beta-\gamma)(w-t) L_1 \\ &= \frac{n}{1+n} \frac{1+r_1}{\beta r_1} (1-\beta-\gamma) p_1 c_1^y L_1. \end{aligned} \tag{11}$$

This is positive so long as $1-\beta-\gamma > 0$ and $n > 0$. We get the following result.

Proposition 1 *If the economy grows at the positive rate, and the consumers derive positive utility from holding money, we need positive budget deficit to maintain full employment under constant prices.*

3 Inflation and Recession

In this section, the variables in each equation are considered to represent their respective nominal values. (11) in the previous section means that we need the budget deficit described in (11), or we need the fiscal spending,

$$G_1 = \frac{n}{1+n} \frac{1+r_1}{\beta r_1} (1-\beta-\gamma) p_1 c_1^y L_1^f + tL_1^f \tag{12}$$

to maintain full employment under constant prices. (10) and (11) mean that if the actual fiscal spending is larger than (12) or the actual budget deficit is larger than (10) with $m_1 = m_0$, $L_1 = L_1^f = (1+n)L_0$ and $r_1 = r_0$, p_1 should increase. Then, inflation is triggered.

From (6) r_1 is increasing in L_1, and we assume that $m_1 L_1$ is increasing in L_1[3]. Then, if the actual fiscal spending is smaller than (12) or the actual budget deficit is smaller than (10) with $p_1 = p_0$, L_1 should be smaller than L_1^f, and recession occurs. Summarizing the results,

Proposition 2 *If the actual budget deficit is larger than the value which is necessary and sufficient for full employment under constant prices, an inflation is triggered.*

Proposition 3 *If the actual budget deficit is smaller than the value which is necessary and sufficient for full employment under constant prices, a recession occurs.*

This paper does not pursue the causes of involuntary unemployment. However, it is believed that deflation has not occurred to the extent that the real balance effect could realistically eliminate the recession.

[3] From (6) r_1 is increasing in L_1. On the other hand, m_1 is decreasing in r_1. However, we assume that $m_1 L_1$ is increasing in L_1.

4 Conclusion

In this paper we have mainly proved that the budget deficit is necessary and inevitable to maintain full employment under constant prices in a growing economy by incorporating consumers' desire to hold money into the overlapping generations model. Although we considered only growth due to population growth, we expect to obtain similar results for growth due to technological progress.

Appendix: Monetary policy by government bonds

We have analyzed only fiscal policy by fiscal spending and taxes. Suppose that the government issues government bonds at the same rate of return as capital. Let b_1 be the government bonds held by each consumer in Period 1. Then, from (2) and (4) the investment in capital is

$$\left(\frac{w-t-m_1-b_1}{p_1}-c_1^y\right)L_1$$

It is smaller than (4) given p_1, w, t and r. By (6) a decrease in the capital increases r given p_1 and L_1. Therefore, issuance of government bonds raises the rate of interest. In this case instead of (4) and (5) we have

$$K_2 = (1-\beta)\frac{w-t}{p_1}L_1 - \frac{b_1}{p_1}L_1 - \frac{m_1}{p_1}L_1,$$

$$p_1 c_1^o L_0 = (1+r_0)p_0 K_1 + (1+r_0)b_0 L_0 + m_0 L_0 = (1+r_0)\gamma(w-t)L_0.$$

b_0 is the government bonds held by each consumer in Period 0. Then, with $b_1 = b_0$, instead of (10), we get

$$G_1 - tL_1 + rb_0L_0 = m_1L_1 - m_0L_0 + b_1(L_1-L_0) + (p_1-p_0)(1+r_1)K_1 + p_0(r_1-r_0)K_1.$$

The left-hand side is the budget deficit including interest payments on government bonds. This is positive when $p_1 = p_0$.

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